THE THREE-PHASE INDUCTION MOTOR WITH UNBALANCED STATOR CONNECTIONS WITH PARTICULAR REFERENCE TO THE KUSA CONNECTION AND SINGLE-PHASE OPERATION

BY

Prof. M. A. EL-KHOLY

AND

MOH. NUR EL-DIN ABDEL HAMID, M.Sc.

1.—Introduction

The characteristics of the three-phase induction motor (3-ph. I.M.) when operated under balanced conditions have received great care and investigation. Nevertheless, its operation with unbalanced stator connections has received comparatively smaller attention. The general case to be discussed here (Fig. 1)

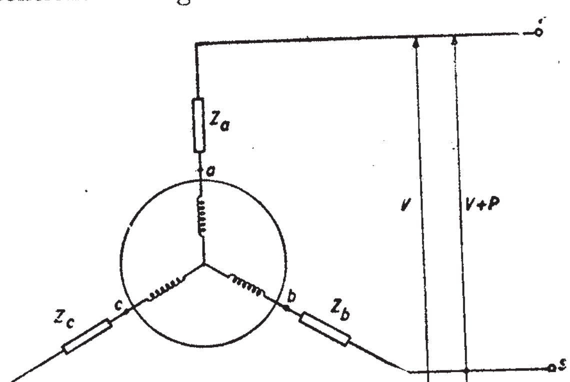


Fig. 1.—The general case of unbalanced stator connections

is the operation of 3-ph. I.M. having symmetrical primary and secondary windings, but the primary external impedances and the supply voltages are unbalanced. This problem will be discussed by the application of the method of symmetrical components developed by Fortescue. General treatment will be explained to obtain all motor operating characteristics of importance. Particular cases of asymmetrical stator connections will be also discussed.

2.—Principles of Symmetrical Components

The theory of symmetrical components method is a particular case of the superposition theorem. A single asymmetrical system of 3-ph. vector quantities I_a, I_b and I_c may be replaced by three symmetrical systems of positive negative and zero sequence. These three systems are represented, respectively, by: (*i*) I_{a1}, I_{b1}, and I_{c1}, where $I_{b1} = a^2 I_{a1}$, $I_{c1} = a\, I_{a1}$, and a is unit vector quantity $e^{j2\pi/3}$; (*ii*) I_{a2}, I_{b2} and I_{c2}, where $I_{b2} = a\, I_{a2}$ and $I_{c2} = a^2\, I_{a2}$; and (*iii*) I_{ao}, I_{bo} and I_{co} where $I_{ao} = I_{bo} = I_{co}$; The asymmetrical system may be expressed mathematically by:

$$\left.\begin{aligned} I_a &= I_{ao} + I_{a1} + I_{a2} \\ I_b &= I_{ao} + a^2 I_{a1} + a\, I_{a2} \\ I_c &= I_{ao} + a\, I_{a1} + a^2 I_{a2} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

It can be proved that;

$$\left.\begin{aligned} I_{ao} &= {}^1/_3\, (I_a + I_b + I_c) \\ I_{a1} &= {}^1/_3\, (I_a + a\, I_b + a^2 I_c) \\ \text{and} \quad I_{a2} &= {}^1/_3\, (I_a + a^2 I_b + a\, I_c) \end{aligned}\right\} \quad . \quad . \quad . \quad . \qquad (2)$$

When the method of symmetrical components is applied to induction machines under unbalanced conditions, we assume the following:

1. The machine itself has symmetrical windings, *i.e.* the unbalance occurs from outside. In practice, a slight unsymmetry of stator windings may occur due to unequal resistances of the three phases, mainly due to unequal average turn length for the

different phases. The effect of such unsymmetry on the behaviour of induction machines is only small.

2. Equivalent circuit parameters are considered linear; magnetic saturation is neglected.

3. Mutual air gap flux is distributed sinusoidally and is proportional to the net m.m.f. of the stator and rotor currents producing it.

4. The unbalance is only on one side of the air gap.

3.—Analysis of the Unbalanced Stator Connections

(3.1) Solution for Currents

The case to be considered is shown in Fig. 1. A star connected stator will be assumed for the purpose of analysis. If, however, the induction motor has delta connected stator, an equivalent star connection can be found. We can, therefore, limit our work to discuss the 3ph. I.M. having a star connected stator winding with unbalanced connection.

Due to unbalanced connections, the stator phase currents are unbalanced. They can have no zero sequence components since their sum is always zero. The positive and negative sequence components of the stator currents induce two oppositely rotating fields which may be defined as rotating synchronously in the clockwise and counterclockwise directions, respectively.

If all speeds are taken as fraction of the synchronous speed, the per unit slip is s and the rotor speed is (1-s). The clockwise rotating field induces in the rotor voltages of slip frequency, while the counterclockwise rotating field induces in the rotor voltages of frequency (2-s) times the supply frequency f. The positive and negative sequence circuits per phase are shown in Fig. 2 in which V_{b1} and V_{b2} are the positive and negative sequence phase voltages of phase b of the motor which is taken as our reference phase. $V_{b1} = I_{b1} Z_{m1}$ and $V_{b2} = I_{b2} Z_{m2}$, where Z_{m1}

and Z_{m2} are the impedances of the I.M. to positive and negative sequence currents, respectively, as shown in Fig. 2.

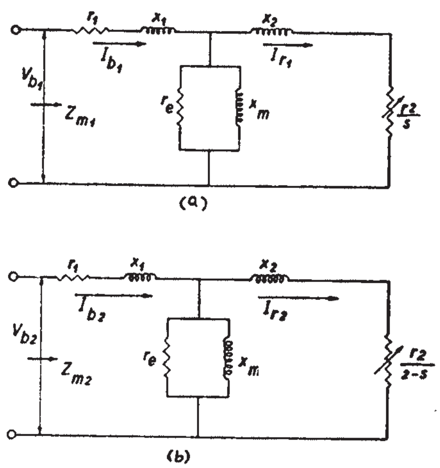


Fig. 2.—Equivalent circuits :
(a) Positive sequence (b) Negative sequeuce

Applying the method of symmetrical components, the expressions for the positive and negative sequence stator currents, I_{b1} and I_{b2}, are obtained in terms of the impedances Z_a, Z_b and Z_c, and the line voltages V and P. The expressions for I_{b1} and I_{b2} are as follows :

$$I_{b1}=\frac{V.(-j a^2\sqrt{3}Z_{m2}-Z_b+aZ_c)+P.(j\sqrt{3}Z_{m2}-a^2Z_a+aZ_c)}{-j\sqrt{3}[Z_{m1}(3Z_{m2}+Z_a+Z_b+Z_c)+Z_{m2}(Z_a+Z_b+Z_c)+Z_a(Z_b+Z_c)+Z_bZ_c]} \quad (3)$$

$$I_{b2}=\frac{V.(-j a\sqrt{3}Z_{m1}+Z_b-a^2Z_c)-P.(-j\sqrt{3}Z_{m1}-aZ_a+a^2Z_c)}{-j\sqrt{3}[Z_{m1}(3Z_{m2}+Z_a+Z_b+Z_c)+Z_{m2}(Z_a+Z_b+Z_c)+Z_a(Z_b+Z_c)+Z_bZ_c]} \quad (4)$$

The stator phase currents can be derived by superposing the positive and negative sequence current components in the different phases. In the rotor, the positive and negative sequence currents have two different frequencies, sf, and (2-s) f, respectively. The rotor phase currents referred to stator are all equal to the root mean square value of the magnitudes of the positive and negative sequence rotor referred currents indicated in Fig. 2 by I_{r1} and I_{r2}, respectively.

