STRESSES AND FAILURE LOADS FOR STATICALLY INDETERMINATE REINFORCED CONCRETE FRAMES

BY

Prof. Dr. W. S. HANNA, Dr. MICHEL BAKHOUM
and ROUSHDY TOSSON (M.Sc.)

Introduction

The aim of the investigation is to find the distribution of moments and stresses, in statically indeterminate R.C. frames when subjected to gradually increasing loads, from zero up to failure loads; and to develop methods for the determination of the safe load, the critical failure load, and the ultimate load which the frame can carry.

The study is divided into the following :—

1. A review for the determination of stresses in R.C. members subjected to flexure.

2. A review for the determination of ultimate loads in R.C. members subjected to flexure.

3. Conception of critical failure load and methods for its determination.

4. A review for some of the principal methods for solving statically indeterminate structures for the cases of homogeneous elastic materials.

5. A discussion of the theory of limit design and its adaptability to R.C. structures.

6. A study of the action of R.C. statically indeterminate frames in the elastic and plastic stages.

7. Experimental verifications on R.C. frames.

8. Results and conclusions.

The study made on some of the main methods of analysis of statically indeterminate structures according to the elastic theory, was a pre-requisite for the study of the main problem in reinforced concrete. Most of other materials reviewed in the thesis were adopted from papers not yet published.

Stresses in R.C. Members subjected to Flexure

A.—*Standard Theory:* Based on the following assumptions : (i). The strain distribution is planer, (ii). Both concrete and steel are perfectly elastic, (iii). Concrete can resist no tensile stresses.

None of these assumptions is strictly correct and this leads to some discrepancies between computed and actual stresses. To find the stresses f_c and f_s we find the position of the neutral axis.

B.—*More Accurate Determination of Stresses:* To reduce the difference between the values of f_c and f_s computed according to the standard theory and the actual values obtained from tests and to get more accurate stresses, it has been proposed :

1. To retain the assumption of planer distribution of strains.

2. To provide for the fact that stress-strain relation for concrete in compression is not linear but a curve which could be obtained experimentally. An equivalent linear distribution is obtained at each stress by drawing a secant line which gives the same area as that bound by the original curve and the base line (Fig. 1). The slope of the secant line of' gives the value E_c to be used in finding n ; and f' gives the equivalent stress which should be converted into the actual value on the original curved stress-strain relation.

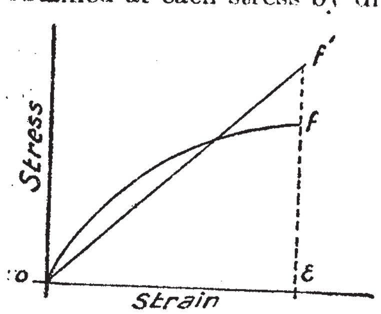


Fig. 1.

3. To take the effect of concrete in tension into account. If the modulus of elasticity of concrete in compression E_c is equal to that in tension E_t, the checking of stressses would be very simple.

Experimental work has shown that the stresses f_c and f_s computed according to the above-amended assumption are in good agreement with the actual stresses than those obtained by the standard theory.

Ultimate Load for R.C. Members subjected to Flexure

Many investigators have contributed to the ultimate load theory among whom are mainly theories of Saliger, Whitney and Jensen, but most of these theories treat special cases of sections. In an investigation at the University of Illinois 1946–1948, Bakhoum attempted to derive general theories applicable for the general case.

The determination of ultimate load and critical failure load in our thesis is based on Bakhoum's work.

The following three methods give simple procedure for the determination of ultimate loads. The case in which the stress in steel at failure exceeds the yield stress f_y, has been excluded, being complicated, and of no practical significance.

(1) *Using an Amended Standard Theory:*

The stress strain curve for concrete is simplified into an equivalent linear distribution as shown by the line of'_p (Fig. 2). The stress strain curve for steel is considered to have a slope E_s (Fig. 3), up to the yield stress f_y and then the stress equals f_y.

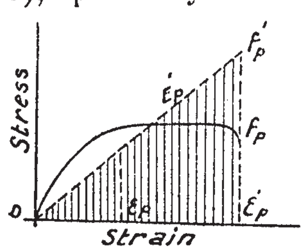


Fig. 2

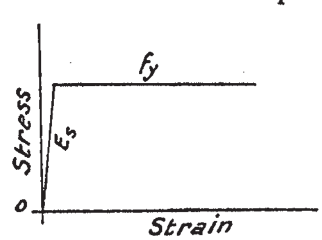


Fig. 3

The derivation of the ultimate loads in this case will be very similar to the determination of stresses according to the standard theory, except that the appropriate value of n should be used.

(2) *Assumption of using Trapezoidal Stress Distribution:*

The stress strain curve for concrete is approximated into an equivalent trapezium (Fig. 4) which shape depends on f_p and α and varies according to concrete strength. The approximate diagram is close to the actual curve. The stress strain curve for steel is also idealized into equivalent trapezium (Fig. 5). For a section subjected to flexure in plane y—y, if we assume a value for Z it can be proved that:

$$h = \alpha Z \text{ and } h_y = h_p \frac{R}{n} \quad \text{where } R = f_y / f_p \text{ and } n = E_s / E_c$$

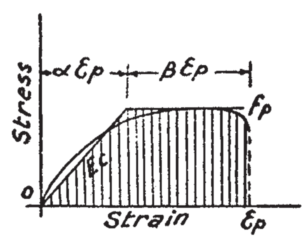


Fig. 4

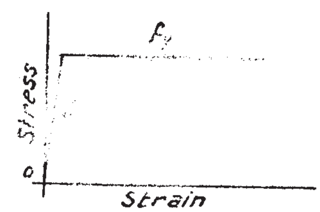


Fig. 5

We denote the area in which the stress distribution is triangular by the index 1, and the area in which the stress distribution is rectangular by the index 2. Thus A_{c1} is the area of concrete in compression within h_p and A_{c2} is the area beyond h_p, etc. .

Formulae for ultimate loads under different conditions of loading were reviewed in the thesis.

(3) *Approximate Method using a Rectangular Stress Distribution:*

The method of trapezoidal distribution is elaborate. It is simplified by converting the stress strain curve of concrete into an equivalent rectangle (Fig. 6).

The equations of ultimate loads in this case are very simple under any condition of loading.

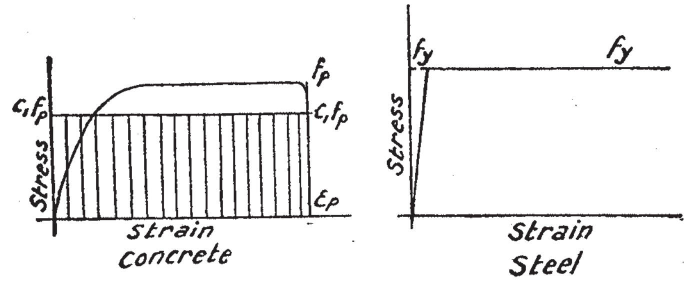


Fig. 6

Theory of Critical Failure Load

The conception of critical failure load arises from the observation in experimental work on R.C. elements subjected to flexure, that at a certain stage prior to failure, strains start to increase to a much higher rate than before, and that most of these strains are of plastic nature and generally lead to wide cracking in concrete. These rapid strains start when either f_{y1} or f_{p1} has been first reached (Fig. 7).

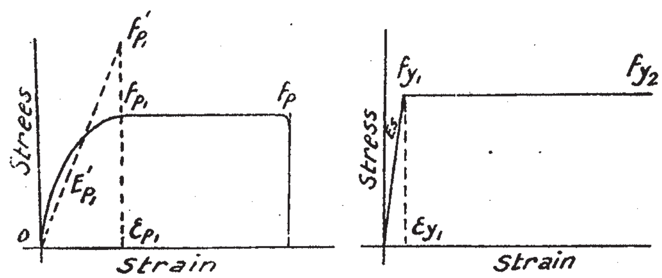


Fig. 7

For the determination of the critical failure load, stress-strain curve for concrete in compression is converted into an equivalent linear stress distribution as shown by the line of f'_{pl}.

The method of determining the critical failure load in all cases is identical with the usual procedures of the standard theory except that the appropriate value of n should be introduced.

THEORY OF LIMIT DESIGN AND ITS ADAPTABILITY TO R.C.

The first one who laid the main doctorines of this theory was Prof. N. C. Kist. Later Prof. V. A. Van Den Broek expanded the theory. He showed that the main emphasis in this theory is shifted from the consideration of permissible stresses to the consideration of permissible deformations. He says: "As a load is gradually applied to a structure, its redundant members, in general, are successively stressed one after another until they reach their elastic or buckling limit strength, etc.".

Consider a case of a R.C. frame (Fig. 8). According to the elastic theory the frame would fail if either section A or B reaches its ultimate capacity.

