THE EFFECT OF PRIMARY LOSSES ON THE PERFORMANCE OF RADIO FREQUENCY TRANSFORMERS WITH TUNABLE SECONDARY

BY

M. A. H. EL-SAID, *Senior Member*, IRE[1]

SUMMARY

It is shown that the primary losses have considerable effect on the overall performance of the radio frequency transformer. A quantitative analysis is given and theoretical results are verified experimentally.

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The transformer circuit analysed here is one form of two inductively coupled LCR circuits, in which the secondary is tunable over a wide range of frequencies and the primary is resonant at one frequency which is outside the secondary tuning range. For this case of coupled circuits, the formulae shown in the literature[2] neglect the effect of primary losses on the circuit performance. In many practical cases these formulae are not sufficiently accurate for use in circuit design. This article shows a more complete quantitative analysis which is illustrated by a practical example.

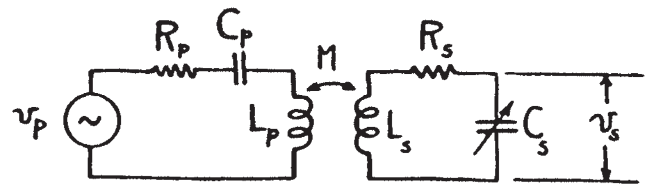

Fig. 1.—Circuit of radio frequency transformer with tunable secondary

(1) Assistant Professor, Faculty of Engineering, Cairo University, Giza, Egypt.
(2) See References.

A.— *Voltage Transfer Ratio:*

Consider the circuit shown in Fig. 1 in which :

L_p = primary inductance

L_s = secondary inductance

$M = k\sqrt{L_s L_p}$ = mutual inductance

k = coefficient of coupling

C_p = fixed primary capacitance

C_s = variable secondary capacitance

R_p = total effective primary resistance

R_s = total effective secondary resistance

v_p = primary voltage supplied from a constant voltage source

v_s = voltage appearing across secondary condenser

$Q_p = w\,L_p/R_p$ = primary circuit magnification

$Q_s = w\,L_s/R_s$ = secondary circuit magnification

$T = v_s/v_p$ = voltage transfer ratio.

The voltage transfer ratio is given by :

$$T = -\,M/C_s \left\{ Z_s Z_p + w^2 M^2 \right\} \qquad (1)$$

where $Z_s = R_s + JX_s$, $X_s = w\,L_s - I/wC_s$

$Z_p = R_p + JX_p$, $X_p = w\,L_p - I/wC_p$

$w = 2\pi f$ = the signal frequency of the applied voltage. Substitution of above values in equation (1) gives:

$$T = -\,M/C_s \left\{ (w^2 M^2 + R_s R_p - X_s X_p) \right\} \qquad (2)$$

It can be shown that (see Appendix I) the voltage transfer ratio reaches a maximum with respect to X_s as a variable when:

$$w_r^2 M^2 = X_{sr} X_{pr} \qquad (3)$$

The suffix r is added to indicate magnitudes at a frequency f_r at which the voltage transfer ratio is maximum Equation (3) shows that detuning of the secondary is necessary in order to obtain maximum gain Under this condition, the required value of C_s is determined from the relation :

$$X_{sr} = w_r L_s - I/w_r C_s = w_r^2 M^2 / X_{pr} \text{ hence,}$$

$$C_s = F/w_{po}^2 L_s \quad \ldots \ldots \quad (4)$$

$$\text{where } F = I/\beta_r^2 (I - P_r k^2) \quad \ldots \ldots \quad (5)$$

$$P_r = \beta_r^2/(\beta_r^2 - I) \quad \ldots \ldots \quad (6)$$

$$\beta_r = w_r/w_{po}, \text{ and } w_{po} = I/\sqrt{L_p C_p}.$$

From equation (4) it follows that: $w_r^2 L_s C_s = F \beta_r^2$ which is a relation expressing the condition of the secondary circuit at the frequency of maximum gain. If $k = o$ or very small, equation (5) shows that $F \beta_r^2 = I$ which means that no detuning of the secondary is necessary. Conversely, if k is not small, the amount of deviation of $F \beta_r^2$ from unity represents the amount of detuning necessary to obtain maximum gain. For values of $\beta_r > I$, the primary circuit will operate above its resonance, and so would also the secondary because $F \beta_r^2 > I$. The transformer in this case is said to have a "large primary". For values of $\beta_r < I$, the primary circuit will operate below its resonance, and so would also the secondary because $F \beta_r^2 < I$; the transformer being of "small primary". The relative resonances of both types of transformer are shown in Fig. 2.