(3.2) *Solution for the Developed Torque and Power*

The positive sequence rotor currents of frequency sf_1 will induce a field rotating at a speed equal to the slip speed with respect to the rotor, and at the synchronous speed w.r.t. the stator. Since the stator positive sequence currents induce a field rotating at the synchronous speed, the clockwise stator and rotor rotating fields are always stationary w.r.t. each other. A clockwise torque will be developed similar to that of a 3-ph. I.M. fed from a balanced supply system. The magnitude of this torque can be calculated from the equivalent circuit of Fig. 2-a in a manner similar to that applied in case of balanced motor.

Similarly, both the counterclockwise stator and rotor fields rotate synchronously in the opposite direction to that of the rotor; a counterclockwise torque will thus be developed. Its magnitude can be calculated applying the equivalent circuit of Fig. 2-b.

The resultant torque is the difference of the two developed torques, its direction will be either clockwise or counterclockwise according to whether the clockwise or the counterclockwise torque is the prevailing one.

The mechanical power developed by the motor in watts is equal to the torque in synchronous-watts multiplied by (1-s). The shaft power is smaller than that developed by the amount of the motor friction and windage losses.

The power taken by the I.M. itself is the sum of the powers taken by its positive and negative sequence circuits given in Fig. 2. The total power consumed is the sum of the power taken by the motor and the copper losses in the added impedances.

4.—Kusa Connection of the Three-Phase Induction Motor

(4.1) *Starting through a Kusa Resistance*

(4.1.1) *Introduction:*

In some industries, as in taxtile industry, smooth starting of the driving motors is necessary. A convenient possible way is to insert a so-called kusa resistance in series with one stator phase, as shown in Fig. 3. This method, mainly used to have

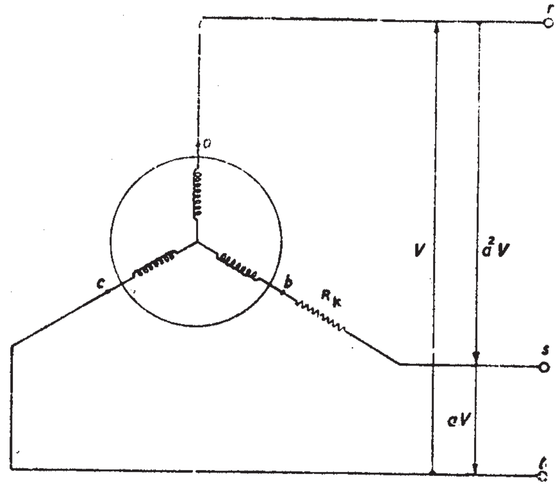


Fig. 3. The stator of the 3.ph. I.M. with a kusa resistance

smooth starting in case of squirrel-cage motors, was discussed by many German authors ([2], [8], [10]). The name kusa is an abbreviation for the German expression "Kurzschlussläufer Sanftanlauf".

(4.1.2.) Starting Characteristics :

The operation of the 3-ph. I.M. with a kusa resistance R_k is a special case of Fig. 1. The solutions for I_{b1} and I_{b2} at starting are obtained from the general equations (3) and (4) with $Z_a = o$, $Z_b = R_k$, $Z_c = o$, $P = a\,V$, and $Z_{m1} = Z_{m2} = Z_s$, where Z_s is the starting impedance of the motor per phase. Having calculated I_{b1} and I_{b2}, the phase currents I_{ak}, I_{bk} and I_{ck} can be computed applying the symmetrical components theory. The locus of each phase current is a circle of parameter R_k. Fig.4–a shows the starting current I_{bk} flowing through the kusa resistance (in terms of the starting current I_s corresponding to $R_k = o$) drawn as a function of the starting torque T_{sk} (expressed in terms of the torque T_s obtained with $R_k = o$), for different motor starting power factors of 0, 0·4 and 0·8.

As regards the effect of the kusa resistance on the starting turque T_{sk}, the behaviour of the 3-ph. I.M. will then approach that of the I-ph. I. M. having an auxiliary winding. In the latter case the starting troque, besides depending on the phase shift

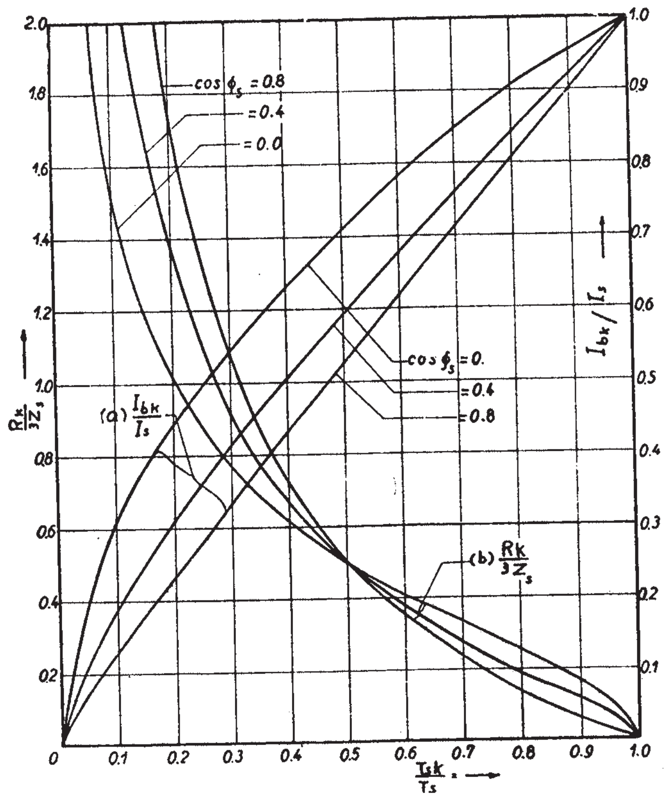


Fig. 4.—Relation between the kusa resistance, its current, and starting torque

between the currents in the main and auxiliary winding, depends also on the magnitude of the current in the auxiliary winding, Phase b including R_k may stand for the axuiliary winding of the I-ph. I.M. The torque T_{sk} will then depend on the current I_{bk}, being zero when $I_{bk} \Rightarrow o$. Fig. 4-b shows the relation between R_k (expressed in terms of the starting impedance of the motor, Z_s) and T_{sk}/T_s, for different motor starting power factors of 0,0·4 and 0·8 The curves are asymptotic with the resistance axis since an infinite resistance is required to have zero starting torque.