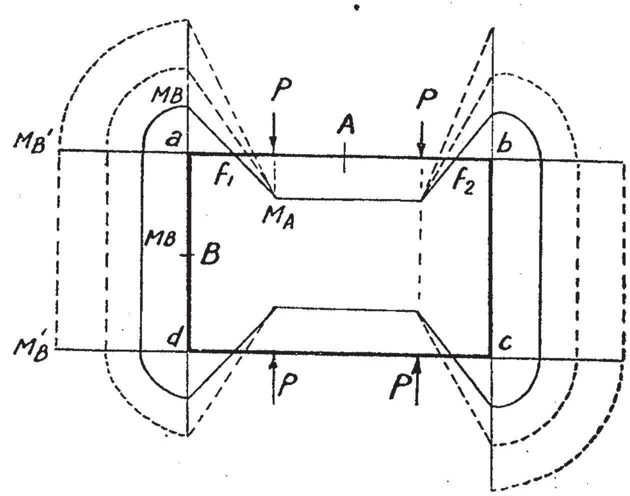


Fig. 8

As the yield stress of steel, for example, is reached at one section, plastic yielding occurs at the section and the B.M.D. is expected to adjust itself in such a manner that that section does not carry higher straining actions, while other sections for which the ultimate stress has not yet been reached carry higher straining actions. If M_A (Fig. 8) is the moment which causes section A to reach the critical failure load due to the yield of steel while the corresponding moment M_B gives stresses at section B below its critical value, then for any lower load, the ratio $M_A : M_B$ is expected to be constant, and the B.M.D. passes through points f_1 and f_2.

After M_A has been reached, the B.M. at a starts to take higher values while M_A keeps almost constant until the critical failure of section B is reached as shown by the dotted line M_A M'_B. The B.M.D. is expected to adjust itself through the partial yielding of the highly stressed sections such that the maximum capacity of the whole frame is reached.

Analytical Determination of Stresses in Frames

Stresses in Elastic and Plastic Stages:

The limit of the elastic stage is considered as the load at which the first section in the frame reaches its critical failure load by attaining f_y or f_p. The plastic stage starts from there on up to the complete failure of the frame.

A.—*Stresses in Elastic Stage:* For an approximate analysis for the accurate determination of stresses, we assume as many factors into consideration as seems practicable, and nearly accurate as follows:—

(I) The value of E_c is assumed constant for all stresses in concrete up to f_{p1} (Fig. 7) and equals the slope of the secant line as shown.

(II) Before the maximum tensile resistance of concrete (ft max.) has been reached in a member, its moment of inertia is assumed as the full virtual area acts. E_t is assumed equal to E_{c1}. The virtual area consists of the full area of concrete in section plus n times the area of steel, n being taken on the basis of E_c obtained in (I).

(III) After ft max. has been reached in any member, the virtual area is assumed on the basis of $A_c + nA_s$. The problem is solved by successive trial.

(IV) The values of f_c obtained in the assumption of a linear stress distribution for the compressive stresses in concrete, is corrected to the corresponding value from the actual curved stress-strain curve for concrete in compression.

B.—*Stresses in Plastic Stage:* The following hypothesis is used :—

When a load on a statically inderminate R.C. frame reaches such a stage as would make any section reach the first yield point of steel or first prismatic stress of concrete, the moment at this section ceases to increase, while the moment in the other sections continue to increase with the load until the stresses at all critical sections of maxima reach either the first yield stress of steel or first prismatic stress of concrete.

The determination of the stresses with the above hypothesis becomes accordingly very simple because the problem becomes a statically determinate one.

Experimental Verification

Fifteen frames were made, each having 2 × 3 metres exterior dimensions. All the frames had a constant cross-section 20 × 25 cms. for all the four members. The frames, however, differed in the amount of reinforcement in the tension and compression sides as shown in Figs. 9 to 11.

The frames were tested in a 500 ton Amsler machine under two concentrated loads acting as shown. The materials used were Portland cement, Pyramids sand and gravel and ordinary round mild steel bars.

Proportions of the mix used were 1 : 2 : 4 by weight and water/cement ratio = 0·5. Mixing was carried out in a mechanical mixer.

Control specimens gave the following results:

Average prism strength in compression at slow rate

$f_p = 240\ kg/cm^2$

Yield stress for steel used . . . . $f_y = 2800\ kg/cm^2$

Young's modulus . . . . $E_s = 2000\ t/cm^2$

Curves of Load—ε_c, load—ε_s for every member of the fifteen frames, and also load—f_c, load—f_s curves are given in the thesis (ε_c is the maximum strain at compression side of concrete and ε_s maximum strain in the tension steel).

Theoretical Computation

A.—*Factor of Safety:* The safe loads for all the frames computed according to the following assumptions are given in Table 1.

TABLE 1

Computed Safe Loads using $n = 15$, $f_c = 60$ kg./cm². and $f_s = 1200$ kg./cm². and Factors of Safety

A / B Frame	Ultimate load from tests in tons (P ult.)	Computed safe load in tons (P safe)	Governing factor for the safe load		Actual factor of Safety $\frac{P\ ult.}{P\ safe}$	Design factor of safety
C_1	7·50	1·95	f_s for Sec.	A	3·85	2·32
C_2	8·00	1·92	f_s	A	4·17	2·32
C_3	9·00	2·45	f_c	A	3·67	4·00
C_4	11·00	2·59	f_c	A	4·25	4·00
C_5	12·75	2·92	f_c	B	4·36	4·00
C_6	8·75	2·88	f_s	A	3·04	2·32
C_7	11·35	3·72	f_c	A	3·05	4·00
C_8	13·15	4·225	f_c	A	3·11	4·00
C_9	6·575	2·10	f_s	B	3·13	2·32
C_{10}	5·90	1·32	f_s	A	4·47	2·32
C_{11}	7·50	1·38	f_s	B	5·43	2·32
C_{12}	7·375	0·772	f_s	A	9·55	2·32
C_{13}	5·475	2·10	f_s	B	2·61	2·32
C_{14}	7·375	2·45	f_c	A	3·01	2·32
C_{15}	7·10	1·95	f_s	A	3·64	2·32

$n = 15$, $f_c = 60$ kg/cm², $f_s = 1200$ kg/cm² and I for the full area of concrete without consideration for the steel.

B.—*Stresses*: The stresses obtained by tests for the 15 frames were compared with those computed according to the following assumptions:

Assumption 1:—

(*a*) The relative values of moments of inertia were taken for the concrete section without steel, i.e. $I_A = I_B$ for our frame.

(*b*) When computing the stresses, concrete in tension was neglected with $n = 15$ according to specifications.

Assumption 2:—

(*a*) $I_A = I_B$ same as above.

(*b*) Concrete in tension was neglected but n is taken for actual values of E_s and E_c as given by the control specimens, i.e. $n = 2000/266 = 7 \cdot 5$

Assumption 3:—

Computation of stresses was divided into two stages:

Stage 1: Before cracking of concrete.

It was assumed that cracking occurred when the tensile stress in concrete f_t reached 20 kg/cm² and we assume:

(i) I of each section = I full concrete area + n I of steel.

(ii) n computed from actual E_s and E_c (i.e. $n = 7 \cdot 5$).

(iii) Youngs modulus for concrete in tension equals that in compression.

Stage 2: After cracks start to develop on the tension side when f_t exceeds 20 kg/cm², concrete in tension is abruptly neglected, the stage of transition in which concrete in tension acts on a part of the section is omitted. For this stage we assume:

(i) I of each section is taken for the virtual area consisting of $A = A_c + n\ A_s$

(ii) n = 7.5 same as above.

The computation in this stage is by trial and error.

(iii) As f_s reaches $f_y = 2800$ kg/cm², or f_c reaches $f_p = 240$ kg/cm² for either section A or B, (see plastic stage).

(iv) The values of f_c obtained by the ordinary computation according to assumption of linear stress distribution is converted to the actual value in the curved stress-strain distribution (Fig. 12).

As an example see Figs. 13 to 20 for frames C_3 and C_6 with the stresses computed according to the three assumptions compared with the actual stresses.

C.— *Ultimate and Critical Failure Loads:*

(1) *Ultimate Load:*

It was computed according to the following three assumptions and the results were compared with the actual load obtained by tests. (Table 2).

Assumption 1: The failure load is considered as the load at which the stresses in either section A or B reach their ultimate values of $f_y = 2800$ or $f_p = 240$ kg/cm². The values of f_y and f_p being computed according to assumption 3 for finding the stresses.

Assumption 2: According to the theory of limit design, the failure load is considered as the load at which the stresses in both sections A and B reach their ultimate values of $f_y = 2800$ and $f_p = 240$ kg/cm². The values of f_y and f_p being computed according to assumption 3 for finding stresses.

Assumption 3: The failure load is considered as the load which makes both sections A and B reach their ultimate capacities as in assumption 2 above, but the ultimate capacity of each section is computed by the plastic theory using the rectangular stress distribution for concrete and steel.

