A VOLTAGE-TRANSFORMER NETWORK ANALYSER (*)

BY

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1.—Summary

The paper deals with the use of voltage transformers as multipliers for scalar and complex quantities and describes how they may be arranged to form the analysing system called "The Voltage-Transformer Analogue". The two-, three- and multi-winding voltage-transformer analogues are developed and it is shown how they are used to solve electrical, mathematical and structural problems.

The accuracy of the analyser depends upon the supression of amplitude and phase errors in the voltage-ratios of the transformers. Each transformer may be supplied with the required exciting current from special compensators, in which case the accuracy of operation of the analyser may be as high as having the minimum of about 98 %, or if we are interested to have a minimum cost of the analyser, we can dispense with these compensators, in which case the accuracy of operation may fall down to about 95 %.

The paper concludes with the description of the construction and method of operation and some experimental results obtained by an uncompansated analyser constructed by the author and having about 95% accuracy of operation and of a much cheaper cost than the two existing voltage-transformer analysers.

(*) Based on a lecture given by the author at the Faculty of Engineering, Giza, on April 11, 1953.

2.—Introduction

Engineers associated with electrical power supply realise the importance of knowing beforehand the exact operating characteristics of a projected power scheme. Even more impo: tant perhaps is to be able to predict with accuracy the effect of an alteration to an existing system, whether it relates to an extension, an interconnection, or an increase in generating plant capacity. Apart from the aspects of technical efficiency and safety, the economics of any new proposal must also be considered ; how to make the best use of available plant, and how the completed system will react to abnormal conditions.

The answers to these problems can be obtained by normal mathematical methods, but the calculations needed to determine the best of many possible arrangements—even in simple cases—can become so protracted and laborious that an accurate solution is often impracticable. Such difficulties have led to the use of various aids to solution, of which the most widely used have hitherto been model networks. Most of these network analysers have consisted of variable resistors, inductors, capacitors and model generators which can be adjusted to represent a particular network.

The second type of network analysers is the analogues-model type using voltage-transformers which does not represent the actual system in a miniature form but rather solves, by analogy, mathematical equations created by the original system. For many problems this analyser has district advantages over the conventional type using variable-impedance elements. Negative resistors and resistance-free inductors can be represented by standard connections between transformers, whilst mutual-inductors, network transformers and other coupling effects may be represented by special connections between transformers. When the analyser is in use, there are four separate networks and the voltages in these correspond to the inphase and quadrature components of the voltages and also of the *currents* in the network under investigation. This feature makes it possible to apply various special restraints which are not possible with a model network, *e.g.* those required to represent a salient-pole synchronous machine by representing its vector diagram.

3.—General Theory of Voltage-Transformer Analogues (V.T.As.)

The V.T.A. method of analysis makes use of the well-known transformation properties of a voltage-transformer to build a valtage-transformer system, the interconnections between the different windings of which are made to realise the governing equations of the system considered.

Depending on the number of windings of the voltage-transformer, we have the two-, three- and multi-winding V.T.As, each of which has a particular field in engineering applications as shown below:

(3.1.) The Two-Winding V.T.A.

(3.1.1) Solution of resistance networks (1) (Figs. 1–9):

Fig. 1 shows a simple resistance circuit with the voltage V applied to it in which case $V = IR$.

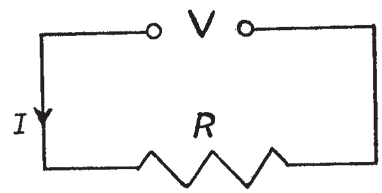


Fig. 1.—V = IR (Resistance)

Fig. 2. gives the general transformer analogue. With the shown turns ratio and indicated voltages and currents, it can be proved that $V_V/V_I = V/I$ and $I_V/I_I = V/I$ *i.e.*, by the transformer analogue, values of V and I of Fig. 1 are obtained to a certain scale. Fig. 2 may be split into Fig. 3 giving the current and voltage transformer analogues of Fig. 1.

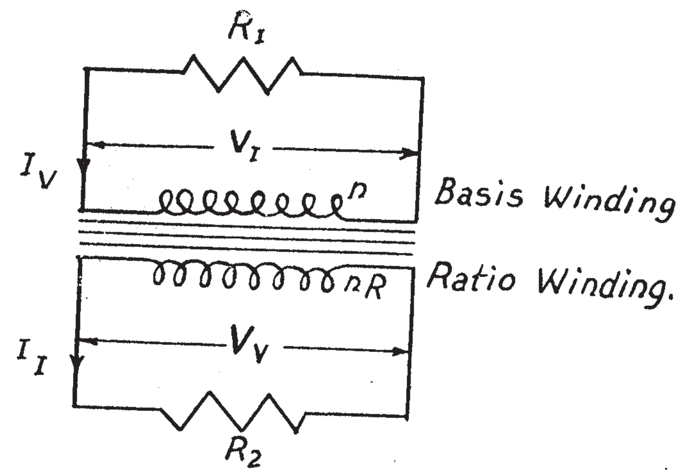


Fig. 2.—Transformer analogue of Fig. 1

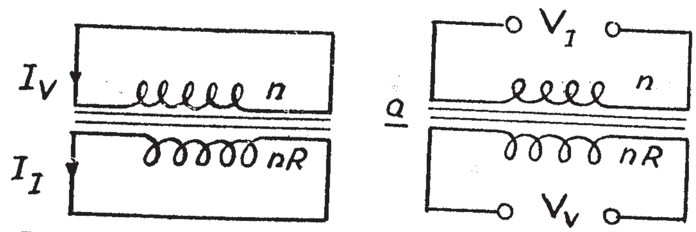


Fig. 3.—Current and (potential) voltage transformer analogues of Fig. 1

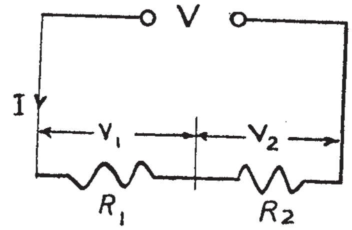


Fig. 4.—Two series resistances

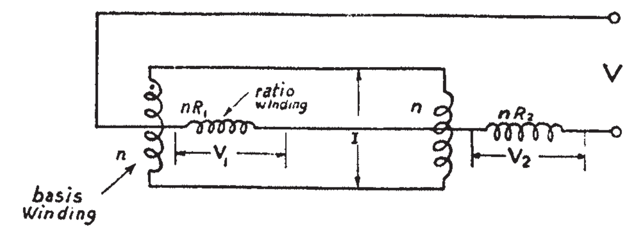


Fig. 5.—V.T.A. of Fig. 4

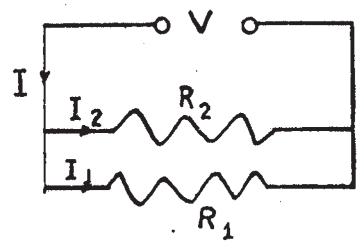


Fig. 6.—Two parallel resistances

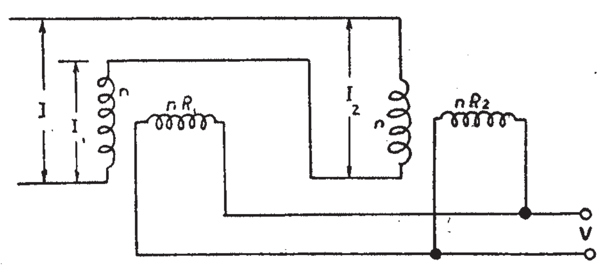


Fig. 7 —V.T.A of Fig. 6

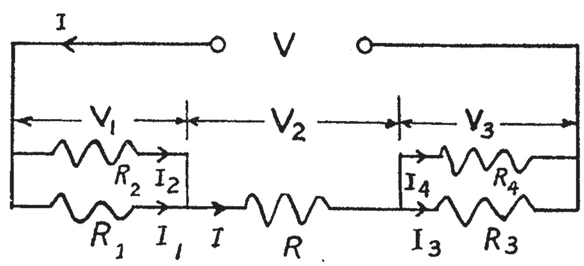


Fig. 8.—Series-parallel resistive network

V.T.A. means Potential transformer analogue.

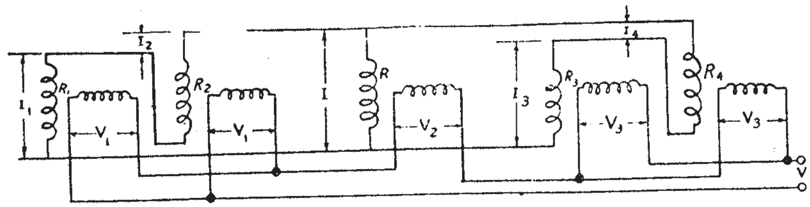


Fig. 9.—V.T.A. of Fig. 8

The 2-winding V.T.A. may be extended to solve two resistances in series as shown in Figs. 4 and 5, where each resistance is represented by a 2-winding voltage-transformer with turns ratio equal to the resistance considered. The interconnections between the transformer windings are made to obey Ohm's and Kirchhoff's laws applied to the problem, *e.g.* for this case: $V_1 = I_1 R_1$, $V_2 = I_2 R_2$, $I_1 = I_2 = I$ and $V = V_1 + V_2$.

Similarly Figs. 7 and 9 give the 2-winding V.T.A. of the resistance circuits of Figs. 6 and 8 respectively.

(*3.1.2.*) *Solution of 2-dimensional structures under steady-state loading (without and with member deflection)* [2, 3]:

Since, for this particular problem, and referring to Table 1 we have the sum of horizontal components of stress in members

TABLE 1

STEADY-STATE DETERMINATE STRUCTURE
(ACTUAL AND ANALOGOUS QUANTITIES)

In actual structure	In voltage-transformer analogue
Basic relationships	
At panel-point: $\Sigma F_x = 0$	In closed mesh: $\Sigma V_x = 0$
$\Sigma F_y = 0$	$\Sigma V_y = 0$
Corresponding quantities	
Stress distribution	Voltage distribution

meeting at a panel point is zero, and this is analogous to the fact that the sum of voltages existing in a closed mesh is zero, *i.e.* a closed mesh in the electrical analogue could be made to represent a panel point in the structure and the voltages appearing at the appropriate terminals of the closed mesh will represent the stress components of the corresponding panel point. It follows that, for two-dimensional structures, we should have two networks the X-network having a number of closed meshes equal to the panel points of the structure and each closed mesh has voltages equal to the number of members meeting at the corresponding panel point in the structure and a Y-network identical to the X-network as far as the number of meshes and the number of voltages of each mesh are concerned.

In the X-network, voltages representing horizontal stress components of vertical members are to be shorted and similarly in the Y-network voltages representing vertical stress components of horizontal members are shorted. This is because the specified stress components are zero.

In both networks, voltages representing the stress components of an inclined member are to be coupled with a voltage-transformer having turns ratio (equal to the number of turns of the winding connected to the Y-network/number of turns of the winding connected to the X-network) equal to vertical stress component/horizontal stress component, *i.e.* equal to the inclination of the inclined member to the vertical. By this means the voltage ratio of the specified transformer will be equal to the stress ratio and the analogy will be complete.

A load may be represented by an alternating voltage applied where required in both networks.

This is illustrated for a simple problem (without deflection) in Figs. 10 and 11—and for the struss of Fig. 12 in Fig. 14.

Fig. 13 gives a schematic diagram explaining the method of representation just outlined.