Fig. 5 shows the locus of motor terminal voltages with variable R_k. The point b_∞ corresponds to infinite value of R_k, and the point b_o corresponds to $R_k = o$; for any other value of R_k, the vertex b lies on the part of the circle o_1 to the right of

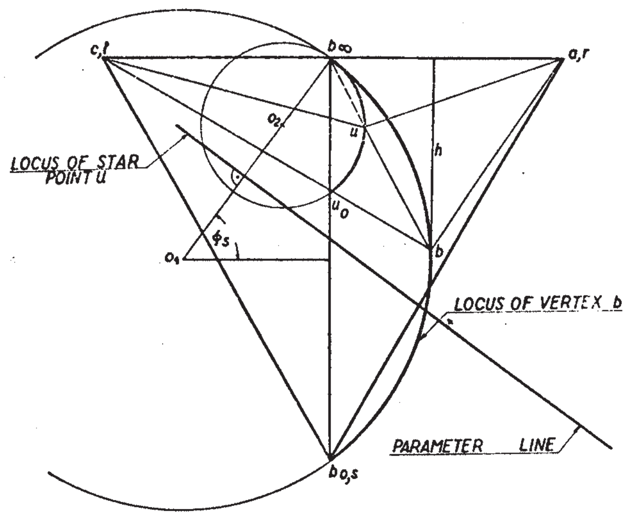


Fig. 5.—Locus of motor terminal voltages,
(cos ϕ_s = starting P.F.)

$b_o\, b_\infty$. The locus of the star point is the circle o_2 whose diameter is equal to 1/3 that of the circle o_1. It may be interesting to note that the starting torque is proportional to the height h of the triangle abc.

(4.1.3) *Torque-slip Curves:*

To discuss the shape of torque slip curves, these curves are drawn for variable values of R_k for a 3-ph., 10 H.P., 220 V., 50 cycle, 6 pole, induction motor. In all the curves of Fig. 6, except the curve corresponding to $R_k = o$, there exists a very small negative torque at synchronous speed. The shape of the curves shows that the method of kusa resistance provides a way to master the starting torque and the shape of the torque-slip curve of the motor ; the starting period and acceleration will be consequently mastered according to the duty of the motor.

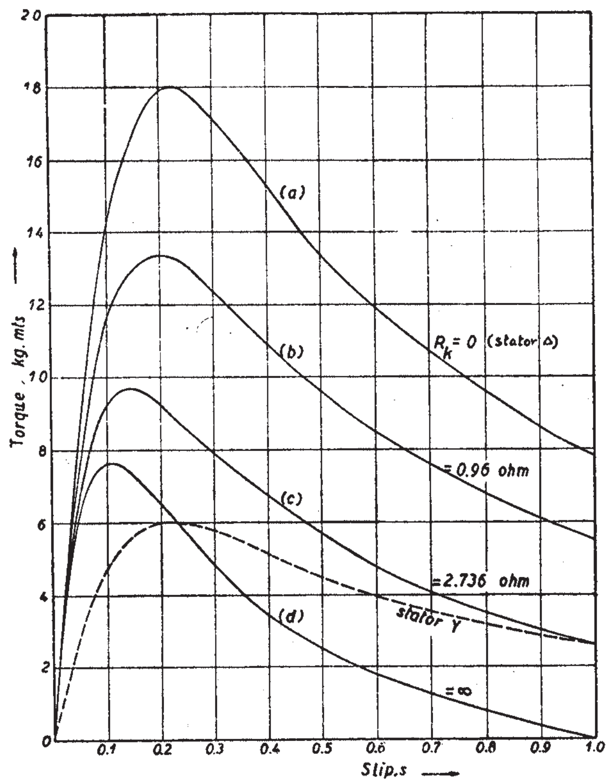


Fig. 6.—Torque-slip curves with different values of R_k

(4.2.) Starting through a kusa inductance

(4.2.1.) Starting Characteristics:

It has been explained how smooth starting of the 3-ph. I.M. can be obtained by the use of a kusa resistance ; a kusa inductance ban be similarly applied. The stator of the motor is connected as in Fig. 3 with R_k substituted by the kusa inductive reactance jx_k. The solution of the positive and negative sequence starting currents, I_{b1} and I_{b2}, can be obtained from the general equations (3) and (4) with $Z_a = o$, $Z_b = jx_k$, $Z_c = o$, $P = aV$, and $Z_{m1} = Z_{m2} = Z_s$. The phase currents I_{ak}, I_{bk} I_{ck}, can then be expressed in terms of I_{b1} and I_{b2}. Fig. 7-a shows the current flowing in the kusa icductance with variable xaiues of tnis inductance drawn against the corresponding starting torques T_{sk}, for different values of

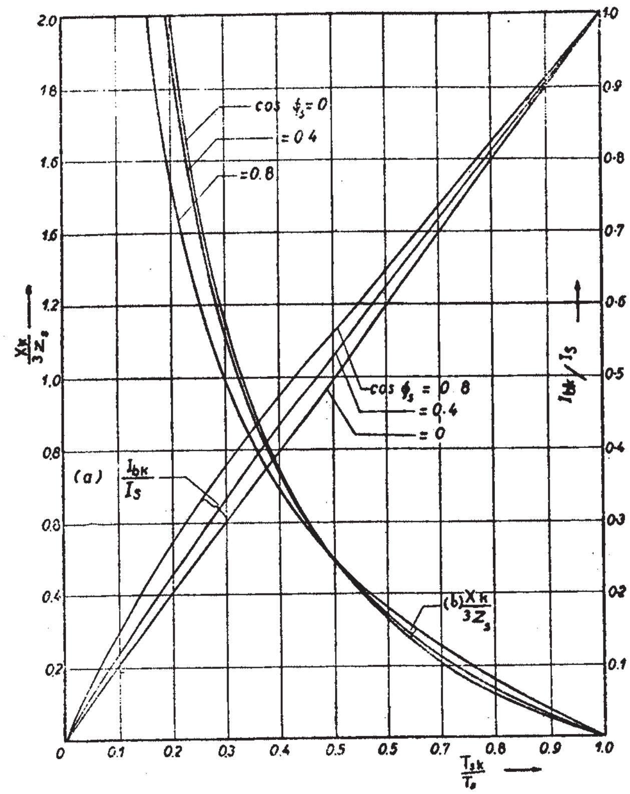


Fig. 7 .Relation between the kusa reactance, current, and starting torque.

starting power factors of 0,0·4 and 0·8. Fig. 7-b shows the relation between X_k |$3Z_s$ drawn against T_{sk}/T_s. These curves are asymptotic with the ordinate since an infinite value of X_k is required to have zero starting torque.