TABLE 2

P ULTIMATE ACCORDING TO ASSUMPTIONS 1, 2 and 3 COMPARED WITH P ULTIMATE FROM TESTS

Frame	P test in tons	P ultimate assump. 1 (P_1 in tons)	$\frac{P\ test}{P_1}$	P ultimate assump. 2 (P_2 in tons)	$\frac{P\ test}{P_2}$	P ultimate assump. 3 (P_3 in tons)	$\frac{P\ test}{P_3}$
C_1	7·500	5·280	1·42	7·350	1·02	7·980	0·94
C_2	8·000	5·175	1·54	7·360	1·09	9·010	0·89
C_3	9·000	7·390	1·22	8·470	1·06	9·170	0·98
C_4	11·000	8·790	1·25	9·400	1·17	12·100	0·91
C_5	12·750	9·160	1·39	9·800	1·30	14·800	0·86
C_6	8·750	7·430	1·21	9·260	0·94	9·570	0·912
C_7	11·350	9·770	1·16	10·900	1·04	13·410	0·84
C_8	13·150	11·600	1·13	12·470	1·05	17·650	0·74
C_9	6·575	6·034	1·09	6·300	1·04	6·750	0·97
C_{10}	5·900	3·780	1·56	5·850	1·01	6·600	0·89
C_{11}	7·500	6·350	1·18	6·650	1·13	7·590	0·99
C_{12}	7·375	2·692	3·65(*)	5·700	1·29	7·250	1·01
C_{13}	5·475	4·886	1·12	5·465	1·00	5·920	0·92
C_{14}	7·375	6·481	1·14	7·370	1·00	7·860	0·937
C_{15}	7·100	5·185	1·37	6·880	1·03	7·71	0·92
		Average	1·28		1·08		0·914

(*) Excluded in average

(2) *Critical Failure Load:*

It was considered as the load which makes both sections A and B reach their critical failure capacities. The results of computation are given in table 3.

TABLE 3

CRITICAL FAILURE LOAD COMPARED WITH P ULTIMATE FROM TESTS

Frame	P ult. from test (in tons)	Critical failure load (in tons)	$\frac{P \text{ test}}{P \text{ critical}}$
C_1	7·500	7·48	1·00
C_2	8·00	7·48	1·07
C_3	9·00	8·95	1·00
C_4	11·00	12·00	0·92
C_5	12·750	14·18	0·90
C_6	8·750	9·14	0·96
C_7	11·350	12·82	0·88
C_8	13·150	17·15	0·77
C_9	6·575	6·33	1·04
C_{10}	5·900	6·25	0·94
C_{11}	7·500	7·40	1·01
C_{12}	7·375	7·30	1·01
C_{13}	5·475	5·56	0·98
C_{14}	7·375	7·48	0·98
C_{15}	7·100	7·42	0·96
		Average ...	0·962

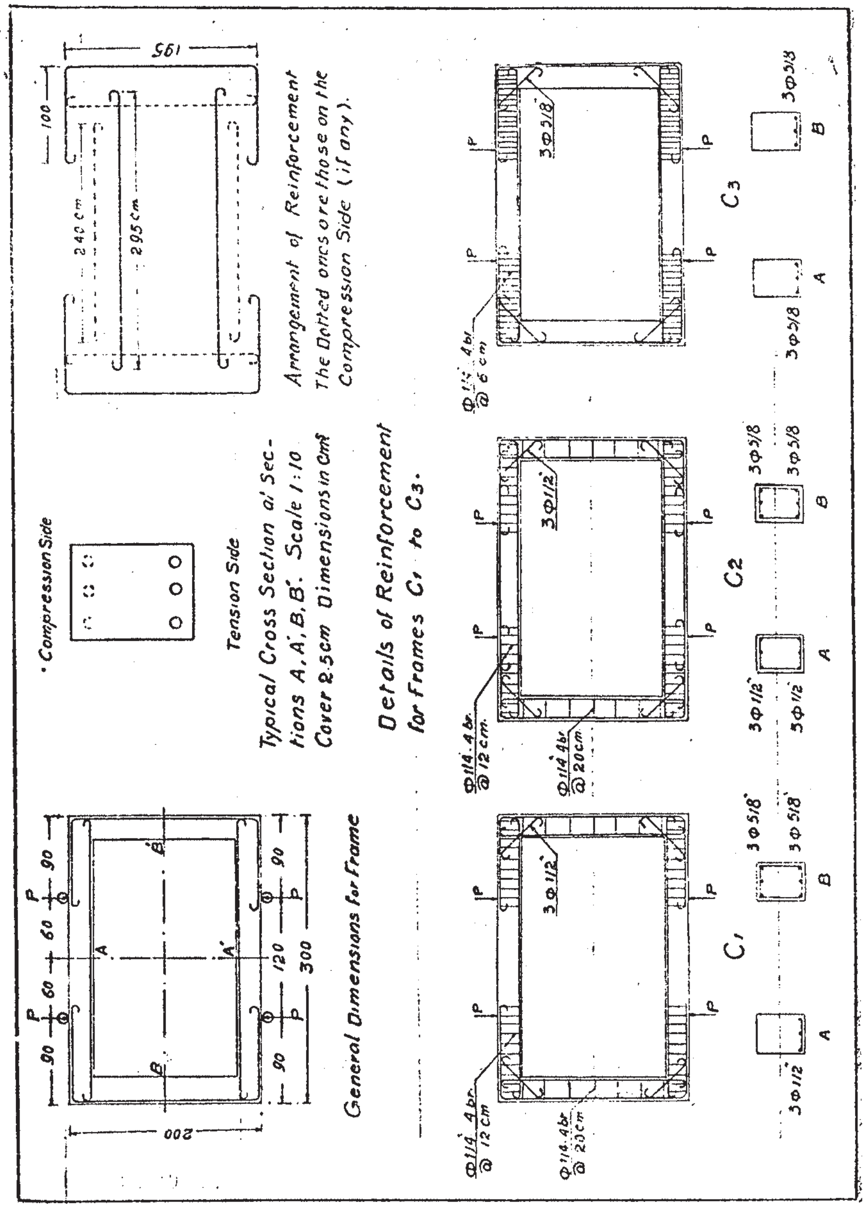

Compression Side
Tension Side
Typical Cross Section at Sections A, A', B, B'. Scale 1:10
Cover 2.5cm Dimensions in Cms
Arrangement of Reinforcement
The Dotted ones are those on the Compression Side (if any).
General Dimensions for Frame
Details of Reinforcement for Frames C1 to C3.
C1
C2
C3

Fig. 9

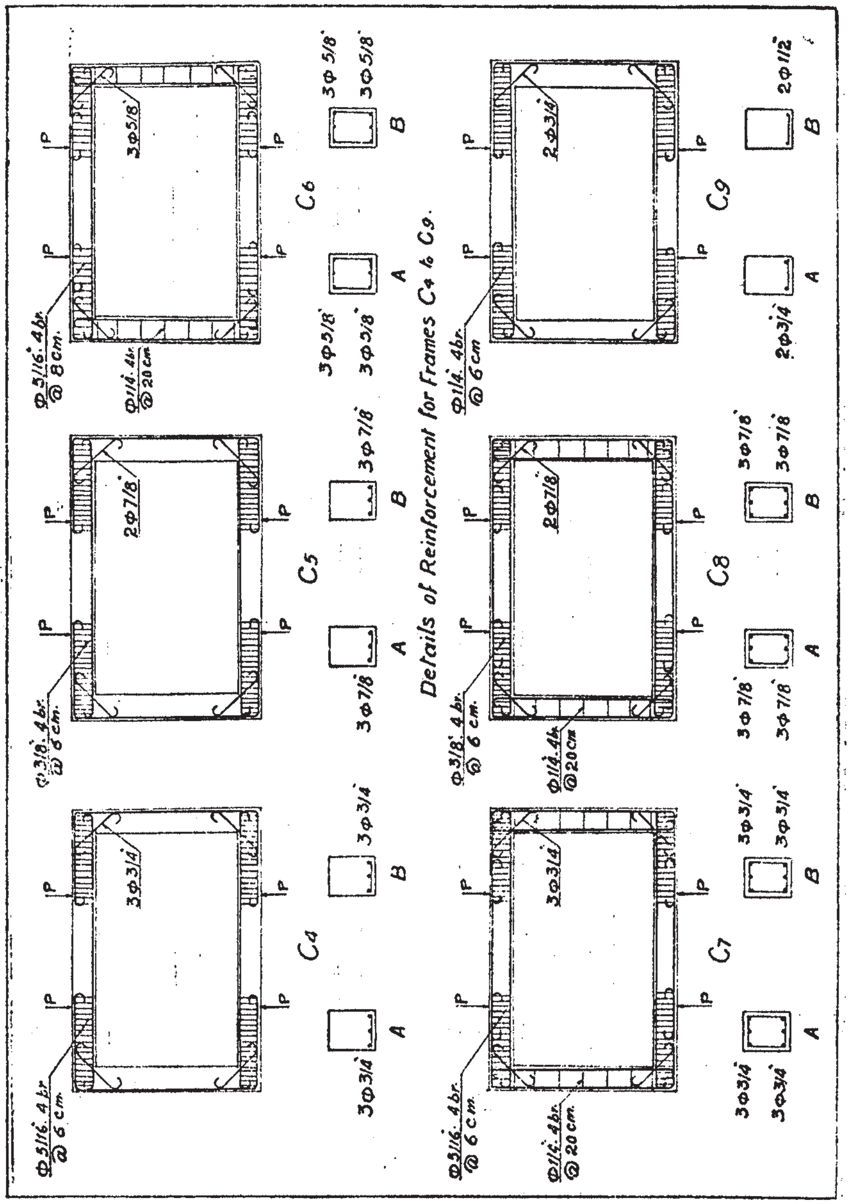

Details of Reinforcement for Frames C4 to C9.
C4
C5
C6
C7
C8
C9
A
B
P
3φ3/4"
3φ7/8"
3φ5/8"
2φ7/8"
2φ3/4"
2φ1/2"
φ5/16" 4 br. @ 6 cm.
φ3/8" 4 br. @ 6 cm.
φ5/16" 4 br. @ 8 cm.
φ1/4" 4 br. @ 20 cm.
φ1/4" 4 br. @ 6 cm