Substituting from equation (3) into equation (2), then :

$$T_r = -M / C_s \left[R_s (R_p + JX_p) + JX_s R_p\right]_{wr}$$

Assuming that the primary is mostly reactive, *i.e.* $R_p \leqslant X_p$, and eliminating C_s, it can be shown that (see Appendix II):

$$T_{max.} = \sqrt{L_s/L_p}\, k\, Q_{sr}\, P_r\, G_r \qquad \text{(2b)}$$

$$\text{where } G_r = (1-P_r k^2) / (1+P_r^2 k^2 Q_{sr}/Q_{pr}) \qquad \text{(7)}$$

When the primary losses are totally neglected or $Q_p = \infty$ the factor $G_r = (1-P_r k^2)$, which for most cases is close to unity. It follows, therefore, that when the primary losses are taken into consideration, the maximum voltage transfer ratio differs from that obtained when these losses are neglected by the factor G_r. This factor depends largely on the coefficient of coupling and the ratio of the secondary to primary magnifications.

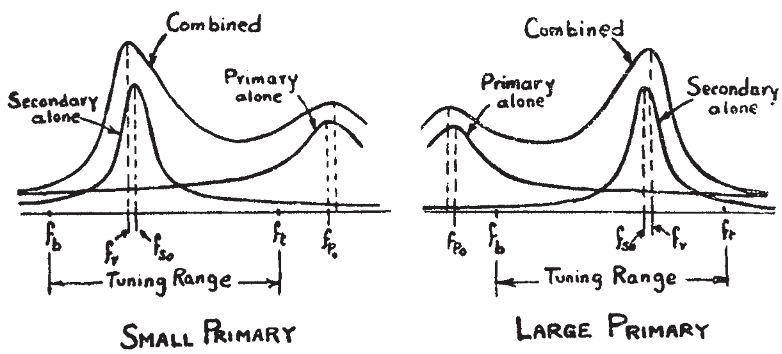

Fig. 2.—Relative resonance curves of small and large primary transformers

Fig. (3) shows the values of the gain factor $(P_r G)$ against β_r for various values of the ratio (Q_{sr}/Q_{pr}) in a transformer having a large primary. The curves are calculated from equation (2b) for the indicated values of the coefficient of coupling. In particular, the curves marked "p" assume that $G_r = 1$ and therefore represent the gain factor when the primary losses are totally neglected. These curves show that when the operating frequency is sufficiently far from primary resonance, the gain factor is substantially independent of frequency. Further, the gain factor varies widely with the ratio of magnification, particularly at the high values of the coefficient of coupling. It is also interesting to note that when the ratio of magnifications is high, the gain is not directly proportional to Q_s.