Fig. 8 shows the locus of motor terminal voltages with variable values of X_k. The vertex b lies on the part of the circle o_1 to the left of b_o b_∞ ; the part to the right of the line b_o b_∞ corresponds to the case of replacing the kusa inductance by condenser.

(4.2.2) Comparison with the Case of Kusa Resistance:

The curreut curves shown in Fig. 7-a have the same shape as those obtained in the kusa resistance case drawn in Fig. 4-a.

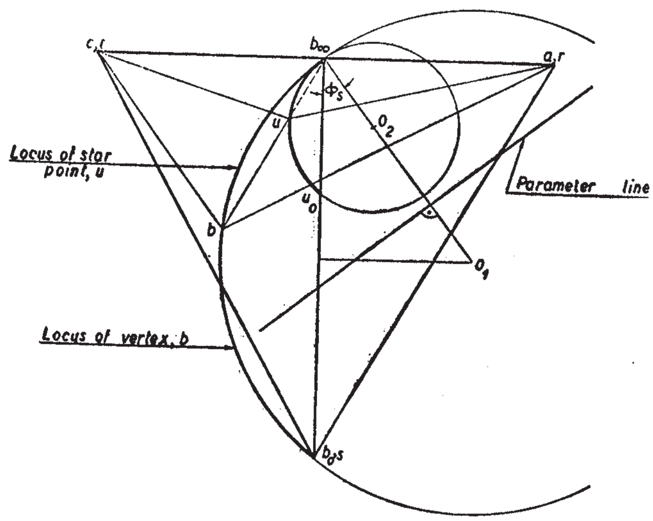


Fig. 8—Locus of motor terminal voltages with a kusa inductance and condenser

As regards the curves of Fig. 7-b and Fig. 4-b, they have clearly the same shape. It is interesting to note that a curve in Fig. 7-b drawn for a motor starting power factor equal to cos ϕ_s is identical with the curve in Fig. 4-b corresponding to the power factor cos $(90 - \phi_s) = \sin \phi_s$.

It has been shown that a kusa inductance, as well as a kusa resistance, may be used to reduce the normal starting torque of the 3-ph. I.M. to the required value. As regards the supply power factor, it is generally improved by the use of a kusa resistance, while, as expected, it deteriorates with a kusa inductance. For law starting power factor motors, below 0·5, a kusa resistance is preferred to a kusa inductance as it is accompanied by the higher power factor. In case of high starting power factor motors, above 0·5, a kusa inductance may be recommended, though accompanied by a slight reduction in supply power factor, in order to avoid troubles due to overheating of kusa resistance if the motor fails to start quickly.

(4.3.) Starting through a Kusa Condenser

Having discussed the operation of the 3-ph· I. M. with a kusa resistance and a kusa inductance, it remains to study its starting characteristics with a kusa condenser. The stator is connected as in Fig. 3 with R_k replaced by a condenser of reactance

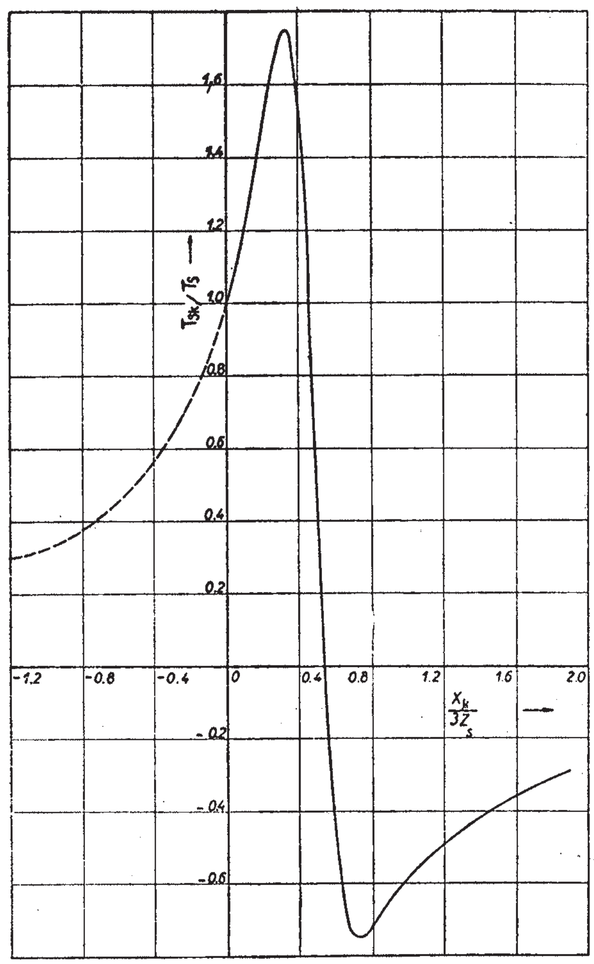


Fig. 9. Effect of kusa condenser on T_{st} ($\cos \phi_s = 0{\cdot}4$)

equal to $-jX_k$. This problem is a particular case of Fig. 1 with $Z_a = o$, $Z_b = -jX_k$, $Z_c = o$, and $P = aV$. Noting that at starting

$Z_{m1} = Z_{m2} = Z_s$ the expressions for the positive and negative sequence currents, I_{p1} and I_{p2}, for the starting phase currents, I_{ak}, I_{pk} and I_{ck}, can be obtained easily.

Fig. 9 shows the starting torque available by the use of kusa condenser, (expressed in terms of the normal starting torque T_s) drawn as a function of the capacitive reactance (expressed in terms of Z_s) for an induction motor whose starting power factor = 0·4. It is seen that the curve possesses both a maximum and a minimum value. The strange fact that the induction motor may develop a negative torque for certain values of condenser necessitates that the phase currents must form together a negative sequence system. The expressions for the maximum positive and maximum negative torques, expresssed in terms of T_s are given by:

$$\text{Maximum positive torque}/T_s = \frac{1 + \cos \phi_s}{2 \cos \phi_s} \quad . \quad . \qquad (5).$$

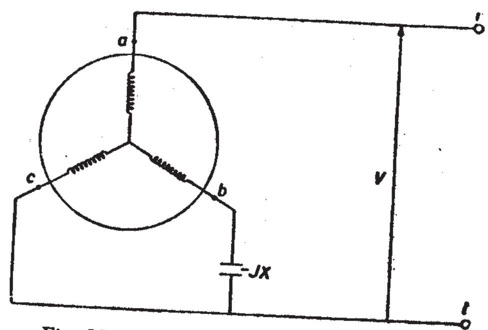
$$\text{Maximum negative torque}/T_s = -\frac{1 - \cos \phi_s}{2 \cos \phi_s} \quad . \qquad (6)$$

It is seen also from Fig. 9 that it is possible to use two different values of condenser so that the motor may develop starting torque. bigger than or opposite to the normal starting torque.