Fig. 10

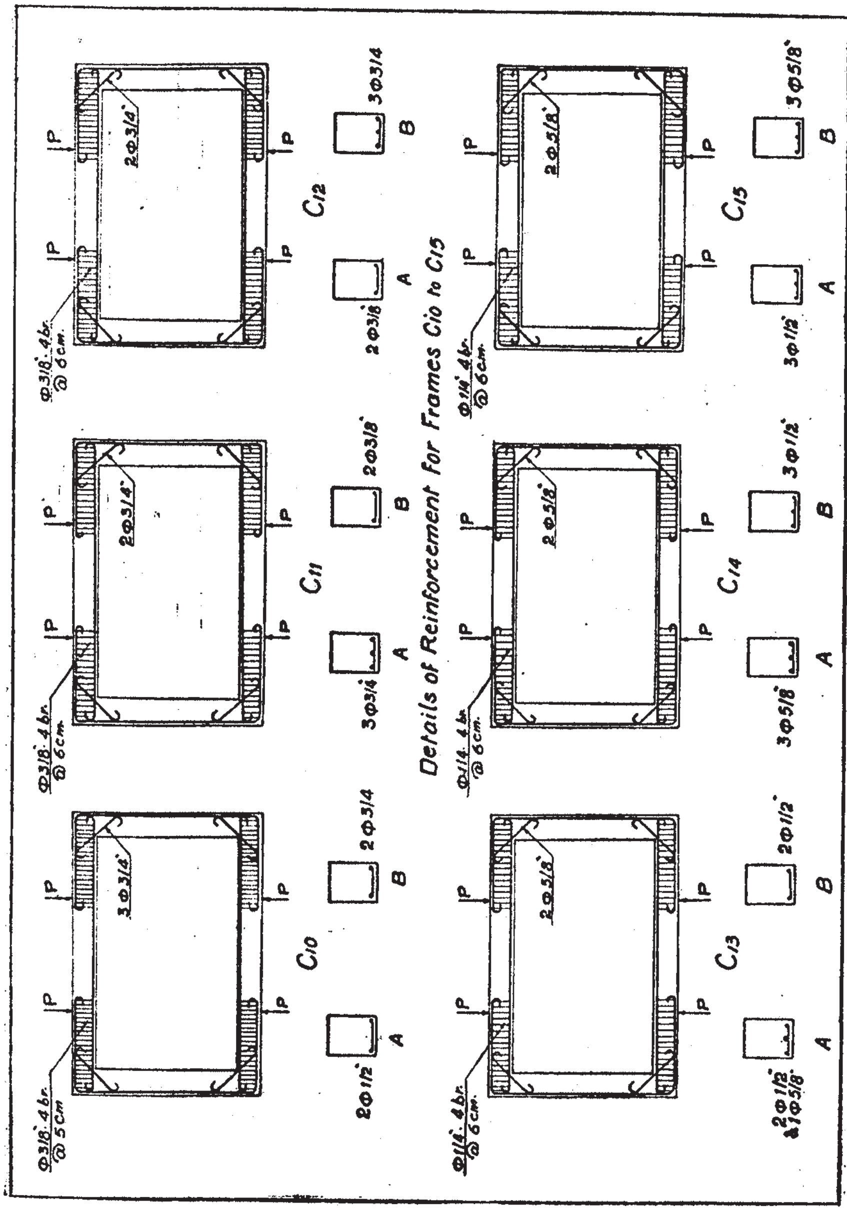

Details of Reinforcement for Frames C10 to C15
C10
C11
C12
C13
C14
C15
P
A
B
Φ3/8" 4 br. @ 5 cm
Φ3/8" 4 br. @ 6 cm.
Φ1/4" 4 br. @ 6 cm.
3 Φ3/4"
2 Φ3/4"
2 Φ5/8"
2 Φ1/2"
2 Φ3/4
3 Φ3/4
2 Φ3/8
2 Φ3/8"
3 Φ3/4
3 Φ5/8"
3 Φ1/2"
3 Φ5/8"
2 Φ1/2" & 1 Φ5/8"

Fig. 11

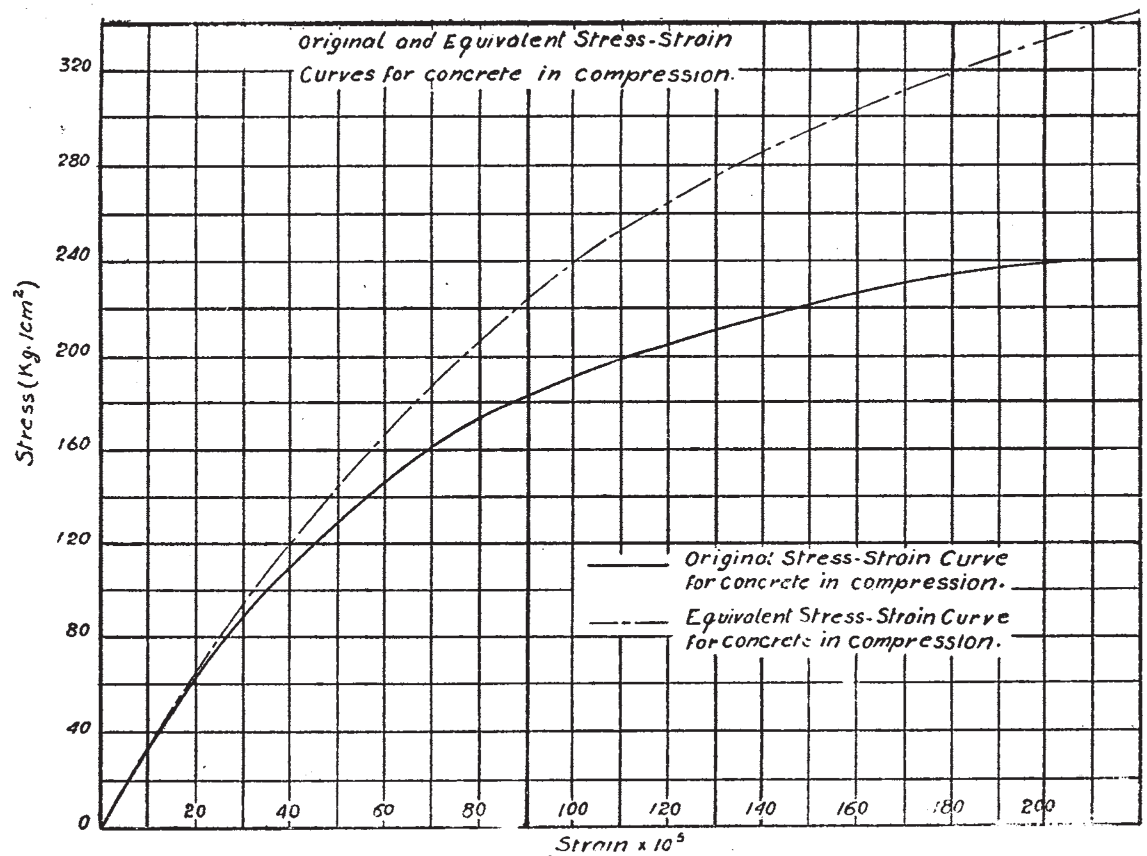
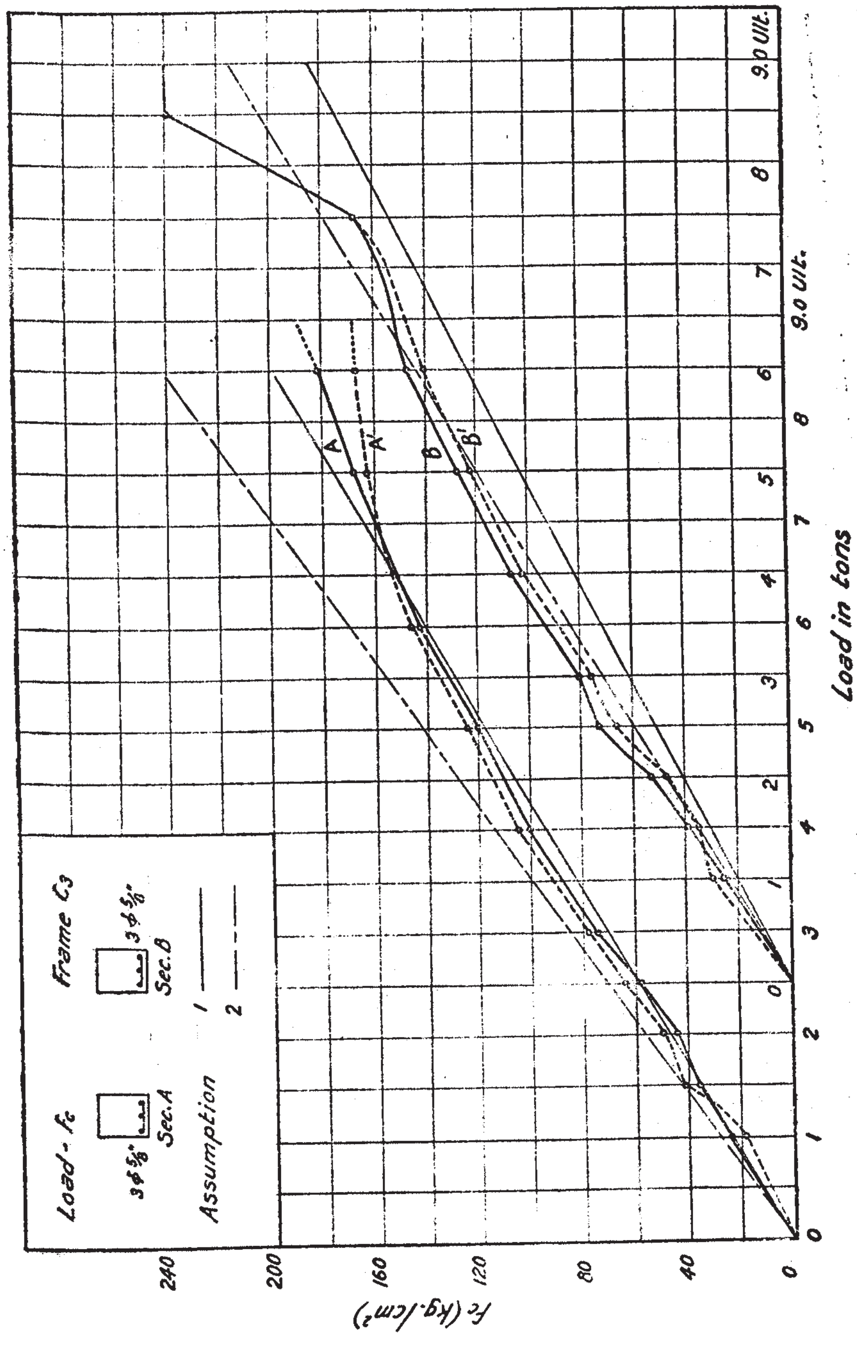