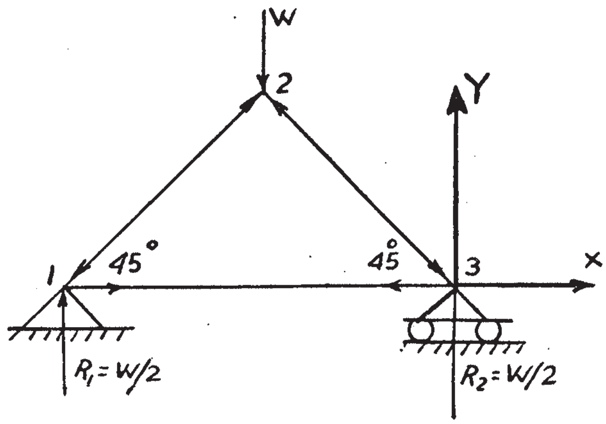


Fig. 10.—Example

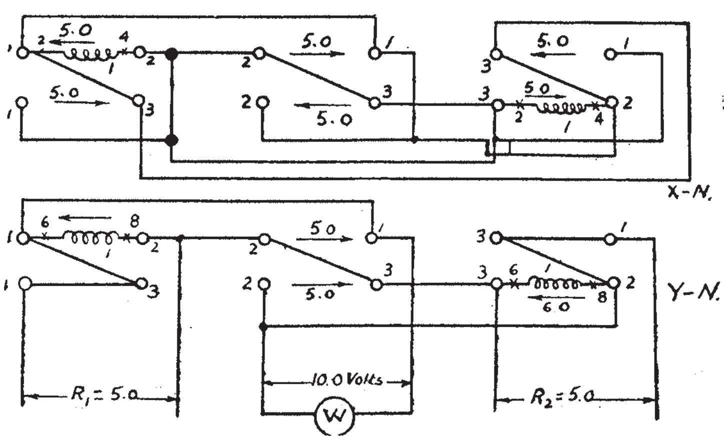


Fig. 11.—Experimental set up of example of Fig. 10

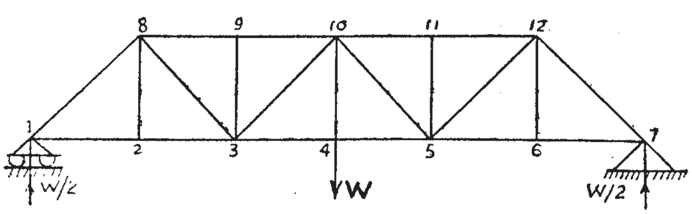


Fig. 12.—Two-dimensional truss with central loading

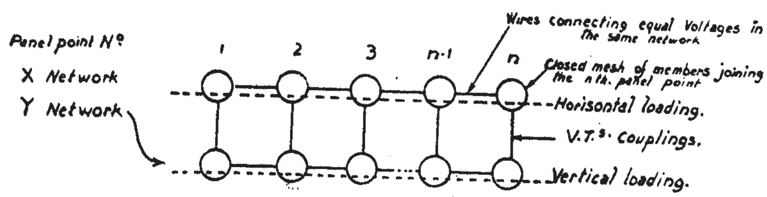
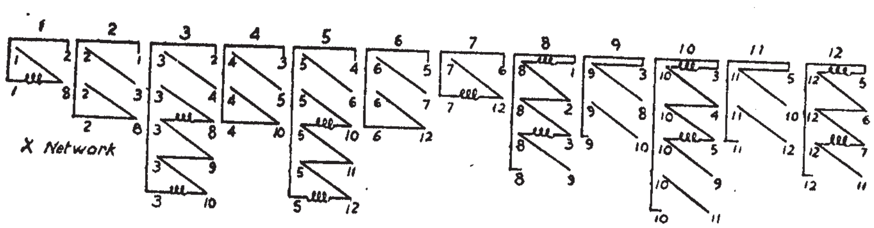


Fig. 13—New method of representing two-dimensional structure with n panel points by P.T.A. using a panel-point method of representation



Important : Coils shown with the same numbers belong to the same transformer Equal voltages with the same numbers must be connected together in the same network : 1/1 V.T.s may be used to avoid S.Cs.

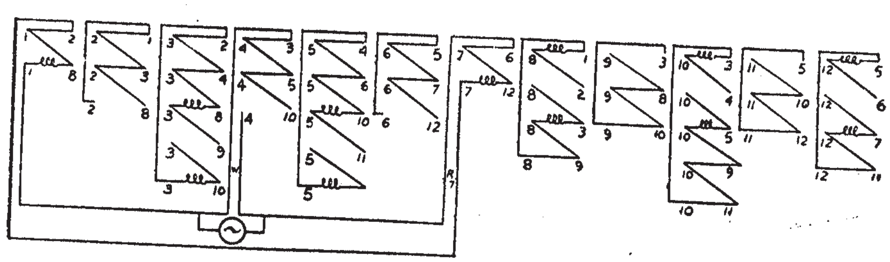


Fig. 14.—V.T.A. of truss Fig. 12 under steady state using the new panel-point method of representation

Taking *member deflections* into consideration and defining compliance r_{mn} of the member joining panel points m, n as the change in length of this member per unit axial stress, then Table 2 gives the basic relationships and the corresponding analogous quantities of a two-dimensional structure with member deflection.

TABLE 2

STEADY-STATE INDETERMINATE STRUCTURE (ACTUAL AND ANALOGOUS QUANTITIES)

In actual structure	In voltage-transformer analogue
Basic relationships	
Potential energy stored in a member $= F^2 r$.	Power dissipated in a network element $= V^2/R$.
Distribution of horizontal stresses is such as to make $\delta \Sigma F^2_x r = 0$.	Distribution of voltages in horizontal network is such as to make $\delta \Sigma V^2_x/R = 0$.
Distribution of vertical stresses is such as to make $\delta \Sigma F^2_y r = 0$.	Distribution of voltages in vertical network is such as to make $\delta \Sigma V^2_y/R = 0$.
Distribution of stress in the structure is such as to make: $\delta \Sigma (F^2_x + F^2_y) r = 0$. *i.e.* $\delta \Sigma F^2 r = 0$.	Distribution of voltages in analogous network is such as to make: $\delta \Sigma (V^2_x + V^2_x)/R = 0$. *i.e.* $\delta \Sigma V^2/R = 0$.
Corresponding quantities	
Stress distribution.	Voltage distribution.
Deflection of a member:	V/R through the corresponding;
Horizontal deflection $= F_x r$.	Horizontal network $= V_x/R$.
Vertical deflection $= F_y r$.	Vertical network $= V_y/R$.
Total deflection $= Fr$.	Corresponding current $= V/R$.
1/Compliance $= 1/r$	Resistance $= R$.

An illustrative example for this type of problem is shown in Fig. 15 with its 2-winding V.T.A. in Fig. 16. It may be noted that also here we have the X- and Y-networks. The resistances r shown are inversely proportional to the compliances of the members specified. Transformation ratios of voltage transformers are as stated before.

In this representation stress components are represented by the voltages in the corresponding network and the components of the deflections of members are represented by the currents flowing through the corresponding resistances in the corresponding network.

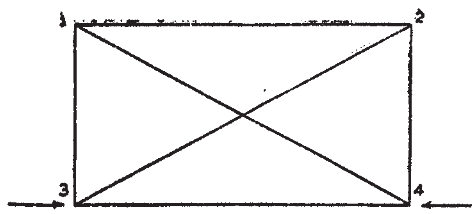
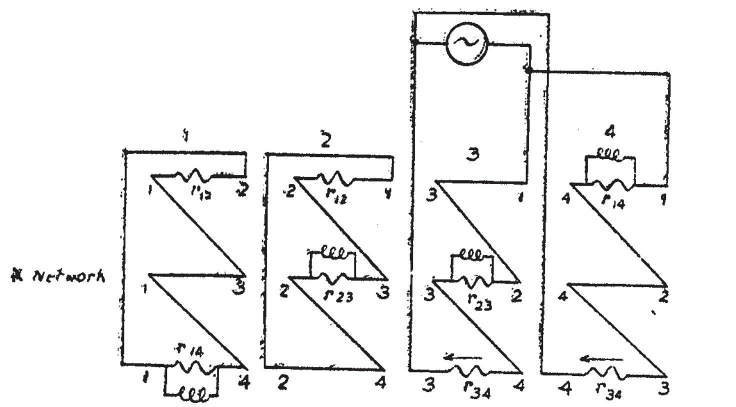


Fig. 15.—A two-dimensional indeterminate framework



Important: Coils shown with the same numbers belong to the same transformer. Equal valtages in the same network with the same numbers must be paralleled together

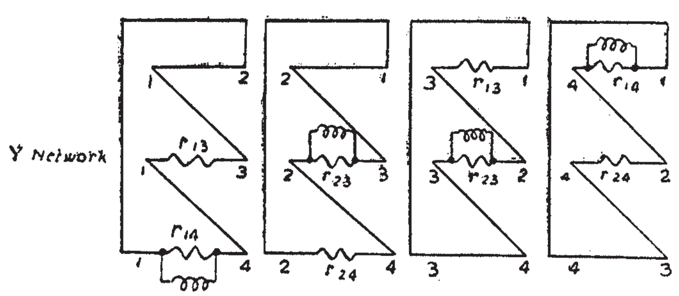


Fig. 16.---V.T.A. of Fig. 15 in steady state using the new panel-point method of representation; 1/1 V.Tsmay be used to avoid short circuits

It may be noted that transformers here are not strictly voltage transformers since currents representing components of member deflections are allowed to flow, in which case either these transformers are (power) transformers or this current is limited within the wire capacity of the voltage-transformer, and this is possible in practice since the scale of resistances (r) and supply voltage are chosen and not imposed by the problem.

Measurements are to be made by a high input impedance valve voltmeter in order not to disturb the system and a sensitive current meter.

(3.1.3) Solution of two-dimensional structures under Transient loading (without and with friction) [2,3] **:**

With reference to the symbols given in Table 3 for this particular problem and loading, the governing equations of the structure are :

$$F = vf + M\frac{dv}{dt} \quad \text{and}$$

$$l = r\,f$$

TABLE 3

VOLTAGE-TRANSFORMER ANALOGUE FOR TRANSIENT LOADING
(ACTUAL AND ANALOGOUS QUANTITIES)

In the structure		In the potential transformer analogue	
Mass	M	Capacitance	C
Compliance	r	Inductance	L
Velocity	v	Voltage on the R-L-C circuit	V
Displacement	l		$\int V\,dt$
Stress	F	Current through a coil = $C\,dV/dt$ + $RCV/L = \int Vdt/L = i$	
Friction	f		RC/L.

(Friction considered)

The two analogous equations of the electrical network are :

$$\int \frac{vdt}{L} = V\frac{Rc}{L} + C\frac{dV}{dt} \quad \text{and}$$

$$\int Vdt = VRC + CL\frac{dV}{dt}$$ hence the analogous quantities of Table 3 follow.

In this representation also we have two networks. In each network, condensers represent masses of members which are considered to be concentrated at the panel points. An inductor inserted in between each two condensers representing two panel points interconnected by a member has a value proportional to the compliance of the specified member. The transformation ratios are such that (referring to Fig. 17 and its 2-winding V.T.A. of Fig. 19):

$$\frac{N_B}{N_A} = \cos\theta \text{ in the X-network and}$$

$$N_E/N_D = \sin\theta \text{ ,, ,, Y- ,,}$$

By doing so and having the interconnection shown, it can be proved that $\int V dt$ flowing through L_{23} is proportional to the net

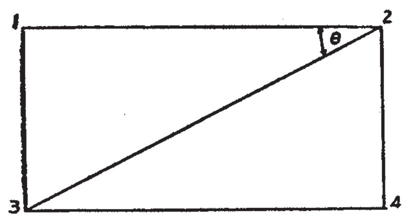


Fig. 17.—A 2 dimensional structural framework

Note:

The following diagrams (Figs. 18 and 19) are drawn ignoring frictional forces; if such forces are considered then: for C.T.A. a resistance proportional to 'f' is to be inserted in series with every L. For V.T.A. a resistance proportional to fL/C is to be connected in series with every C.

change in length of member 2-3 owing to the horizontal and vertical deflections of both points 2 and 3 acting simultaneously.