5.—The Three-Phase Induction Motor on Single Phase Pupply

(5.1) *Introduction*

The 3-ph. I.M. is sometimes required to run on 1-ph. supply, which is the case when such a supply or a badly balanced 3-ph. supply is only available. It is well known that the 3-ph. I.M. on 1-ph. supply cannot start by itself and that auxiliary impedances may be used to shift the phase currents and terminal voltages in order to obtain a starting torque (1). Quite often the auxiliary impedances are employed for starting purposes only. When the proper speed is attained they are cut off and the motor will then be capable oı operating alone. Of practical interest is the case of a 3-ph. I.M. run as a capacitor motor on 1-ph. supply (Fig. 10).



Eig. 10.—Capacitor start of the 3-ph. I.M.

The two common ways applied to run the 3-ph. I.M. on 1-ph. supply are either to let it run with one stator phase open [6, 12, 17], or to connect its primary circuit so that to consist of two parallel connected stator phases in series with the third phase [9, 12, 17], latter connection is only used as a braking connection.

(5.2) Capacitor Start Motor

(5.2.1) Starting Characteristics:

The stator of the 3-ph. I.M. is connected as in Fig. 10. The solution for the starting currents I_{p1} and I_{p2} is obtained from the general equations (3) and (4) by substituting the values: $Z_a = o$, $Z_p = -jX$, $Z_c = o$, $P = o$, and $Z_{m1} = Z_{m2} = Z_s$. Having obtained I_{p1} and I_{p2}, the phase currents I_a, I_b and I_c can then be obtained.

The locus of motor terminal voltages is as shown in Fig. 11. This figure can be used to find the phase currents, the starting torque, motor terminal voltages and phase voltages for any value of capacitance applied.

It is evident that the starting torque of the motor depends on the reactance X. To obtain the maximum available starting torque, the condenser applied must have a reactance equal to $^3/_2$ times the magnitude of the starting impedance of the motor per phase, *i.e.* $X = 1 \cdot 5\ Z_s$. The maximum torque (expressed in terms of the torque T_s obtrined on 3-ph. supply) is expressed by

$$\text{Maximum Torque } T_s = \frac{1}{2\sqrt{3}} \frac{\cos \phi_s}{1 - \sin \phi_s} \quad . \quad . \quad (7)$$

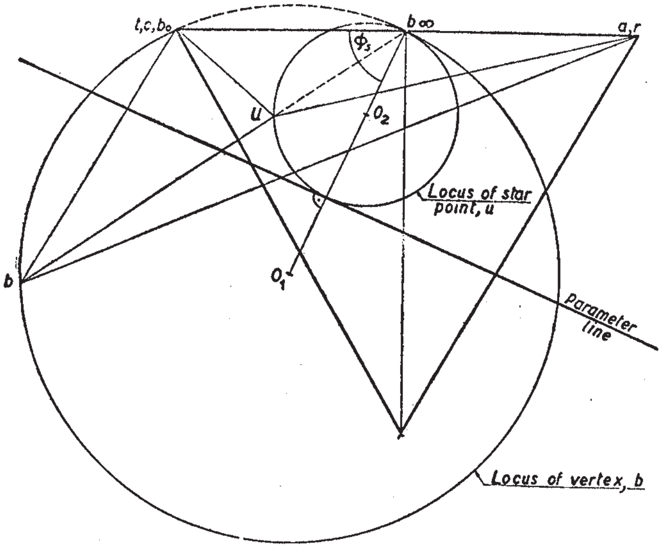


Fig. 11.—Locns of motor terminal voltages connected as in Fig. 10

It is easy to see that as the starting power factor of the motor increases, the maximum available torque decreases till it has a value= $0 \cdot 288\ T_s$ at unity power factor (this is a limiting case but not a practical one).

Fig. 12 shows the starting characteristics of an I.M. having a starting power-factor $= 0 \cdot 3$. It is seen that the starting P.F. has attained a value $= 0 \cdot 9$. In choosing the optimum valume of capacitance required for successful starting, the torque per ampere curve is really of practical help ; it is clear that the maximum possible value of torque per ampere should be obtaived. Fig. 12 shows that when $X = 1 \cdot 8\ Z_s$ the torque per ampere and the power factor are very near to their maximum values, while the torque is slightly reduced below its maximum value occurring at $X = 1 \cdot 5\ Z_s$. Actually the small reduction of the starting torque is of no impotance so that with X = 1·8 Z_s, the starting conditions are pereferable to those obtained with X = 1·5 Z

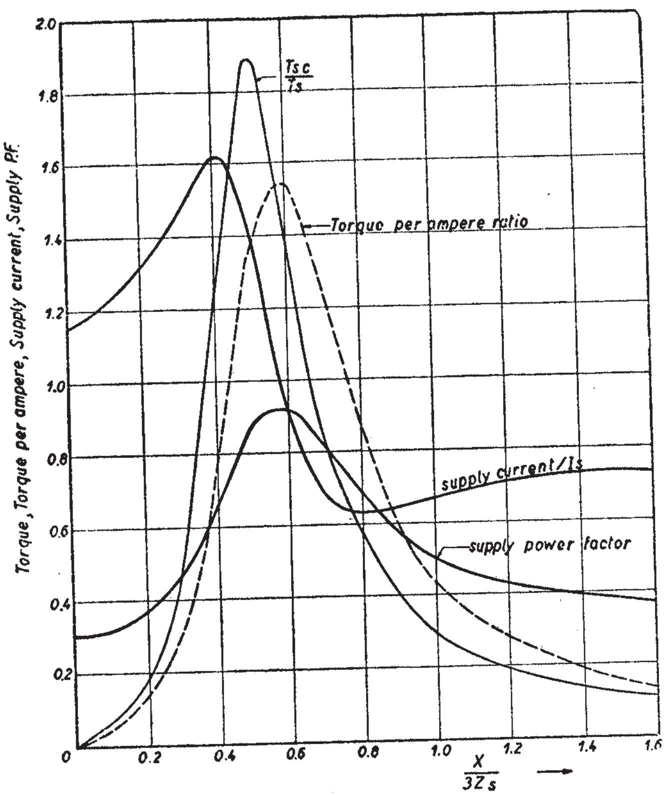


Fig 12. Starting characteristics of the capcitor motor. Cos ϕ_s = 0·3
(Values are expressed in terms of corresponding values on 3.ph, supply)

(5.2.2) Split Phasing by means of Inductance or Resistance

An inductance or a resistance may be used instead of the condenser connected as shown in Fig. 10 The solution for the currents can be obtained from the general equations (3) and (4) in a manner similar ta that applied in case nf condenser.