Original and Equivalent Stress-Strain
Curves for concrete in compression.
Original Stress-Strain Curve
for concrete in compression.
Equivalent Stress-Strain Curve
for concrete in compression.
Stress (Kg./cm²)
Strain x 10⁵
0
40
80
120
160
200
240
280
320
20
60
100
140
180

Fig. 12



Load - fc
Frame C3
3φ5/8"
Sec.A
Sec.B
Assumption
1
2
A
A'
B
B'
9.0 Ult.
9.0 Ult.
0
40
80
120
160
200
240
fc (kg./cm²)
0 1 2 3 4 5 6 7 8
0 1 2 3 4 5 6 7 8
Load in tons

Fig. 13

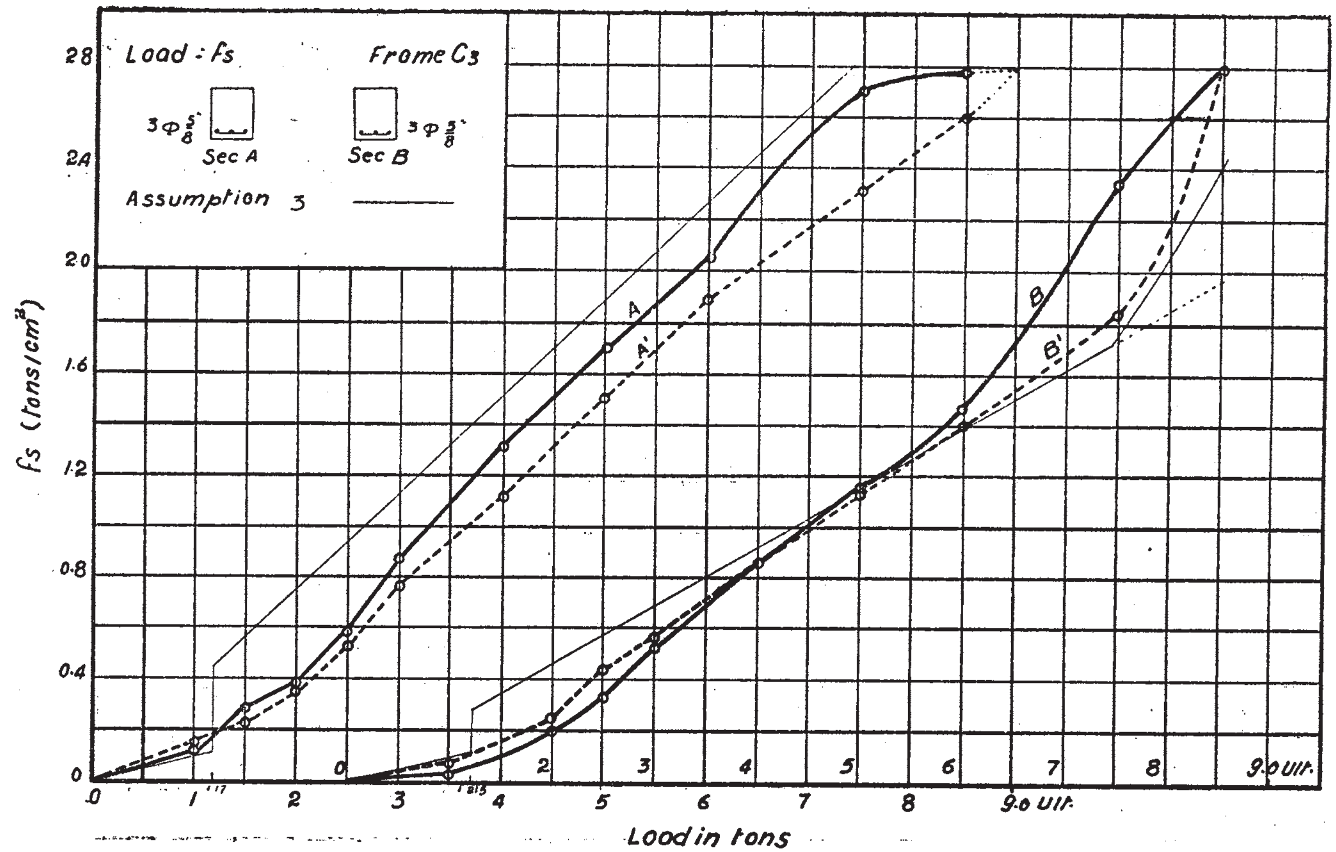

Load : fs
Frame C3
3 Φ 5/8"
Sec A
Sec B
3 Φ 5/8"
Assumption 3
fs (tons/cm²)
2.8
2.4
2.0
1.6
1.2
0.8
0.4
0
A
A'
B
B'
0
1
2
3
4
5
6
7
8
9.0 Ult.
Load in tons