A sudden horizontal application of load between panel points 3 and 4, say, is equivalent, in the V.T.A., to a sudden application of a d.c. source between the condensers C_3 and C_4 in the X-network.

Fig 18 gives the Transformer analogue of the problem-using current transformers.

Measurements in the V.T.A. are to be made by:

A high input impedance valve-voltmeter to measure voltage V representing velocity of displacement.

A sensitive current meter to measure the current i representing stress

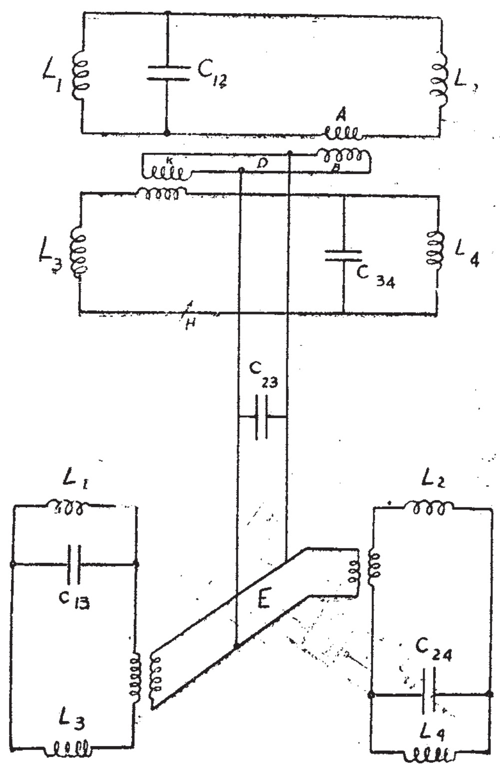


Fig. 18.—C.T.A. of Fig. 17

X networks

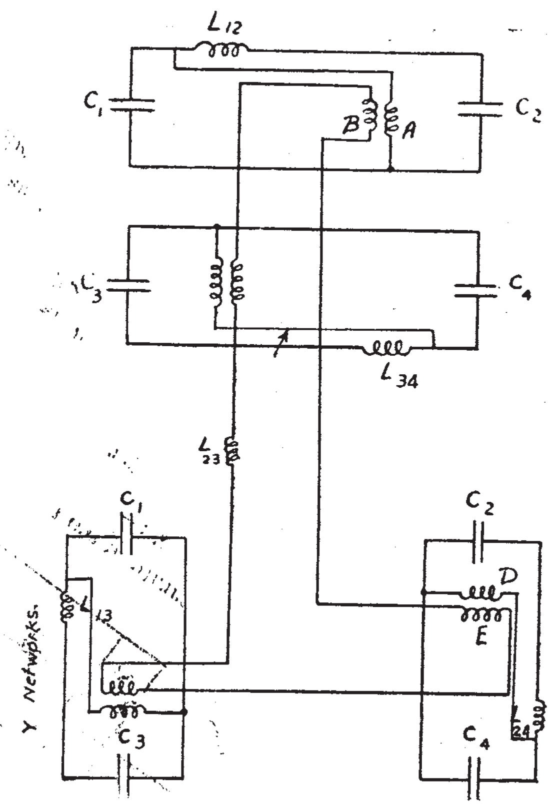


Fig. 19. -V.T.A. of Fig. 17

A cathode ray tube to give $\frac{dv}{dt}$ and hence $\int v \, d t$ may be obtained representing the displacement.

Table 3 and Fig. 19 give the corresponding quantities and the V.T.A. with friction. If this is ignored the frictional force f in Table 3 will be zero and the additional resistance Rc/L mentioned in Fig. 17 will be also zero.

(3.1.4.) Solution of mentioned stuctural problems in m-dimensions [4] *:*

The procedure just outlined for two dimensions with the two networks X and Y may by extended to m dimensions with m networks and n panel points. This is indicated in Figs. 20 and 21.

Fig. 20 gives the schematic way of interconnections for steady-state problems. We notice the existence of m networks and n panel points.

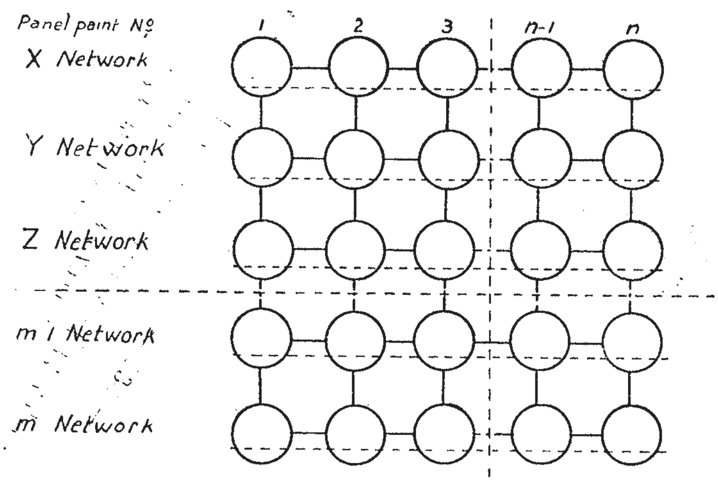


Fig. 20.—Solution of m-dimensional structure with n panel points in steady state using the new panel-point method of representation

Fig. 21 gives the same case but with transient loading, each rectangle represents a network

In both figures the horizontal dotted lines represent the darticular type of loading and the vertical solid lines represent the transformer couplings. In Fig. 20 each circle is a closed mesh representing a panel point.

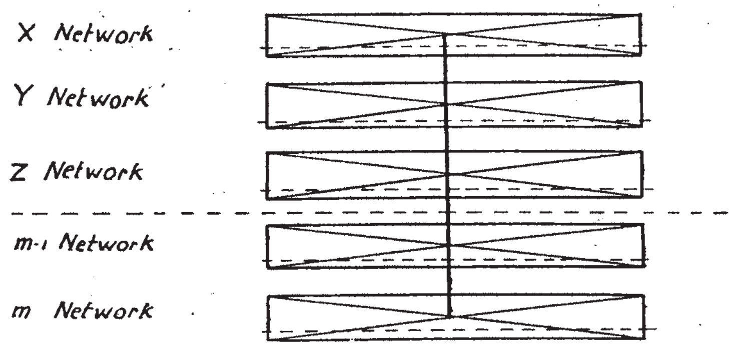


Fig. 21.—Solution of m-dimensional structure (with n members) in transient state using the new member method of representation

(3.1.5) Scalar mathematical calculations:

Simple mathematical calculations may be made easily as in Fig. 22 (multiplication and division) and in Fig. 23 (multiplication, division, addition and subtraction) using V.TS.

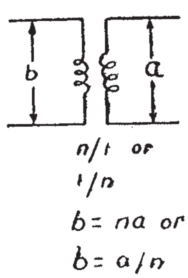


Fig 22—Scalar multiplication and division

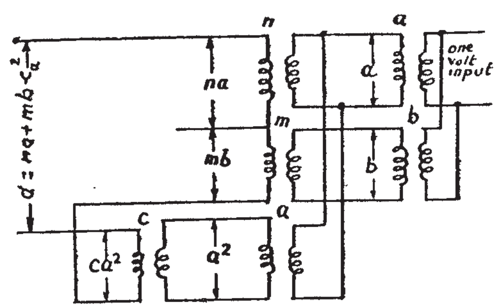


Fig. 23.—General calculations involving scalar quantities

(3.2) The Three-Winding V.T.A.

(3.2.1) Solution of a.c. power networks (1,5):

With reference to Figs 24 and 25 it could be seen that the governing equations of voltage and current applied to the impedance shown in Fig. 24 and written on Fig. 25 can be represented using the two three-winding voltage-transformers with turns ratios equal to R and X, the impedance elements, in the manner shown. Hence if V_p and V_q the inphase and

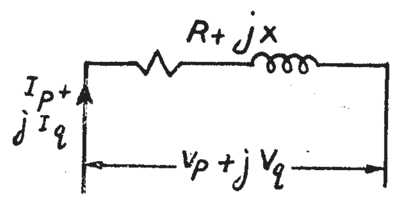
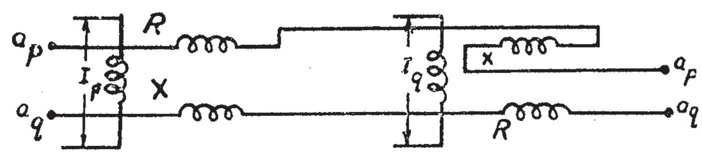


Fig. 24.—Single impedance Circuit

Fig. 25.—V.T.A. of Fig. 24

Vap ap=Ip R—Iq X=Vp

Vaq aq=Ip X+Iq R=Vq



quadrature voltages are applied at the terminals a_pa_p and a_qa_q of Fig. 25 respectively, then the current components I_p and I_q will appear at the shown terminal indicating that Fig. 25 is the three-winding V.T.A. of Fig. 24.

Similarly Figs. 27 and 29 are the three-winding V.T.A.s of the a.c. power networks shown in Figs. 26 and 28 and accordingly

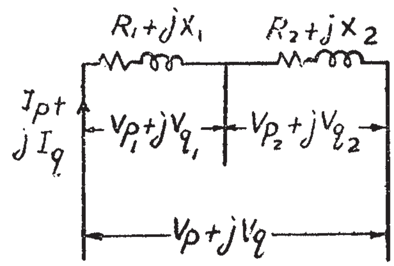


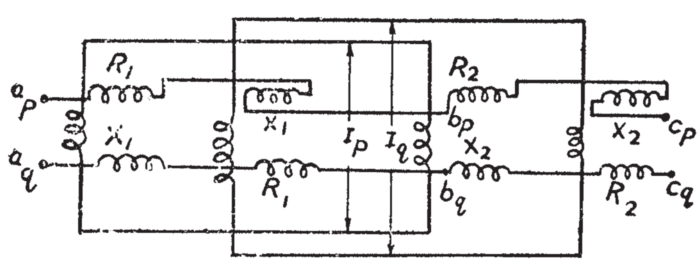
Fig 26.—Series impedance circuit

Fig. 27.—V T. A. of Fig. 26

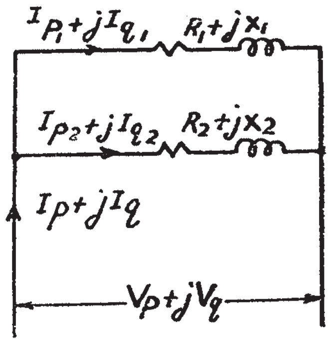
Vap bp=Vp1, Vaq bq=Vq1

Vbp cp=Vp2, Vbq cq = Vq2

Vap cp=Vp, Vaq cq = Vq

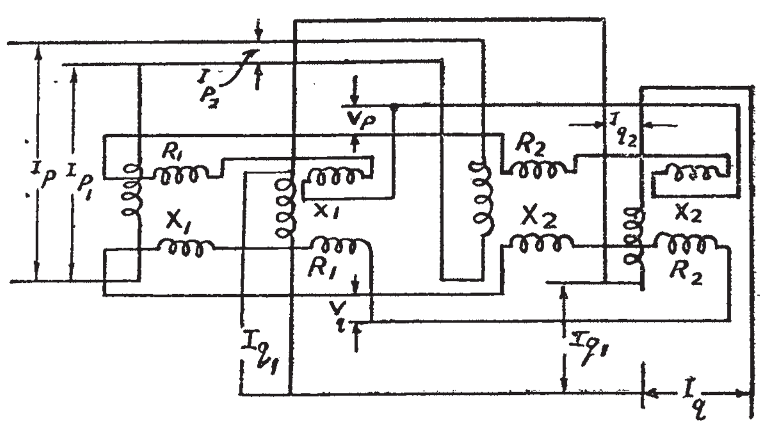


any impedance power circuit can be set up and hence analysed by the three-winding V.T.A. in which each impedance is represented by two three-winding voltage-transformers having the interconnections between the different windings made to realise the governing equations of the circuit. It may be noted



← Fig. 28.—Parallel Impedance circuit

Fig. 29.—V.T.A. of Fig. 28



that we have four separate circuits being V_p, V_q, I and I_q. It is impotant to notice that :—

(i) Negative resistances are represented as shown in Fig. 25 with the connections to the R-windings reversed.