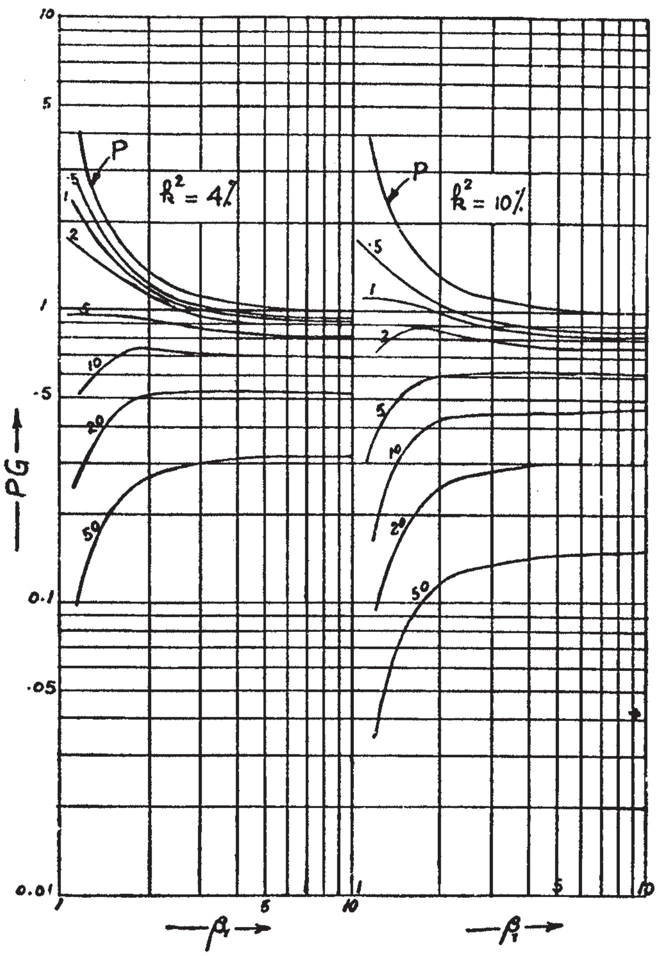

Fig. 3.—Calculated performance curves of large primary transformers

Fig. 4 shows similar curves calculated for a transformer having a small primary. These curves indicate that when the operating frequency is sufficiently far from primary resonance, the gain factor is substantially independent of primary losses and coefficient of coupling Further, the gain factor varies widely with and is closely proportional to the square of frequency.

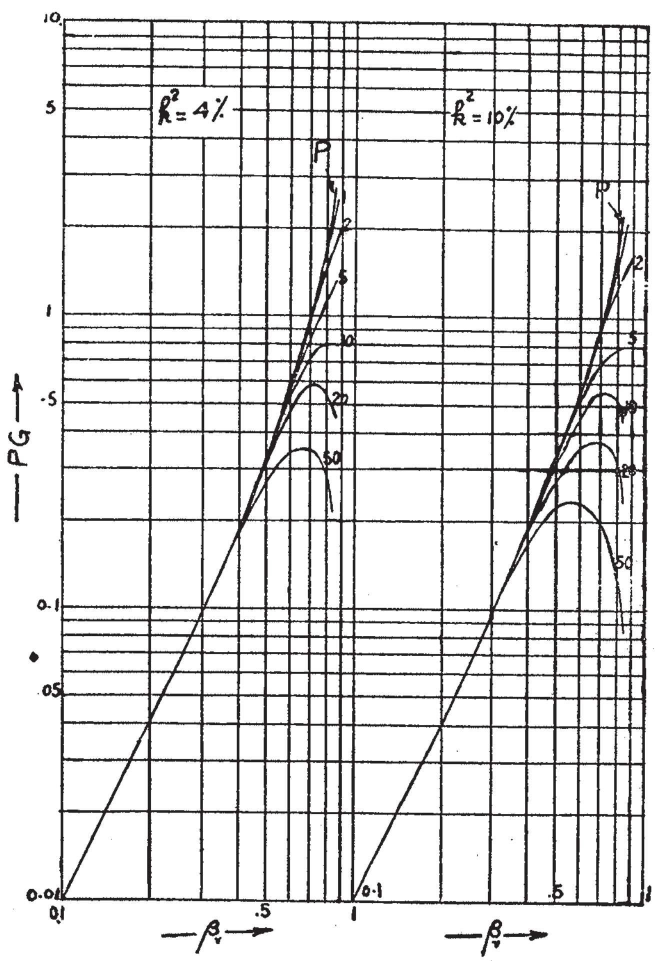

Fig. 4.—Calculated performance curves of small primary transformers

In order to illustrate the value of these results, an experimental transformer was constructed with the following measured particulars : $L_p = 560$ uH , $L_s = 140$ uH , $k^2 = 2{\cdot}62\%$. In the test, the transformer was connected as one with large primary.