Fig. 14

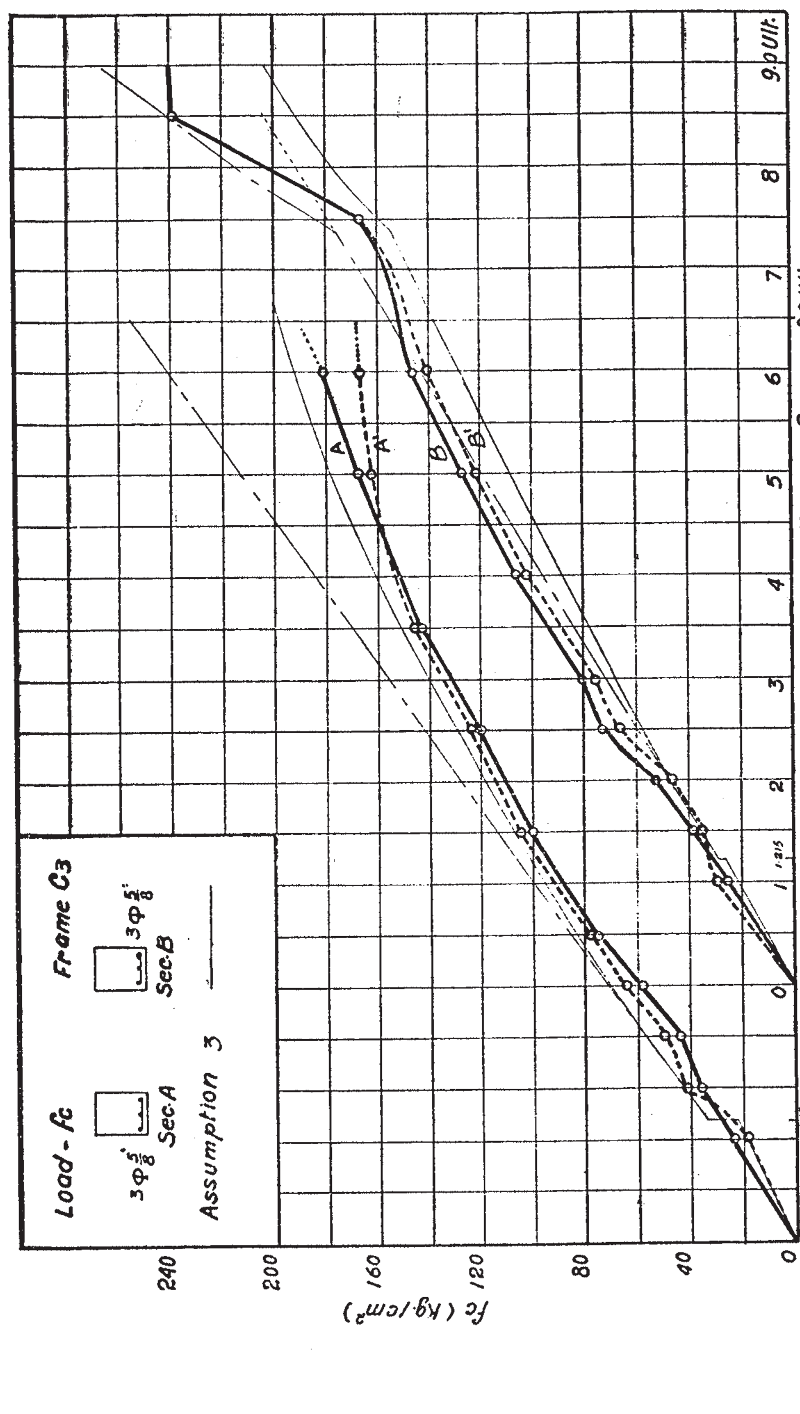

Load - fc
Frame C3
3φ5/8"
Sec.A
3φ5/8"
Sec.B
Assumption 3
A
A'
B
B'
9.0 Ult.
9.0 Ult.
0
40
80
120
160
200
240
fc (kg/cm²)
0
1
2
0
1
2
3
4
5
6
7
8
1
2
3
4
5
6
7
8
Load in tons

Fig. 15

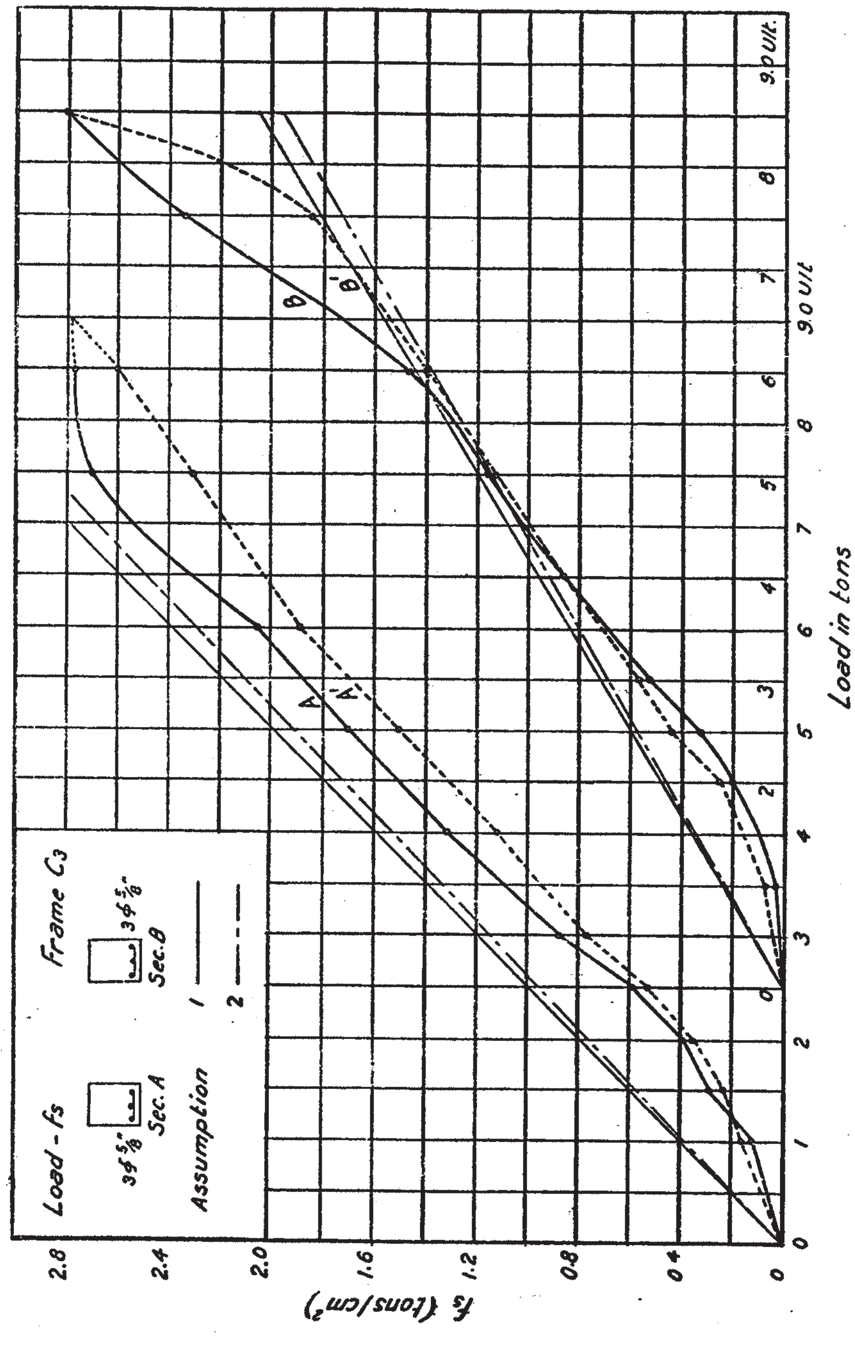

Load - fs
Frame C3
3φ5/8" Sec.A
3φ5/8" Sec.B
Assumption
1
2
A
A'
B
B'
9.0 Ult.
9.0 Ult
0
1
2
3
4
5
6
7
8
Load in tons
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
fs (tons/cm²)