(ii) Condensers are also represented as shown in Fig. 25 with the connections to the X-windings reversed.

(iii) Resistance-free inductor is represented as shown in Fig. 25 with the R-windings having zero ratios.

(3.2.2) Power system analysis (6) :

Many power system problems can thus be set up by the 3-winding V.T.A. method of analysis and studied thoroughly only if its equivalent circuit is known. Amongst these problems which have actually been studied by the 3-winding V.T.A. are the following :—

(i) Solution of faulted networks under symmetrical short circuits by setting up the V.T.A. of the equivalent single-phase circuit with fault.

(ii) Solution of faulted networks under unsymmetrical short circuits by setting up on the V.T.A. the equivalent single phase positive, negative- and zero -sequence networks with the corresponding necessary connections made between them as imposed by the presence of the unsymmetrical fault.

(iii) Steady state power stability studies under different types of short circuits by setting up the V.T.A. of the equivalent positive-sequence network only with the other two sequence networks reduced to their equivalent impedances and their combination shunted to the positive—sequence network at the points of fault.

(iv) Transient state power stability studies under different types of short circuits by setting up the V.T.A. as for steady-state stability but using the transient state impedances and using the step-by-step method of computation *i.e.* taking the results obtained at the end of one interval and applying them at the beginning of the second interval and so on.

(v) Load-flow studies by setting the V.T.A. of the power system considered.

(vi) Power transformers, mutual inductors and other coupling effects by setting the V.T.As of their equivalent circuits.

(3.2.3) Electrical plant and machinery problems (6):

Amongst such problems which may be solved by the V.T.A.s are :

(i) Induction motor circle diagram by setting the V.T.A. of its equivalent circuit.

(ii) Long transmission line characteristics by setting the V.T.A. to represent its governing equations (Fig. 30-a). It may be noted that the governing equations,

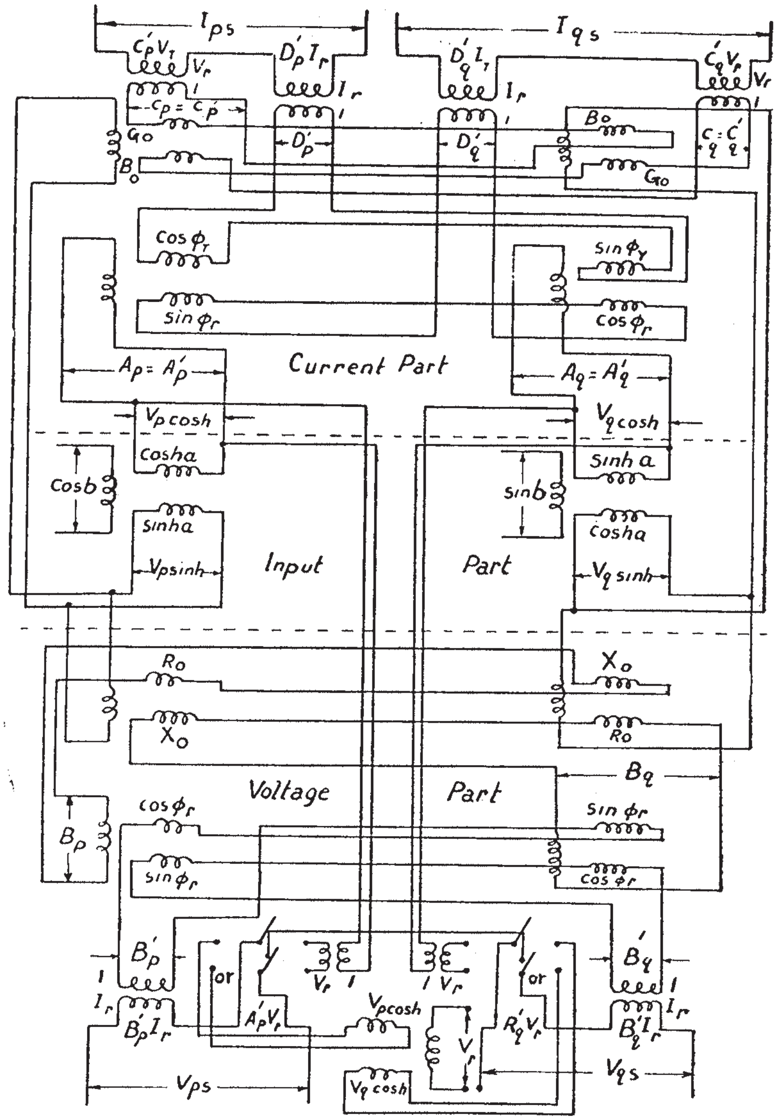


Fig. 30a.—A new representation of a long transmission line by the V.T.A.

set up on the V.T.A., are as follows, using the normal symbols:

Sending end voltage $= V_s = V_{ps} + JV_{qs}$

$= V_r\ (V_p \cosh + JV_q \cosh) + I_r\ (\cos \Phi_r + J \sin \Phi_r)\ (R_o + JX_o)\ (V\ \sinh + JV_q \sinh)$

$= (A'_p\ V_r + B'_p\ I_r) + J\ (A'_q\ V_r + B'_q\ I_r)$

and Sending end current $= I_s = I_{ps} + JI_{qs}$

$= I_r\ (\cos \Phi + J \sin \Phi_r)\ (V_p \cosh + JV_q \cosh) + V_r\ (V_p \sinh + JV_q \sinh)\ (G_o - JB_o)$

$= (C'_p\ V_r + D'_p\ I_r) + J\ (C'_q\ V_r + D'_q\ I_r)$

Where:

$$\text{Cosh}\ \theta = V_p \cosh + J\ V_q \cosh$$
$$= \cos b \cosh a + J \sin b \sinh a$$
$$\text{and Sinh}\ \theta = V_p \sinh + J\ V_q \sinh$$
$$= \cos b \sinh a + J \sin b \cosh a.$$

From the V.T.A, of Fig 30-a sending end conditions say, may be be found for :

Different receiving end conditions for one fixed line.

Fixed receiving end conditions for variable lines.

(iii) Syn chronous machine characteristics by setting the V.T.As of their vector diagrams as shown in Fig. 30-b to 36 for the non-salient-pole machine and in Figs. 37-40 for a salient pole machine.

It may be noted for both cases that the solution is not fully automatic unless the unknown networks (U.N.) indicated are obtained. The function of these unknown networks is to obtain an output voltage having the relation of the magnetisation curve to the input voltage or the opposite. The S.P.S. are static-phase shifters whose function is to shift the quadrature component of

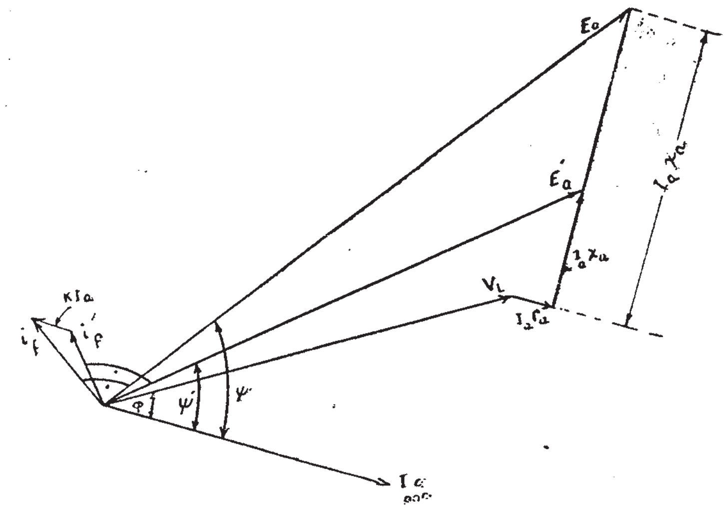


Fig. 30*b*.—Typical vector diagram of a non-salient-pole synchronous generator with a lagging p. f load

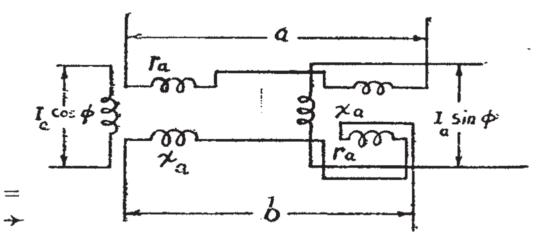


Fig. 31.—First step : $E_a' = V_t + a + jb$.

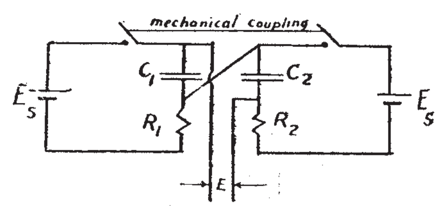


Fig. 32.—Second step : $i'f_1$ = time taken until E equals E'_a

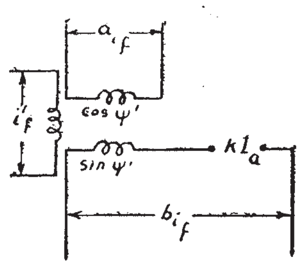
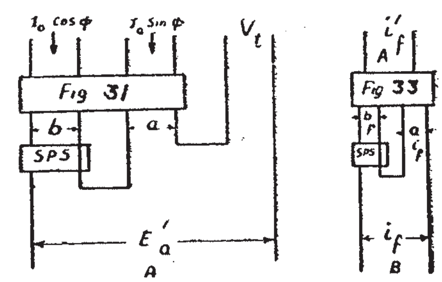


Fig. 33.—Third step : if is equal to ai f + j bi f



Sps : static phase-shifter

Figs. 34 and 35.—Semi-automatic method for non-salient-pole syn. machines. Use is to be made of Fig. 32 to get i'f from E'_a and E_a from if (V T.A. method)

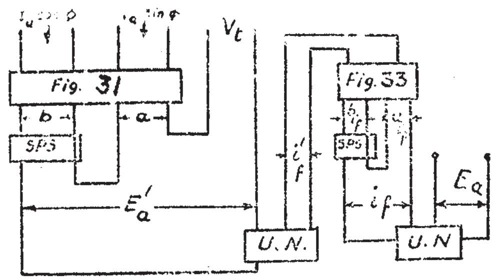


Fig. 36.—Fully automatic method for non-salient-pole syn. machines (V.T.A. method) U.N.: Unknown network