As expected, the maximum starting torque available by the use of condenser is much higher than that obtained by the use of inductance or resistance- The maximum starting torque in the latter two cases can never be higher than 0·288 T_s, and are available when the value of the inductive reactance or resistance is equal to 1·5 times the magnitude of Z Fig 13 shows the maximum starting torque available, drawn as a function of motor starting power factor, in the three cases of applying condenser, inductance and resistance.

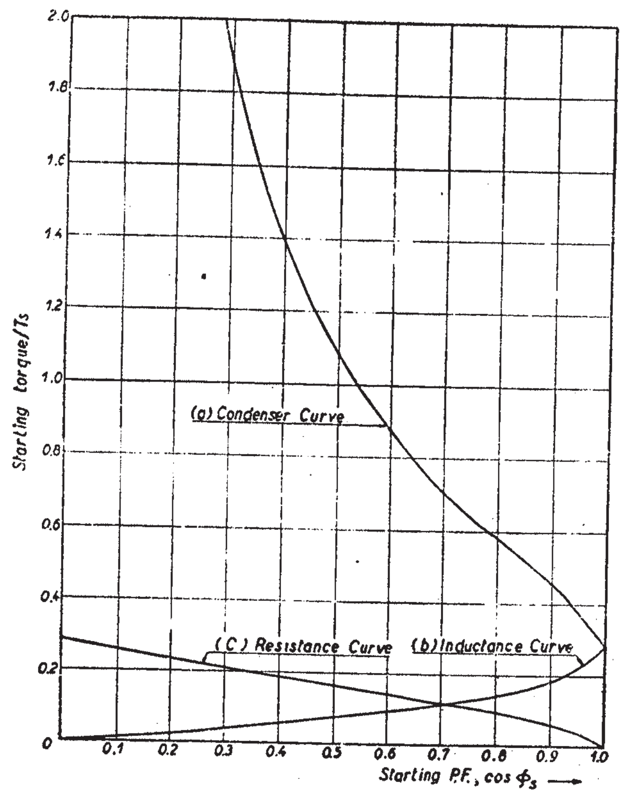


Fig. 13.—Maximum Tst using condenser, inductance, or resistance

(5.2.3) Conclusion:

It is clear from Fig. 13 that the maximum torque available in the resistance case is always bigger than that available in case of inductance, for motor starting power factors below 0·707. The reverse is true for higher power factors which may occasionally be met with in case of small induction motors. In most cases the starting of the 3-ph. I.M. on 1-ph. supply can be accomplished by the use of a resistance of magnitude equal to 1·5 times the magnitude of Z_s, provided that the starting torque required is sufficiently small. If the starting torque required is high, the use of a condenser is incomparable. It was found that a condenser of reactance equal to 1·8 times the magnitude of the starting impedance of the motor per phase, Z_s, is preferable to obtain good starting characteristics regarding supply current, power factor, and torque per ampere, than the condenser whose reactance is equal to 1·5 times the magnitude of Z_s which is necessary to provide the maximum starting torque.

(5.3) The Operation with One Stator Phase Open

(5.3.1) Solution for Currents:

The solution for the positive and negative sequence currents for this extreme case of unbalance shown in Fig. 14 may be obtained

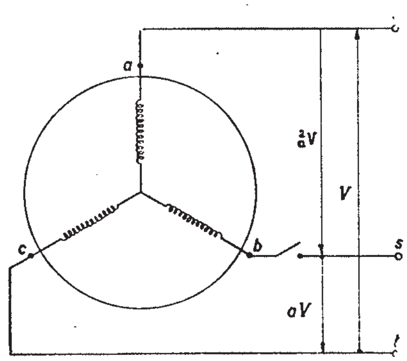


Fig. 14.—Operation of 3-ph. I.M. with one stator phase open

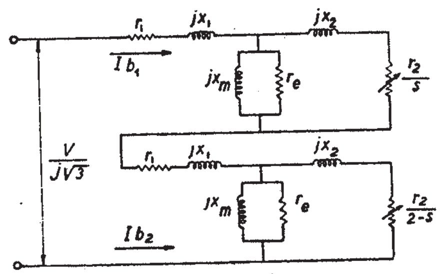
from the general equations (3) and (4) with $Z_a = o$, $Z_b = \infty$, $Z_c = o$, and $P = a\,V$. The following result will be obtained:

$$I_{b1} = -I_{b2} = \frac{j\,V}{\sqrt{3}} \frac{1}{Z_{m1} + Z_{m2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

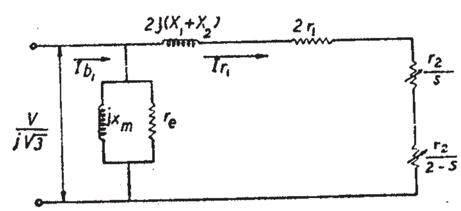
This result shows that the magnitude of either current is equal to the phase valtage divided by the sum of the impedances of the motor to positive ond negative sequedce currents. It follows, therefore, that the 1-ph. I.M. is equivalent in some respects to two identical polyphase motors whose stator windings are connected in series in such a way to produce oppositely rotating fields and whose rotors are rigidly coupled mechanically. The distribution of voltage between the two motors depends upon the slip. As the slip decreases, the motor whose torque is in the direction of the rotor gains more and more voltage till at synchronous speed it nearly utilises all the voltage, *i.e.* the other motor will be practically inactive.

(5.3.2) *Equivalent Circuit and Torque-slip Curve*:

According to equation (8) the exact equivalent circuit for the positive sequence stator current is as shown in Fig. 15-a.