Fig. 16

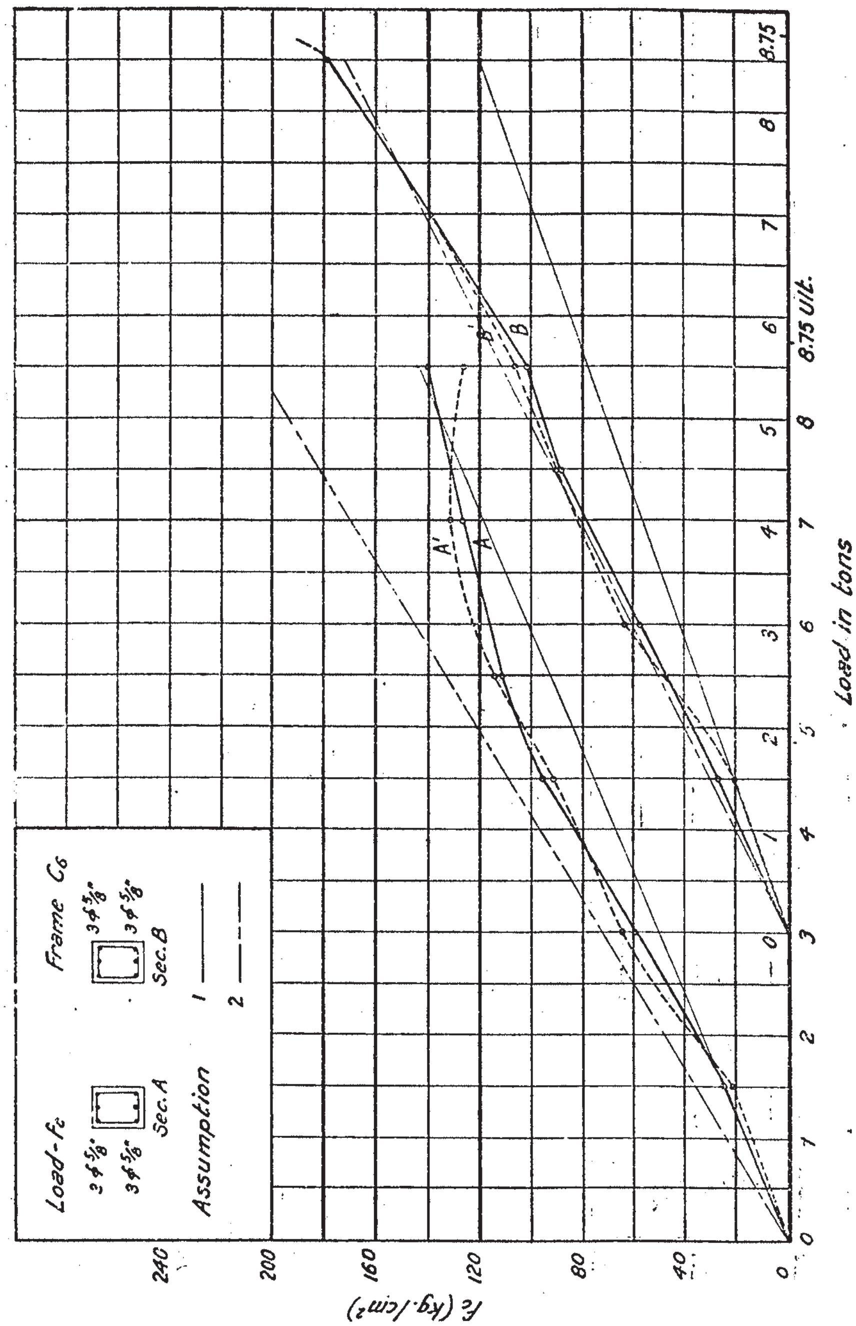

Load-fc
Frame C6
3φ5/8"
3φ5/8"
3φ5/8"
3φ5/8"
Sec.A
Sec.B
Assumption
1
2
A
A'
B
B'
8.75 Ult.
8.75
fc (kg./cm²)
240
200
160
120
80
40
0
Load in tons

Fig. 17

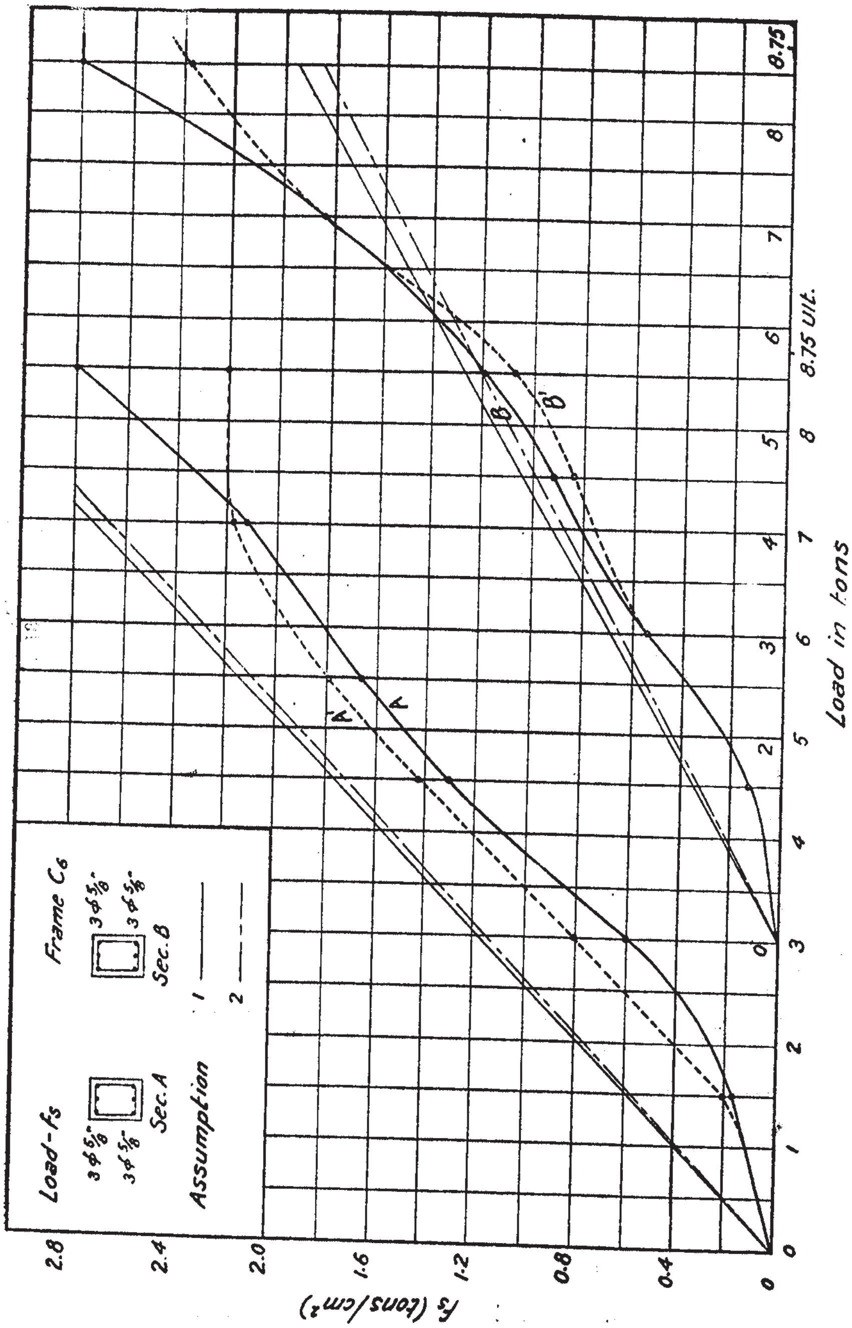

Load-fs
Frame C6
3φ5/8"
3φ5/8"
Sec. A
3φ5/8"
3φ5/8"
Sec. B
Assumption
1
2
A
A'
B
B'
8.75
8.75 ult.
fs (tons/cm²)
0
0.4
0.8
1.2
1.6
2.0
2.4
2.8
0
1
2
3
4
5
6
7
8
0
1
2
3
4
5
6
7
8
Load in tons

Fig. 10

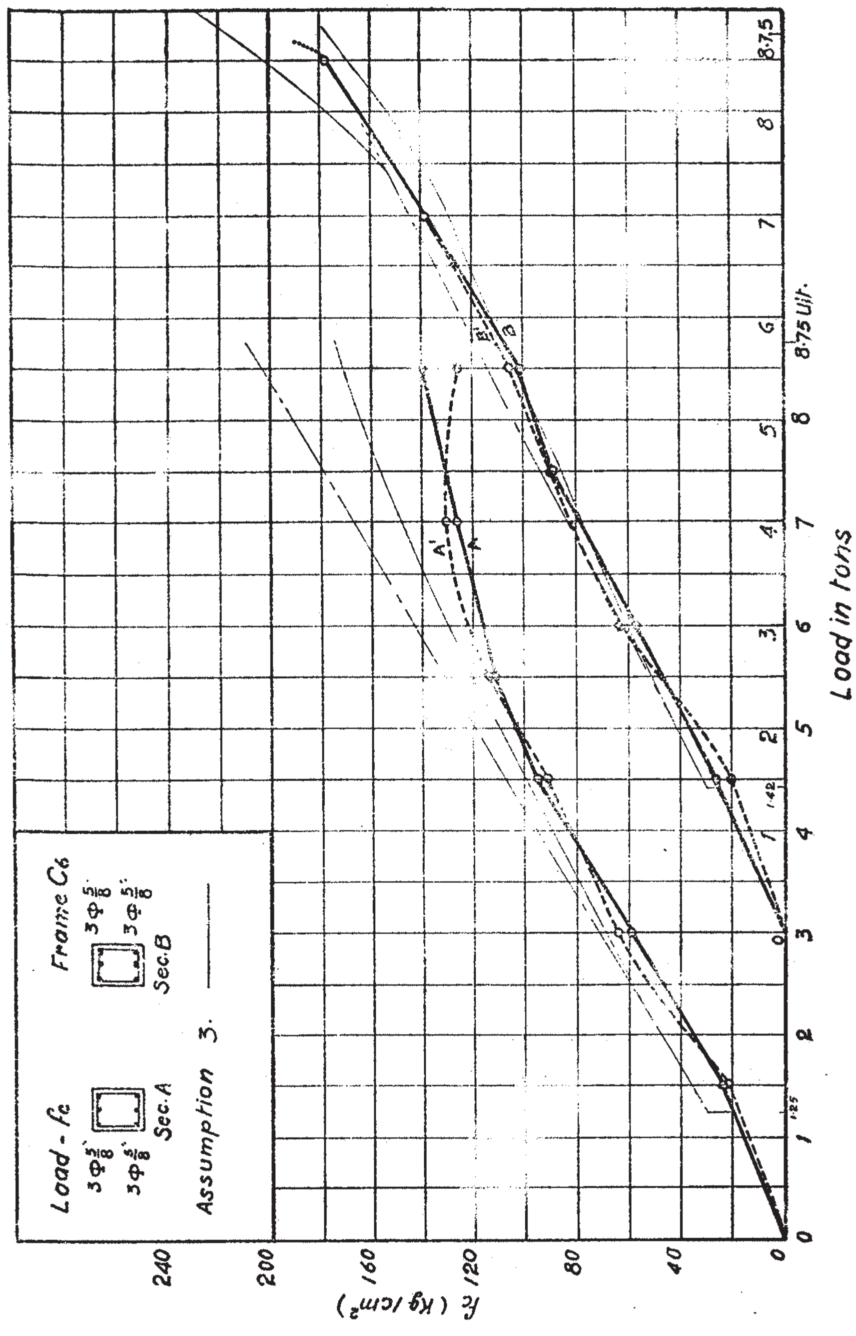

Load - fc
Frame C6
Sec. A
Sec. B
Assumption 3.
fc (kg/cm²)
Load in tons
8.75 Ult.