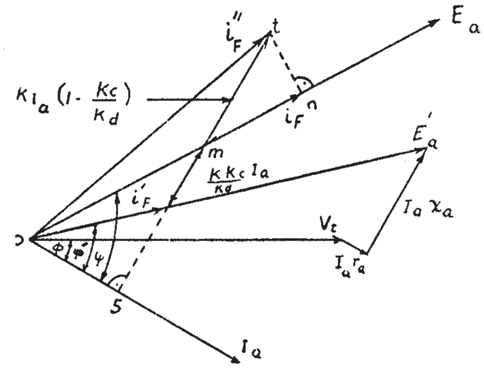
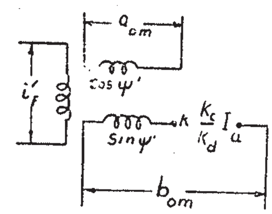


Fig. 37.—Typical vector diagram of salient-pole synch. generator with lagging p f load



First step is shown in Fig. 31

Fig. 38.—Second step ; om is equal to $a_{om} + j\ b_{om}$

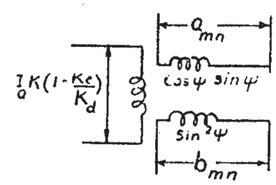


Fig. 39.—Third step ; mn is equal to $a_{mn} + j\ b_{mn}$→

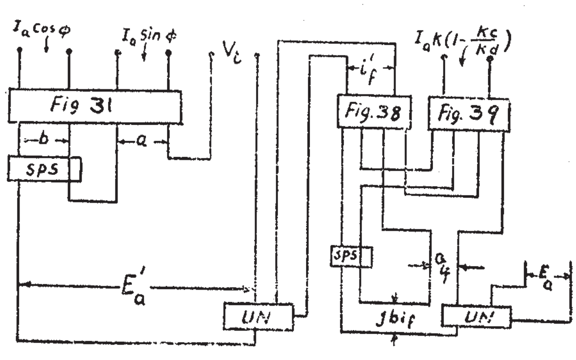


Fig. 40.—Fully-automatic method for solving salient-pole synchronous machines by V.T.A.

voltage by 90° electrical degrees keeping its magnitude constant so that when it is added to its corresponding inphase components (both originally being inphase in the V.T.A.), then the sum of this addition will be the actual magnitude of the voltage concerned.

(3.2.4.) General calculations involving vector quantities [7] :

These calculations may be made as shown in Fig. 41 where the output voltages a and b are given by :

$$a + jb = \frac{c + jd}{e + jf} + \frac{g + jh}{l + jm} - (n + jp)(v + js).$$

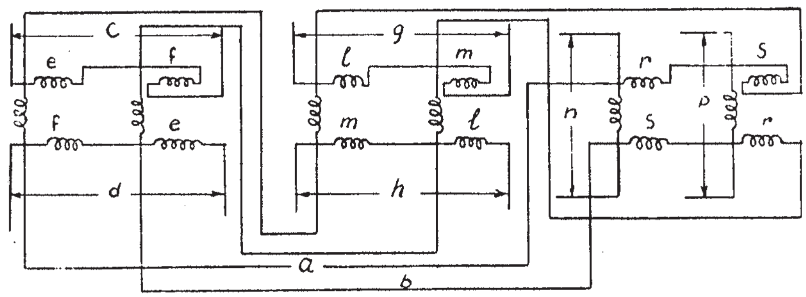


Fig. 41.—General calculations envolving vector quantities in two dimensions

It may be noted that for all previous 3-winding (V.T.A.) measurements are to be made by a high input impedance valve voltmeter.

3.3. The general multi-winding V.T.A.

(3.3.1.) Solution of linear simultaneous algebriac equations (Fig. 42) :

The use of the general multi-winding V.T.A. for the solution of linear simultaneous algebriac equations is explained by Mallock [5] whose basic idea is to use (n+1) multi-winding voltage-transformers and n-windings (11,12,13...etc.) capable of adjustment as regards the number of turns in each winding proportionally to the n-coefficients of the unknown quantity in the n-equations represented by that transformer

If a magnetic flux is produced in one or more of the transformers through additional windings (14, 24...etc) and the several transformer windings relating to each equation are short circuited, then the flux in each transformer will automatically adjust itself so that the fluxes in the transformers (proportional to the

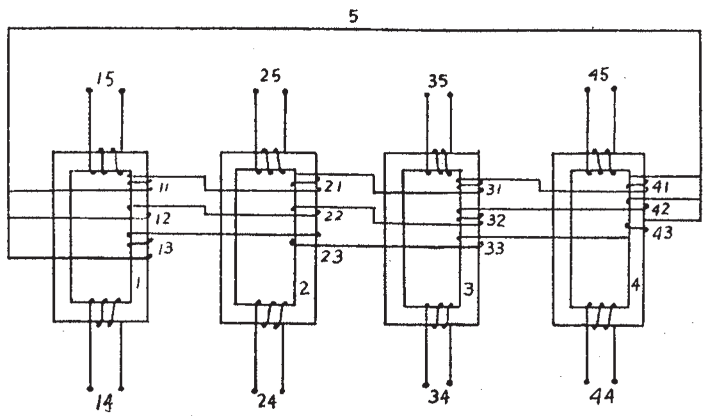


Fig. 42.—Solution of three simultaneous linear algebriac equations by multi-winding V.T.A.

volts/turn measured across 15, 25,... etc., and turns of these windings) are proportional to the values of the unknown quantities in the several equations.

(3.3.2.) *Solution of determinants* (3) :
Scalar determinants:

It can be shown that Fig. 43 gives the solution of the 2nd order determinant given below by the 2-winding V.T.A :

$$D_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

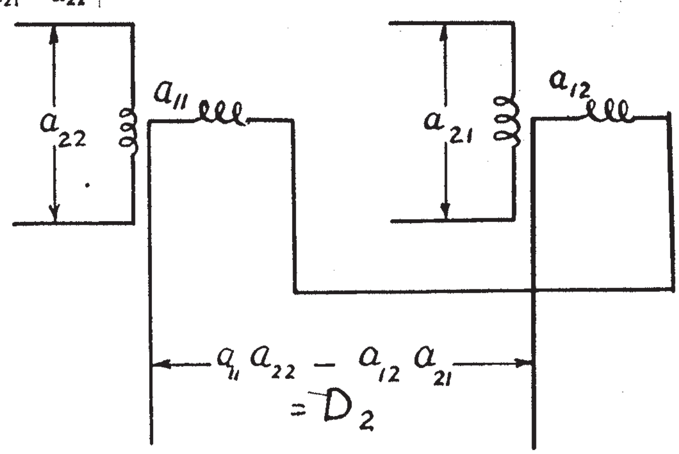


Fig. 43
Solution of
Second order
scalar determinant by V.T.A.

It may be noted that:

elements of first row represent transformation ratios of 2-winding transformers

elements of 2nd row represent voltages applied to 2-winding transformers.

Also Fig. 44 give the 2-and 3-winding V.T.A. giving the solution of the third order determinant shown:

$$D_3 = \left|\begin{array}{ccc} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{array}\right| \begin{array}{l} \text{ratios of 2-winding transformers} \\ \text{,, 3- ,,} \\ \text{voltages on 3- ,,} \end{array}$$

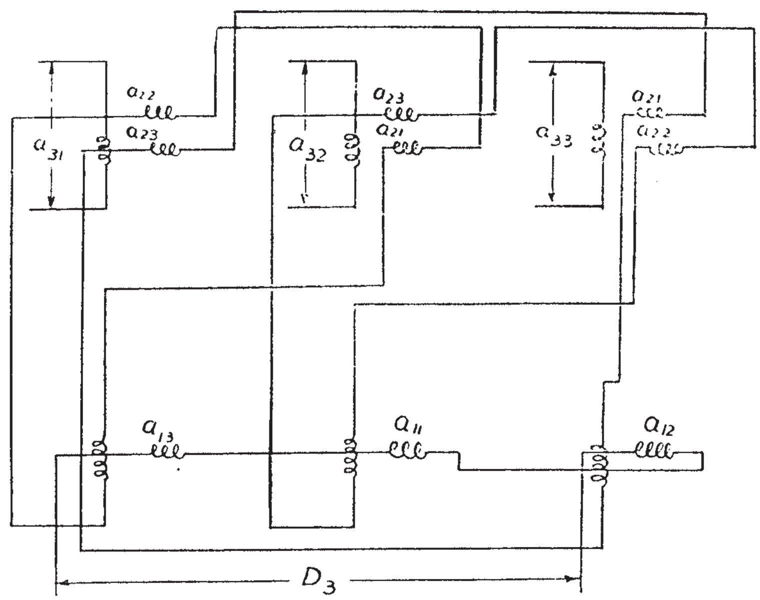


Fig 44.– Solution of third order scalar determinant by V.T.A.

Similarly Fig. 45 is the multi-winding V.T.A. giving the solution of the fourth order determinant shown:

$$D_4 = \left|\begin{array}{cccc} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{array}\right| \begin{array}{l} \text{ratios of 2-winding transformers} \\ \text{,, 3- ,,} \\ \text{,, 4- ,,} \\ \text{voltages on 4- ,,} \end{array}$$

Similarly for the fifth order determinants the ratios of the different transformers and the voltages applied to the five winding transformers are as shown below:

$$D_5 = \left|\begin{array}{ccccc} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{array}\right| \begin{array}{l} \text{ratios of 2-winding transformers} \\ \text{,, 3- ,,} \\ \text{,, 4- ,,} \\ \text{,, 5- ,,} \\ \text{Voltages on 5- ,,} \end{array}$$

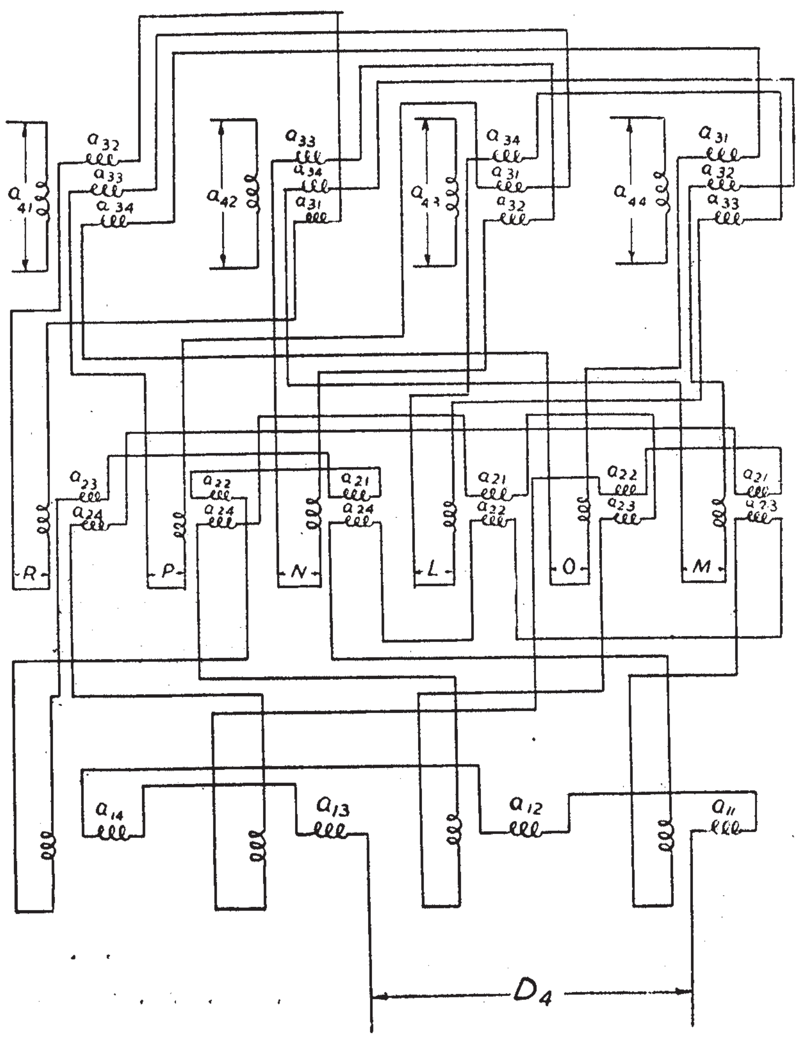


Fig. 45.—Solution of fourth order determinant (scalar) by V T.A.