(a) Exact equivalent circuit

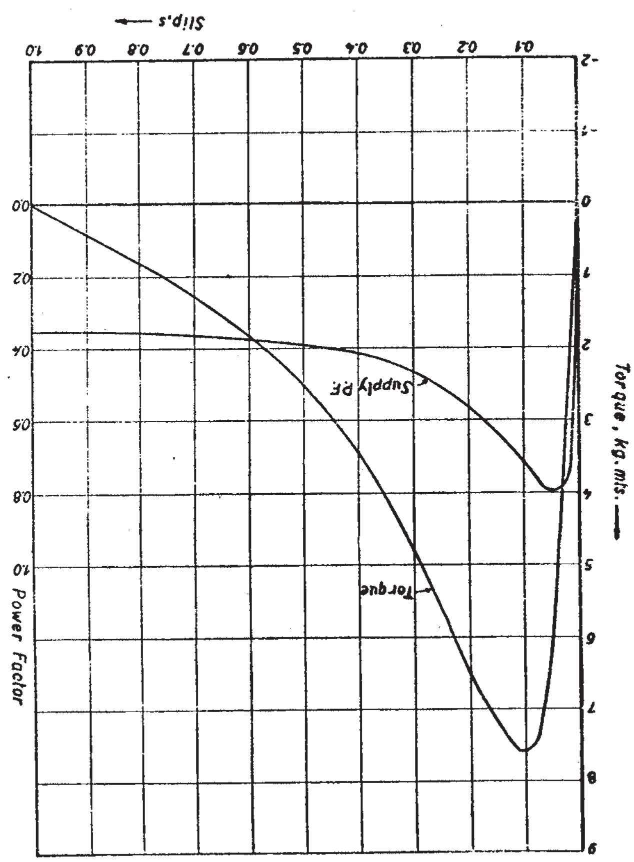


(b) Approximate equivalent circuit near syn.speed

Fig. 15.—Equivalent circuit of the 1-ph. I.M.

The approximate circuit of Fig. 15-b may be applied for the calculation of motor characteristics near synchronous speed. Applying the exact equivalent circuit, the locus of the positive sequence current is a circle of parameter s (2-s). The exact circle diagram for 1-ph. running has a smaller diameter than that for 3-ph. running. The starting current in the former case is $\frac{\sqrt{3}}{2}$ times that in the latter case. Also, the magnetizing current near synchronous speed in the former case in about $\sqrt{3}$ times that in the latter case ; this contributes to the low power factor of the 1-ph. motor. The reduced starting current and the increased magnetizing current in case of 1-ph. running are responsible for the smaller diameter of the circle diagram.

As regards the torque in the case of 1-ph running, the resultant torque developed is the difference between the positive and negative sequence developed torques. The resultant torque is zero at two slips, namely, at starting and at a slip very near to



Fig, 16.– Torque and P. F. curves of the 1-ph. I.M

zero—at which the positive and negative sequence torques are exactly equal. At synchonous speed, a very small negative torque is developed ; thus the motor can never work at synchronous speed even with no rotational losses. The torque-slip curve, together with the supply power factor curve, are drawn against slip in Fig. 16 for the 3-ph. I.M. mentioned in Section (4.1.3). The maximum torque and the slip at which it is developed are about one-half the corresponding values in case of 3-ph. running. Also, the magnitude of the maximum torque in case of 1-ph. running decreases as the rotor resistance is increased symmetrically in three phases while in 3-ph. running the rotor resistance can only affect the value of the break-down slip.

The 1-ph. motor can have no braking region because the 1-ph. motor has no definite direction of rotation at standstill. The 1-ph. motor can, however, operate as a generator when driven at a supersynchronous speed. The mechanical power, efficiency, and power factor in case of 1-ph. running are smaller than the corresponding values in case of 3-ph. running.

(5.4) The Operation of the 3-ph. I.M. on 1-ph. Supply with Two Stator Phases Connected in Parallel

(5.4.1) Solution for Currents:

The motor is connected as in Fig. 17. The solution for the

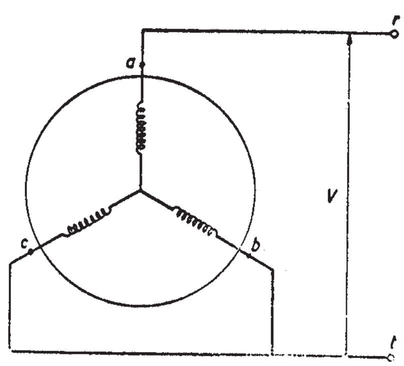


Fig. 17.—The 3-ph. I.M. with 2 stator phases connected in parallel

positive and negative sequence stator currents can be concluded from equations (3) and (4) with $Z_a = 0$, $Z_b = 0$, $Z_c = 0$, and $P = 0$.

The positive and negative sequence currents, I_{p1}, and I_{b2}, and the supply current I_a are expressed by :

$$I_{p1} = a^2 V/3 Z_{m1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$I_{p2} = aV/3 Z_{m2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$I_a = V/3 Z_{m1} + V/3 Z_{m2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

According to equations (9) and (10), the positive and negative sequence currents at any slip s are equal in magnitude to $1/\sqrt{3}$ times the phase currents flowing on balanced 3-ph. voltages at slip values of s and (2-s), respectively.

The loci of I_{b1} and I_{b2} are circles of parameter s, and the locus of I_a is a circle of parameter s (2-s). To get quantitative results, the supply current locus is drawn in Fig. 18 for the I.M.

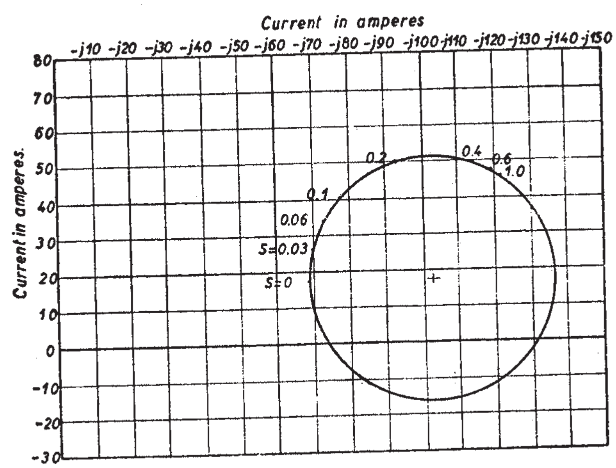


Fig. 18.—Supply current locus of the motor connected as in Fig. 17.

whose particulars are given in Section (4.1.3). The starting current in our case = $2/\sqrt{3}$ times the starting current on balanced 3-ph. voltages ; also, the supply current at synchronous speed is about $1/\sqrt{3}$ times the latter current. The high current at synchronous speed is the cause of the small diameter of the circle diagram and the low power factor of the motor.