Fig. 19

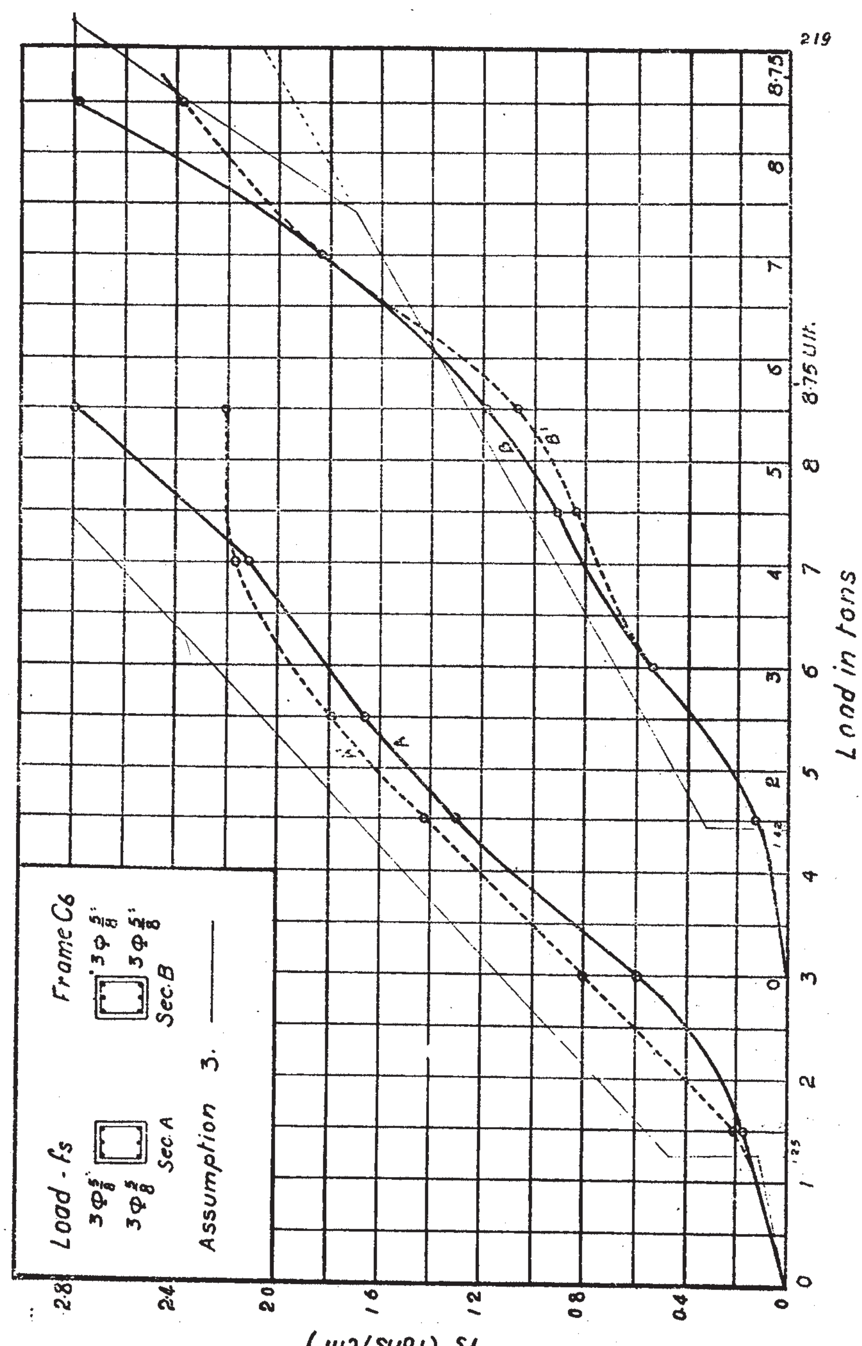

Load - fs
Frame C6
3Φ5/8"
3Φ5/8"
Sec. A
Sec. B
Assumption 3.
A
B'
8.75 ult.
219
Load in tons
fs (tons/cm²)

Fig. 20

CONCLUSIONS

As far as the once statically indeterminate reinforced concrete frames tested in that research are concerned, the following conclusion can be made :

1. The use of the standard theory of reinforced concrete design, results in a wide discrepancy between the computed and actual stresses. The difference becomes salient if the relative ratios of steel in the members were not in conformity with the requirements of the elastic theory.

The use of the appropriate value of n obtained from the properties of the control specimens, leads to some improvement in the computed stresses, but not for all the stages of loading from zero up to failure load.

2. Good agreement between the computed and actual stresses for all stages from zero up to failure load could be obtained by the use of more accurate assumptions for computation of the relative moments of inertia as well as the stresses, than the rough assumptions of the standard theory as shown below :

(*a*) It has been shown that the consideration of the relative moments of inertia, on the basis of the effective virtual section, leads to better results than when assuming the full concrete section.

(*b*) regarding the stresses, better results could be obtained if:

(i) the action of concrete in tension is taken into consideration, particularly before the maximum tensile stress of concrete is exceeded.

(ii) the proper value of n is introduced.

(iii) when one member reaches its yield stress or prism strength, the stresses in the other members are computed on the assumption that the moment in the first member remains constant, while the straining actions in the other member increase gradually.

(iv) the computed values of fc should be reduced to their corresponding values of the curved stress-strain curve for concrete in compression.

The comparison between the computed stresses according to the above amended assumptions and the test results, was satisfactory.

3. The use of the standard theory and the conception of the safe load results in a wide variation in the factor of safety against failure, and a wide difference between the design and obtained factors of safety.

4. The use of compression reinforcement of percentages as high as 2·32 % in the different members did not result in any noticeable increase in the ultimate load. In some cases, even the ultimate load with compression reinforcement was smaller than without compression reinforcement.

The effect of the compression reinforcement on the stresses, was as follows:

(*a*) Negligible reduction in f_s.

(*b*) Small reduction in f_c which sometimes attained about 15 %.

Thus the actual value of compression reinforcement for such frames in which the maximum percentage of tensile steel was 2·32 % is in reducing the deformations and deflections, but not as an addition to safety against failure.

5. The conception that a statically indeterminate frame fails when just one section reaches its maximum capacity, proved to be incorrect. As far as the start of wide cracking in concrete is concerned, however, this first ultimate load gives the best limit.

6. The theory of limit design gives the ultimate capacity of the frame to a satisfactory degree of accuracy. The ultimate load is arrived at when the maximum capacity of each member is reached. Even the use of a rough method for the determination of the ultimate capacity of reinforced concrete members, based on rectangular stress distribution for concrete stresses together with

the theory of limit design, gave quick and satisfactory results for the ultimate capacity of the frames

7. It has been generally found that better agreement between the computed and actual ultimate capacities of the frames could be obtained if we were to use a slightly reduced prism strength than the actual value obtained from the control specimens.

8. The critical failure load is that at which all critical sections of maxima have reached either the yield stress of steel or the prism strength of concrete. It is limited by a strain of 140×10^{-5} for steel and 105×10^{-5} for concrete, at which strains of plastic deformations of excessive values start to occur in the frame, leading to its ultimate failure. It is believed that the conception of critical failure load gives a better criterion of failure than the actual ultimate capacity of the frames. The use of a value for the prism strength of concrete of about 90 % of the test value, would lead to better results for the computed critical failure loads.

9. A wide variation in the ratio of reinforcement in the different members proved to affect the ultimate strength only slightly.

10. A design based on the principal that all critical sections of maxima reach their critical failure loads at the same time, would give the same factor of safety against excessive deformations as well as against wide cracking. The ultimate crushing load of the frame would be slightly higher, but in good proportion with the critical failure load.

11. It, therefore, appears reasonable to make the recommendation that the design should be made on the basis of the critical failure load together with the theory of limit design rather than on the conception of a safe stress as is usually assumed in the standard theory.

12. The theory of limit design is merely hypothetical.

The stresses in the " plastic stage " have also been computed on a hypothetic basis. Experimental work has confirmed, to a fairly good degree of accuracy, the hypothetic assumptions made. This, however, does not dispense with a sound theoretical basis.

The writer believes that an extension of the theory of least work or virtual work would give a good basis for computation of the stresses in a reinforced concrete statically indeterminate frame prior to, and at failure.

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