TABLE 4

SOLUTION OF SECOND TO FIFTH ORDER SCALAR DETERMINANTS BY V.T.A.S.

Order of determinant	Number of transformers	Windings	Total Nu. of transformers
2	2	2	2
3	3	3	6
	3	2	
4	4	4	14
	6	3	
	4	2	
5	5	5	30
	10	4	
	10	3	
	5	2	

Hence in order to solve the n^{th} order scalar determinant and by comparison to Table 4, the required number of transformers and windings on each transformer are shown in Table 5: which is more illustrated in the determinant form D_n given :

TABLE 5

SOLUTION OF THE NTH ORDER SCALAR DETERMINANT IN ITS GENERAL FORM USING THE V.T.A.S

Number of transformers	Number of windings	Total Number of transformers
n/1 !	n	n/1 ! +
n (n—1)/2 !	(n—1)	n (n—1)/2 ! +
n (n—1) (n—2)/3 !	(n—2)	n (n—1) (n—2)/3 ! +
n (n—1) (n—2) (n—3)/4 ! . .	(n—3)	n (n—1) (n—2) (n—3)/4 ! +
. . . .		. . . .
. . . .		. . . .
(n—1) terms	2	(n—1) terms.

$$D_n = \left|\begin{array}{cccccc}
a_{11} & a_{12} \cdots & a_{1x} \cdots & a_{1y} \cdots & a_{1(n-1)} & a_{1n} \\
a_{21} & a_{22} & a_{2x} & a_{2y} & a_{2(n-1)} & a_{2n} \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
a_{(n-2)1} & a_{(n-2)2} \cdot\cdot & a_{(n-2)x} \cdot\cdot & a_{(n-2)y} \cdot & a_{(n-2)(n-1)} & a_{(n-2)n} \\
a_{(n-1)1} & a_{(n-1)2} \cdot\cdot & a_{(n-1)x} \cdot\cdot & a_{(n-1)y} \cdot & a_{(n-1)(n-1)} & a_{(n-1)n} \\
a_{n1} & a_{n2} \cdots & a_{nx} \cdots & a_{ny} \cdots & a_{n(n-1)} & a_{nn}
\end{array}\right|
\begin{array}{l}
\leftarrow \text{Transformation ratios of two-winding transformers.} \\
\leftarrow \text{Transformation ratios of three-winding transformers.} \\
\\
\\
\leftarrow \text{Transformation ratios of (n—1)-winding transformers.} \\
\leftarrow \text{Transformation ratiosof n-winding transformers.} \\
\leftarrow \text{Voltages applied to basis windings of n- winding transformers.}
\end{array}$$

Complex determinants:

Taking the complex determinant in its general form, (*i.e.*) n rows and n columns and each element is complex in itself, then it can be shown that the number of pure real and pure imaginary component determinants split from the n^{th} order determinant are equal to 2^n determinants, (*i e.*) the number of transformers needed will be $2^n \times$ the number of transformers to solve the nth order scalar determinant, (*i.e.*) 2^n $(2^n\text{-}2)$ transformers as may be deduced from Table 5.

Experimental procedure of set-up

(Refer to the nth order determinant).

(i) Voltages equal to the elements of the n^{th} row are obtained either from the tapped secondary of a transformer or by means of appropriately arranged potential dividers.

(ii) These voltages are then applied to the n-winding transformers, whose transformation ratios are equal to the elements of the (n-1) row in the following manner:

A voltage equal to any element of the n^{th} row, say, a_{nx} is applied to the n winding transformers whose transformation ratios are adjusted to be equal to all the elements of the n-1 row *except* the element $a_{(n-1)x}$ and so on for all other n-winding transformers, *i.e.* the transformation ratios of any n-winding transformer should be equal to all the elements of the (n-1) row except the element, of the same column, belonging to the voltage applied on the basis winding of the n-winding transformer under consideration.

(iii) The connections between the secondaries of the n-winding transformers, to correspond to the expansion of the determinant, are best illustrated by the following example:

For any n-winding transformer having the voltage on its basis winding equal to a_{nx}, then for any column y, say, this n-winding transformer will have a secondary winding having $a_{(n-1)y}$ transformation ratio, *i.e.* the voltage obtained across this secondary will be $a_{nx}\, a_{(n-1)y}$. This secondary should be connected to the secondary of the n-winding transformer on whose basis winding a voltage equal to a_{ny} is applied and whose secondary under consideration is adjusted to have the transformation ratio of $a_{(n-1)x}$ in a manner such that the voltage obtained from the combination (provided that x is less than y in the usual manner of representation of determinants) will be:

$$a_{nx}\, a_{(n-1)y} - a_{(n-1)x}\, a_{ny}$$

and this should be repeated to obtain all other possible combinations from the secondaries of the n-winding transformers.

(iv) The voltage thus obtained is to be impressed to the (n-1) winding transformers whose transformation ratios are equal to all the elements of the (n-2) row *except* for the columns x and y and this should be repeated for all the (n-1)-winding transformers.

(v) Secondaries of the (n-1)-winding transformers, having in correspondence with the determinant expansion *different three* transformation ratios of the n-2 row elements, are to be connected together in an addition form *except* the *middle* voltage (obtained from the secondary whose transformation ratio is an element of the (n-2) row which lies, in the usual way of determinant representation, in the column between the two columns from whose elements, with the (n-2) row, the voltage at the secondaries of the two other (n-1)-winding transformers are obtained) which should be subtracted.

The voltages thus obtained are to be impressed on the primaries of the (n-2)-winding transformers whose transformation ratios are of the n-3 row and of all the columns *exeept* those used in the (n-1)-winding transformers.

All other possible combinations should be made and the voltages thus obtained from the secondaries of the (n-1)-winding transformers in the manner just described should be applied to the primaries of the (n–2) winding transformers.

(vi) Secondaries of the (n-2)-winding transformers having in correspondence with determinant expansion *different four* transformation ratios of the (n-3) row are to be connected in the manner (+ — + —) in the way these ratios come in the determinant.

The resultant voltage is to be applied to the primary of the (n-3)-winding transformers whose transformation ratios are equal to the elements of the (n-4) row and all the columns except those whose elements with the (n-3), row have already been used as transformation ratios.

(vii) This should be repeated, in the same way, until a final output voltage which is equal to the magnitude of the determinant of the nth order (D_n) is obtained from the outputs of the two-winding transformers.

(4) *Voltage—Transformer Network Analyser (General).*

4 1 General:

From previous theory it could be seen that the main function of, and the requirements needed from a voltage transformer network analyser are :

(i) Representation of the system constants to a good degree of accuracy and for a wide range of variations in magnitudes.

(ii) Means of flexible and reversible interconnections between transformer windings.

(iii) A high impedance valve voltmeter for voltage measurements.

(iv) Beside all these requirements, the interconnections should be made quickly and easily and the analyser be compact and of a reasonable price.

4·2 History:

The first network analyser using voltage transformers was constructed by Merz and McLellan [8] at Newcastle-on-Tyne based on Blackburn's [1] and [7] patent specification in 1938 (Figs. 46 and 47).

The second analyser was built by the author at the City and Guilds Colleges in London in 1950 [9] (Photograph 48).

A general specification comparsion table between the above two analysers and the third one built in the Laboratories of the Faculty of Engineering, Giza, in April 1953 (Photograph 49) is given in Table 6.

4·3·1. Construction (under registration):

The simplicity of the construction of the Giza analyser in comparison with the other two existing similar systems may be