(5.4.2) *Torque-Slip Curves :*

According to equations (9) and (10), the positive and negative sequence torque developed by the motor at any slips can be easily seen to have values equal to 1/3 the torques developed on balanced supply at motor slip values of s and (2-s), respectively. The resultant torque is zero at starting and at a speed near to synchronous speed. Fig. 19 (a) shows the torque-slip curve of our

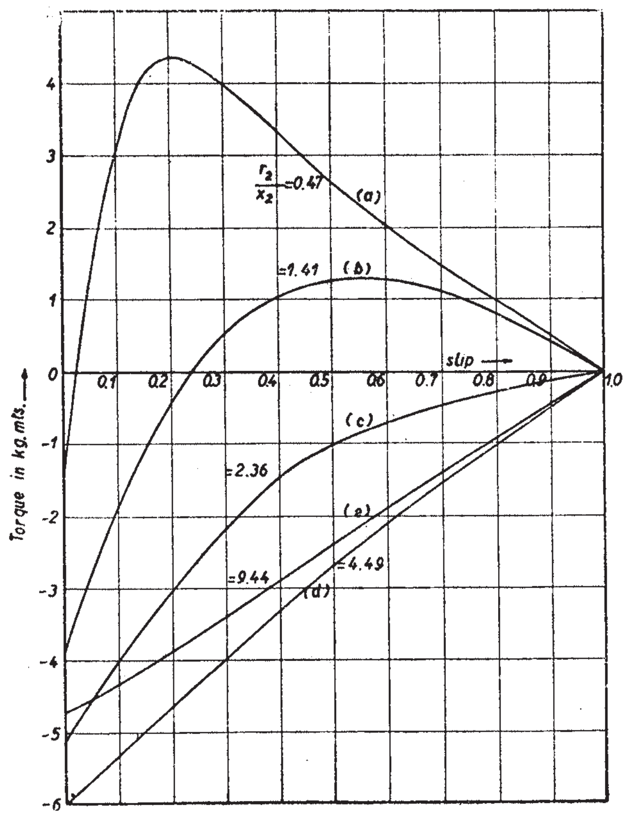


Fig. 19.—Torque-slip curves, with different rotor resistance

experimental motor. It can be seen that the motor, even with no rotational losses, will run at a slip of about 2·5%. The maximum torque in our case is about 1/4 times the maximum torque developed on balanced 3-ph. voltages, and about 1/2 times that obtained with one stator phase open.

The torque-slip curve (a) of Fig. 19 shows that with this asymmetrical connection the motor can drive the load if the load torque is sufficiently low, but at a high slip, high current, and low power factor and efficiency. Compared with the motor operating with one stator phase open, the latter is superior. In this latter case, the maximum torque is nearly double the corresponding one in the former case, and the negative sequence torque is very nearly equal to zero at synchronous speed. If a 3-ph. I.M. is required to operate on 1-ph. supply, the operation with one stator phase open is definitely applied.

In fact, the high negative sequence torque at and near synchronous speed has led to the application of this asymmetrical connection for braking of 3-ph. wound rotor induction motors (9). If the rotor resistances are increased symmetrically in the three rotor phases, the negative sequence torque at synchronous speed increases up to a maximum value of 1/3 the maximum torque available on balanced 3-ph. supply. The rotor resistances which yield this maximum torque at synchronous running are those required to develop the maximum torque of the balanced 3-ph. I.M. at slip of s = 2. A further increase in the rotor resistances will result in reducing the value of the negative torque at synchronous speed until it attains a zero value at open rotor circuit. Torque-slip curves with different values of rotor resistances are drawn in Fig. 19 for our experimental motor. It is seen that they approach a linear shape for high rotor resistances.

6.—Tests on the Induction Motor

Tests were made on a 3-ph., 10 H.P., 220 V., 50 cyles, 6 pole induction motor. Its shaft could be coupled, through a flexible coupling, to a direct current machine. Meters were connected to measure all electrical quantities. Besides this I.M.—d.c. machine set, another set was available to determine the torque-slip curves (7).

The tests were made at a reduced line voltage, about one-half the rated value, since this gave maximum permissible current loading at high values of slip. Test results were then converted to values for rated voltage assuming the current proportional to the voltage and the torque proportional to the voltage square. The experimental and theoretical results showed reasonable agreement. A probable source of error was due to the change of motor constants with temperature and saturation.

7.—Bibliography

1. Ager, R. W., "The Use of Auxiliary Impedances in the Single-Phase Operation of Polyphase Induction Motors". *A.I.E.E. Trans.*, 60 (1941), p. 494.

2. Bohmann, J., "Untersuchung über des Verhalten von Drehstrommotoren in der Kurzschluss-Sanftanlauf-(Kusa-) Schaltung". *Archiv für Elek.*, 28 (1934), p. 759.

3. Brehn, H. and H. Juckenack, "Anlassgeräte für sanften Anlauf von Kurzscheussläufermotoren". *Siemens-Zeitschrift*, 13 (1933), p. 224.

4. Burlando, M. F., "Le Freinage Electrique des Moteurs Asynchrones avec Connexion Asymétrique des Enroulement". *Revue Générale de l'Electricité*, 58 (1949), p. 198.

5. Edward O Lunn., "Induction Motors under Unbalanced Conditions". *Elect. Engineering*, 55 (1936), p. 387.

6. Franken, H., "Single-Phase Operation of Polyphase Induction Motors". *Elektrotechnik und Maschinen bau*, 47 (1929), p. 1127.

7. Hans Rahmann, "Ein einfaches Auswertungsverfahren für die Aufnahme der Drehstromlinien von Asynchronmotoren mit Geeichten Gleichstrommaschinen". *Elektrotechnische Zeitschrift*, 46 (1935), p. 295.

8. Heinz Jordan, "Anwendung der Methode der symmetrischen Komponenten auf unsymmetrische Ständerchaltungen von Drehstromasynchronmotoren". *Archiv für Elek.*, 30 (1936), p. 812.

9. John Reiser, "Die Strom und Momentverhaltnisse der untersynchronen Bremsschaltung von Siemens". *Archiv für Elek.*, 28 (1934). p. 447.

10. Leonhard, A., "Anlassen von Asynchronmotoren über Kusawiderstand, Bemessung des Kusawiderstandes". *Elektrotechnik und Maschinenbau*, 61 (1943), p. 122.

11. Lunn, E. O., "Induction Motors under Unbalanced Conditions". *A.I.E.E. Trans.*, 55 (1936), p. 387.

12. Lyon, W. V., "*Application of the Method of Symmetrical Components*". New York: McGraw-Hill Book Co, Inc., 1937.

13. Lyon, W. V. and Charles Kingsly, "Analysis of Unsymmetrical Machines". *Electrical Engineering*, 55 (1936), p. 471.

14. Reed, H. R. and R. J. W. Koopman, "Induction Motor on Unbalanced Voltages". *Electrical Engineering*, 55 (1936), p. 1206.

15. Schwisky, W., "Das Bremsmoment der Bremsschaltung von SSW". *Archiv für Elektrotechnik*, 30 (1936), p. 552.

16. Seiz, W. and A. Drehmann, "Dreiphasenasynchronmaschine mit Unsymmetrischen Schaltung". *Archiv für Elektrotechnik*, 30 (1936), p. 58.

17. Wagner, C. F. and R. D. Evans, "*Symmetrical components*". New York: McGraw-Hill Book Co., Inc., 1933.