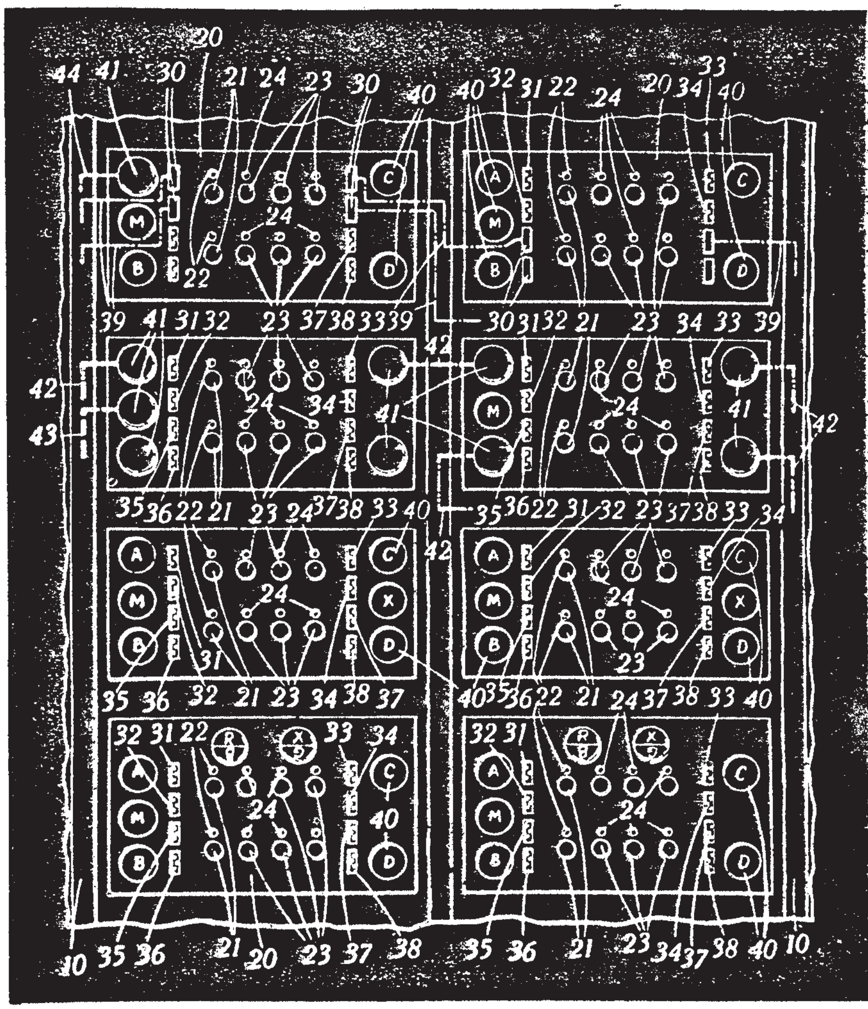


Fig. 46.—Merz and McLellan Network Analyser

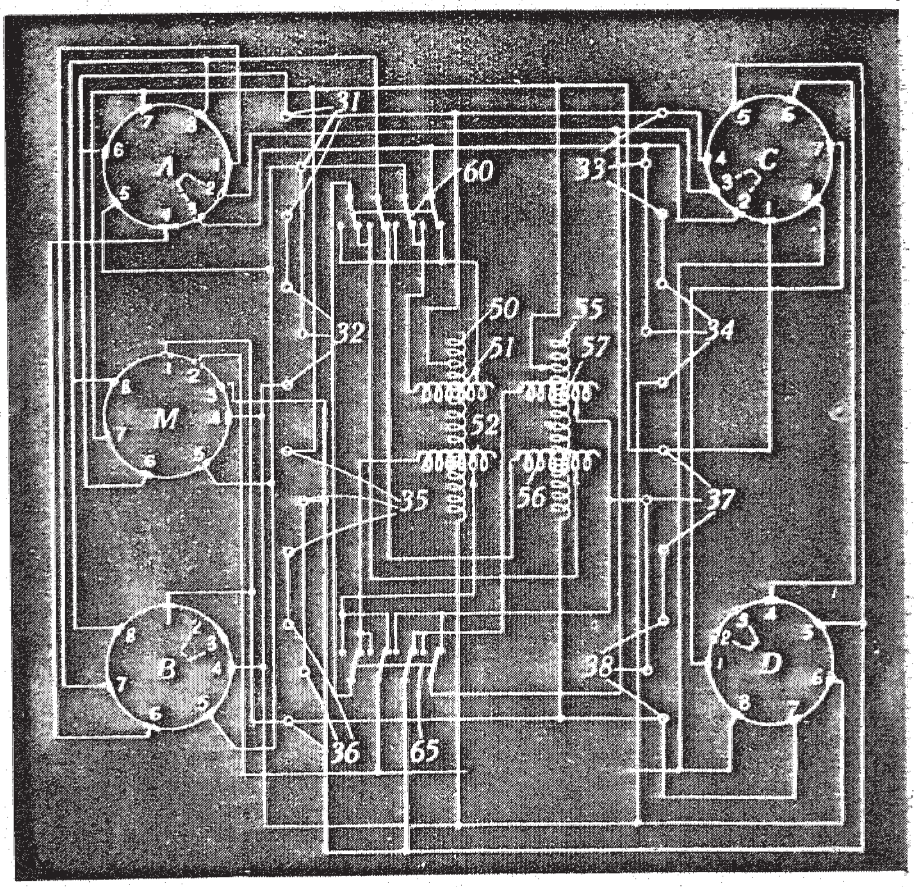


Fig. 47.—Internal connections of Fig. 46

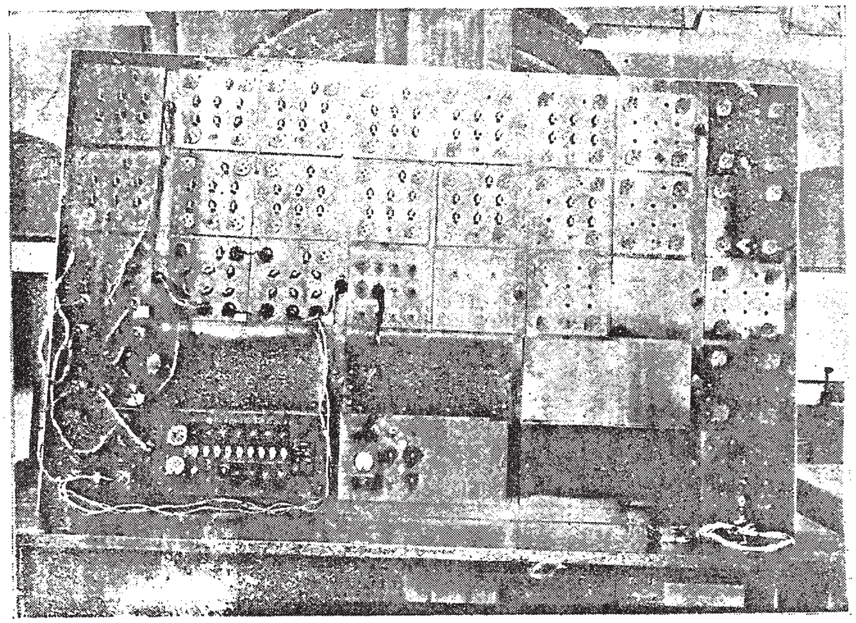


Fig. 48.—City and Guilds Network Analyser.

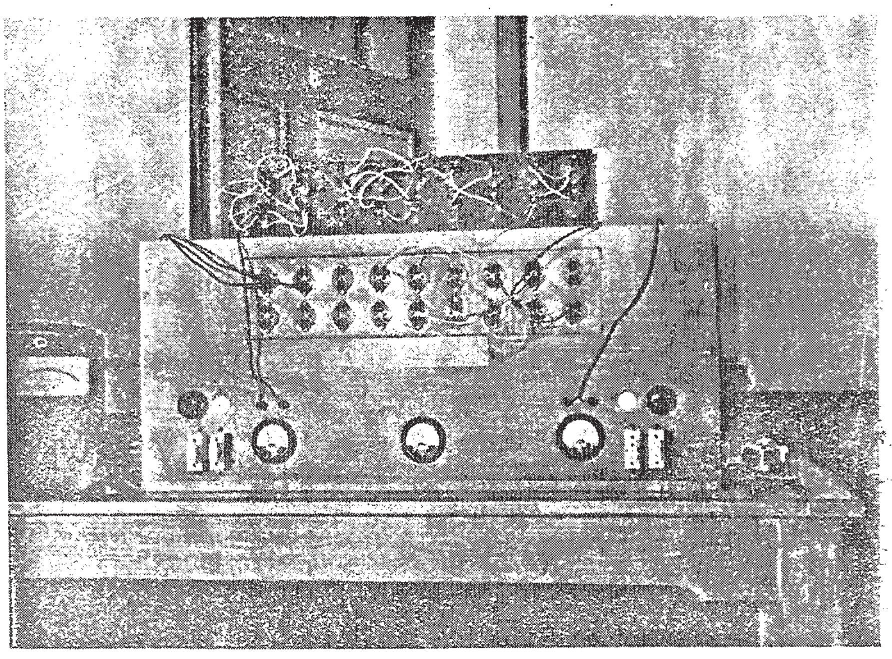


Fig. 49.—Faculty of Engineering, Giza, Network Analyser.

noted partially by reference to Table 6. See also references (8) and (9) for full comparison.

The basic constructional systems are :—

(i) 18 two-winding calculator transformers, each with a high voltage winding standing about 130 volts and two low voltage windings having a common point of interconnection. The two-low voltage windings, in the existing construction, stand about 4 and 7 volts respectively. In the future, it is hoped instead of having all transformers of nearly the same turns ratio, to change them by other two-winding transformers having variable ratios according to a certain scheme but still using 18 transformers, this will enable the extension in the field of application of the analyser to cope with problems having a wider variety of circuit constants.

(ii) The five terminals of each two-winding transformer are connected to two five-pin sockets in front of the operator ; one in the lower backalite panel for complementary connections and measurements and the other socket on the upper inclined backalite panel for basic interconnections. Hence each of these panels has 18 five-pin sockets connected to the 18 two-winding transformers.

(iii) There are also 2 similar two-winding transformers and each serves, through two fuses, single-pole switch, red lamp connected in the normal way, as a voltage source to excite the transformers of the analyser on working. The input voltage (110 volts) to each transformer is controlled by a potential divider and the secondary voltage (about 11 volts maximum) is measured by a voltmeter fixed to the analyser front with its two terminals brought outside the analyser above the voltmeter for interconnections. The ammeter fixed to the analyser front reads the total input current to the analyser.

4·3·2. Operation:

To set up any problem on this analyser the following steps have to be made:—

(i) The data of the problem to be arranged and organised and voltage and impedance scale factors to be chosen within the analyser capacity.

(ii) Single-wire jumpers and plugs to be used between the five-pin sockets of the top panel to build the analyser units having transformation ratios equal, or to scale, to the circuit elements of the problem investigated. Also the connecting system considered, is to be used to make the (internal) connections between the pins of the upper panel sorkets to realise the nature of the circuit element *i.e.* positive or negative resistances and inductors.

(iii) External connections by the single wire system are then to be made between the pins of the upper panel sockets and (if necessary) of the lower panel sockets to realise the governing equations of the problem specified.

(iv) Voltages, using the terminals above the voltmeters, to be connected at the appropriate terminals of the set-up.

(v) Plug in the 110 volts supply using the white plug and put on the two switches on the analyser front, the two red lamps will go bright. Using the two potential dividers the voltages read by the voltmeters to be adjusted to be equal, or to scale, to the applied voltages. Plug in the valve voltmeter terminals (at the same time in parallel connection to a cathode ray tube) where required to measure the voltage and current components in magnitude and direction.

4·3·3. Solved examples and accuracy:

D.C. problem (Fig. 50):

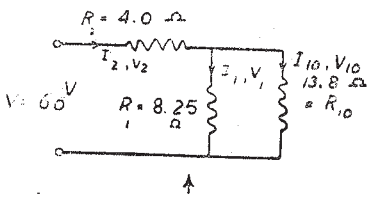
Taking the following scale factors of the problem (see, Fig. 50-a):

Impedance scale-factor = 1/100
Voltage ,, = 1/10
∴ Current ,, = 10/1

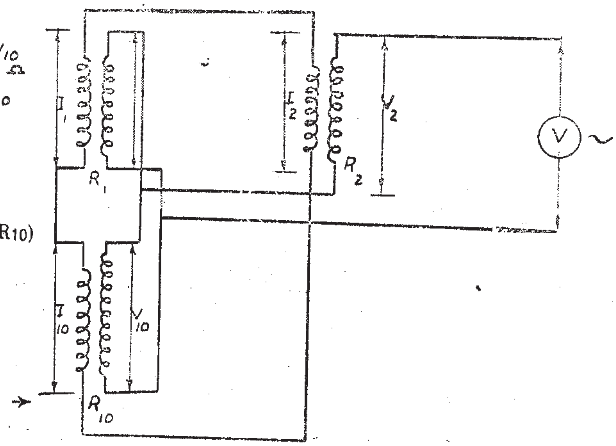
The 2-winding V.T.A. is given in Fig. 50-b and the set up on the analyser front is shown in Fig. 50-c realising the remaining steps of set up.

MEASURED AND CALCULATED RESULTS

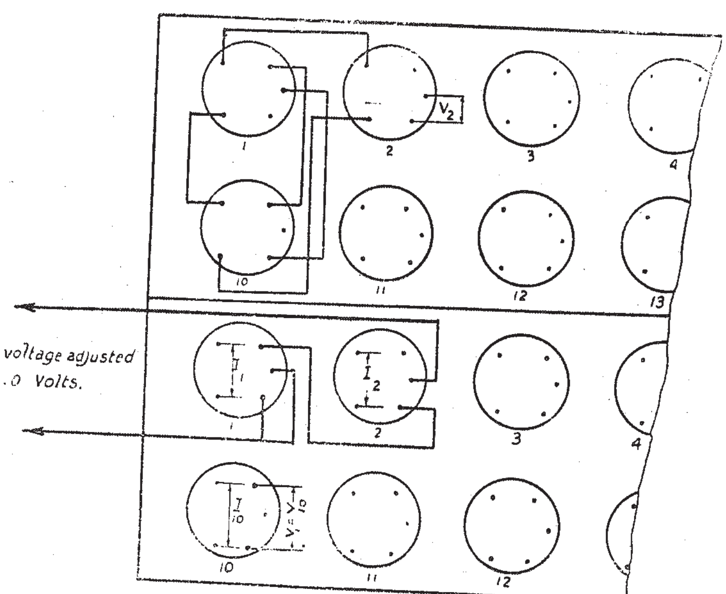
Quantity	Measured volts	Measured reduced to scale (M)	Calculated (C)	% Accuracy $\left(1 \pm \frac{C-M}{C}\right) 100$ %
V	+ 6·0	+ 60·0 volts	+ 60·0 volts	
I_2	+ 65·5	+ 6·55 amperes	+ 6·54 amperes	99·85
I_1	+ 41·0	+ 4·10 ,,	+ 4·10 ,,	100
I_{10}	+ 24·5	+ 2·45 ,,	+ 2·44 ,,	99·59
V_2	+ 2·60	+ 26·0 volts	+ 26·16 volts	99·38
V_1	+ 3·40	+ 34·0 ,,	+ 33·84 ,,	99·42
V_{10}	+ 3·40	+ 34·00	+ 33·84 ,,	99·42



a.—D.C. Problem.
($V_2 = I_2 R_2$, $V_1 = I_1 R_1$, $V_{10} = I_{10} R_{10}$)



b.—The 2-winding V.T.A. of figure a



c.—Part of analyser sockets solving the D.C. problem
(Voltage measurements made by a valve voltmeter and Cathode ray tube)

Fig.—50 Solution of D.C problem by the network analyser

A C. problem (Fig. 51) :

The following scale-factors are chosen for the A.C. problem :

Impedance scale-factor = 1/1000
Voltage " = 1/10
∴ Current " = 100/1

MEASURED AND CALCULATED RESULTS

Quantity	Measured volts	Measured reduced to scale (M)	Calculated (C)	% Accuracy $\left(1 \pm \frac{C-M}{C}\right)$ 100 %
V_p	+ 7·00	+ 70·00	+ 70·00	———
V_q	0·00	00·00	00·00	———
I_{p1}	+ 10·70	+ 0·107	+ 0·108	99·074
I_{q1}	— 42·0	— 0·420	— 0·430	97·68
I_{p2}	+ 7·30	+ 0·073	+ 0·0740	98·65
I_{q2}	— 4·10	— 0·041	— 0·0406	99·01
I_{p3}	+ 3·40	+ 0·0340	+ 0·0340	100
I_{q3}	— 37·9	— 0·379	— 0·3894	97·43
V_{p1}	+ 6·27	+ 62·70	+ 63·00	99·93
V_{q1}	— 0·240	— 2·40	— 2·34	97·44
V_{p2}	+ 0·73	+ 7·30	+ 7·00	95·72
V_{q2}	+ 0·24	+ 2·40	+ 2·34	97·44
V_{p3}	+ 0·73	+ 7·30	+ 7·00	95·72
V_{q3}	+ 0·24	+ 2·40	+ 2·34	97·44

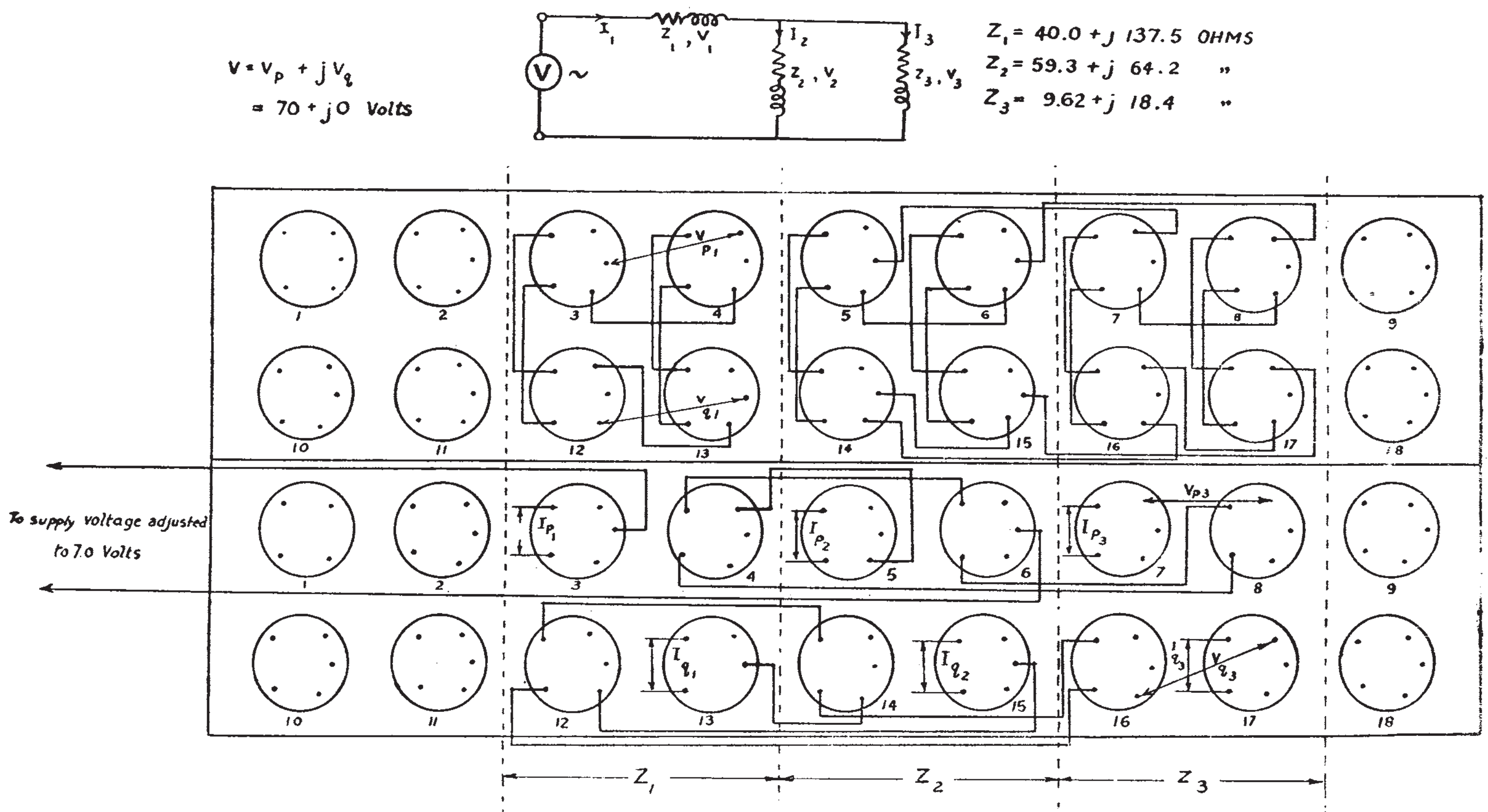


Fig. 51.—Set-up on network analyser of A.C. problem

(5) *Discussions and conclusions*

It follows that a simple A.C. voltage-transformer network analyser has been constructed and operated with a sufficient acurracy for many practical purposes.

To compare its main specifications with the two existing alternative analysers, Table 6 is given :

TABLE 6

GENERAL COMPARISON BETWEEN THE DIFFERENT 3 EXISTING ANALYSERS OF THEIR OWN TYPE

Specification	New Castle N. A.	City and Guilds N A	Faculty of Engineering N. A.
Number of standa d transformers used.	96	48	18
Total Number of windings on each transformer	active . . . . 3 compensating 1 Total . . 4	active . . . . 3 compensating 2 Total . . 5	active . . . 2 compensating 0 Total . . 2
Frequency used cycles/second	50	50	50
Contents/unit	2 calculator Transformer.	2 calculator transforms	4 Calculator Transformers
	2 compensating circuits	2 compensating circuits	—
	2 reversing switches	1 reversing switch	Made by external plugging
	6 tapping Control switches	6 tapping control switches	Made by external plugging
	8 three-pin plugs and sockets for interconnections	—	—

TABLE 6 (*contd.*)

Specification	New Castle N. A.	City aud Guilds N. A.	Faculty of Engineering N. A.
Contents/unit	4 eight-pin plugs and sockets for interconnections 1 eight-pin plug and socket for measurements	4 eight-pin plugs and sockets for interconnections 1 eight-pin plug and socket for measurements	4 five-pin plugs and sockets for interconnections 4 five-pin plugs and sockets for complementary interconnections and measurements
Measurements	Indirect and non-indicating by transformation ratios	Direct and indicating by valve voltmeter	direct and indicating by valve voltmeter
Advantages	First analyser of its type	Highest accuracy	Very cheap, small and compact
Disadvantages	Expensive a bit complicated in operation	Expensive	lower accuracy
Min Accuracy about	06 %	98 %	95 %

From Table 6 the main differences in construction between the three analysers are clear. It remains only to stress the following points :—

(i) 2-winding transformers are used instead of 3 active winding transformers for both Newcastle and City and Guilds analysers. *This leads to using double the number of transformers for A.C. networks and using all analyser transformers, instead of leaving half of them dummy, for D.C. problems and this in itself is a basic change in the method of operation of the analyser.* This can be made classified further clearer, with reference to the general theory of the V.T.A. previously given, by noting that a resistance is represented by a two-winding transformer and an impedance is represented by three-winding transformers (equivalent to 2 two-winding transformers) and to the fact that we are using 2-winding transformers instead of three-winding transformers at Newcastle and in London.

(ii) *Using two-winding transformers at Giza instead of 3-winding transformers abroad means a change in the standard method of interconnections between transformers from four eight-pin plugs and sockets to four five-pin single-wire plugs and sockets* (Table 6).

(iii) *In the Giza analyser, the fact that we have three terminals only on the secondary side of each transformer instead of 42 terminals abroad allows the elimination of all switches and replacing them by the simple external single wiring non-permanent interconnections. This, beside eliminating cold joints and other switching troubles, means a reduction in cost of the analyser.*

This is done at the expense of reducing the number of circuit elements that can be dealt with but this is partially compensated as shown in item (v).

(iv) The standardisation of the method of interconnections between transformer windings for solving standard (impedance) problems abroad means that special problems either cannot be solved by their analysers or special plugging systems are to be made for each special problem, whereas the use of the single-wiring method in Giza analyser abolishes any possibility of using special plugging systems for special problems, *i.e., which means the ease by which not only standard* *problems may be solved but also special problems may be set up on the Giza analyser (Table 6), which is not existing in the two alternative analysers.*

(v) The advantage mentioned in item (iv) is further assisted by the fact that the terminals of *each* transformer are brought in front of the operator and connected to *two* five-pin sockets being both in parallel. *This also allows the use of one transformer in two V.T.A.s., each of which is to be connected to one of the five-pin sockets. This method of doubling the number of V.T.A.S cannot be made in the alternative systems because, as seen in Table 6, there is only one way of transformer interconnections completely making use of, using their method of intereonnections, all the pins of the four eight-pin plugs and sockets.*

(vi) *In addition to the advantages mentioned associated with the Giza analyser, its accuracy of operation is comparable with accuracies of the other systems, although its cost is much lower.*

(6) *Acknowledgement*:

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(7) BIBLIOGRAPHY

1. C. L. Blackburn and Electrical Improvements Ltd., Patent specification No. 476164, 1937.

2. V. Bush: Structural analysis by electric circuit analogies, Journal of the Franklin Institute, p. 289, Vol. 217, 1934.

3. Author's thesis, pp. 2 and 20, Part I, 1950.

4. Author's thesis, p. 6, Part I, 1950.

5. R. R. M. Mallock: Patent specification No. 389524, 1933.

6. Author's thesis, pp. 73 and 82, Part III, 1950.

7. C. L. Blackburn and Electrical Improvements Ltd., Patent specification No. 476165, 1937.

8. No. published reference is available, only some printed papers and a visit to the place in 1950 are the references.

9. Author's thesis, p. 33, Part II, 1950.