The resonance frequency of the primary was adjusted at $f_{po} = 425$ KC, by means of a primary capacity $C_p = 250$ pf. The details of the experimental work are given in Appendix III, and only the observed results are plotted in Fig. 5. In this figure, both calculated and observed data are shown for comparison. The curves marked (1) were taken with the resistance of the primary coil constituting the entire primary losses, and those marked (2) with 100 ohms added in series with the primary coil. Both theoretical and experimental results agree closely and differ markedly from those calculated with the primary losses neglected.

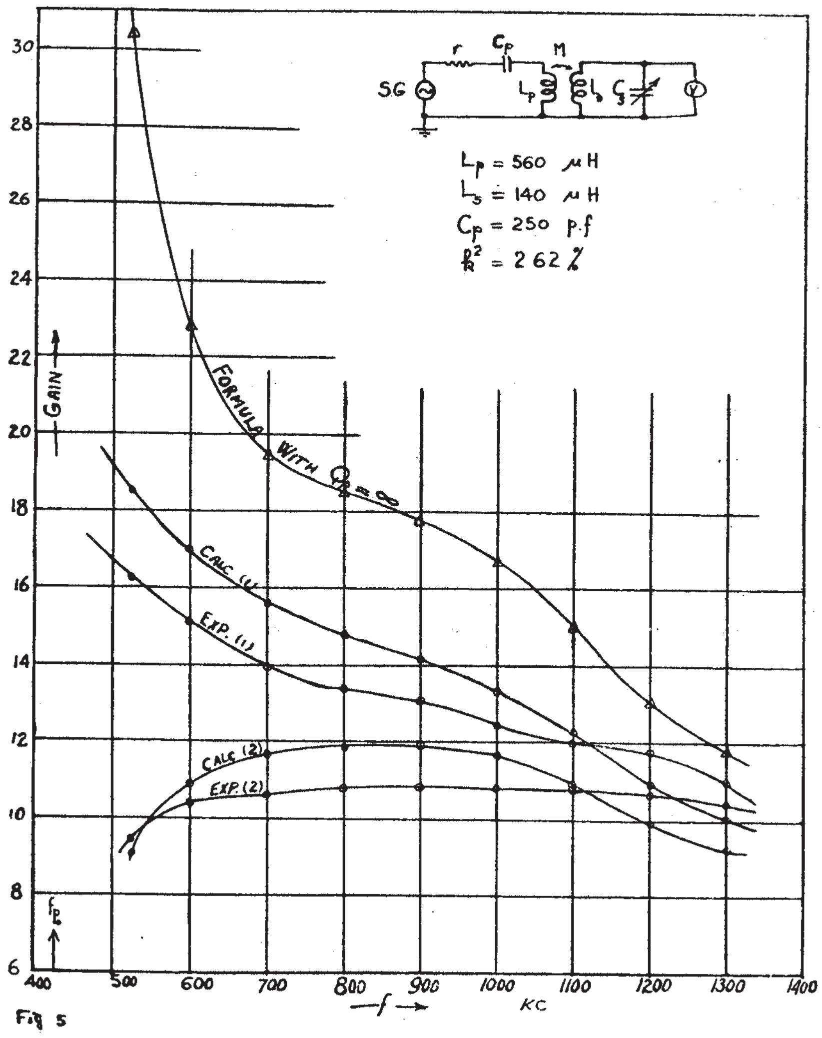

Fig. 5.—Calaculted and observed performance of experimental transformer

Thecircuit of Fig. 1 is widely used in radio receivers as an aerial coupling circuit. A large, rather than a small, primary is almost always used because it gives a sensibly uniform gain and signal to noise ratio over the entire tuning range. It also provides a better image rejection as will be discussed later. A similar circuit is also commonly used in the radio frequency voltage amplifier, particularly in the long, medium, and short wave bands up to about 5 megacycles. Above this frequency, the circuit becomes detrimental to the gain, and the direct coupled tuned circuit is preferred.

Consider the circuit in Fig. 6 (a), in which a pentode is connected to a radio frequency transformer. Under class A tube operating conditions, the equivalent circuits are shown in Fig. 6 (b) and 6 (c), assuming that the self bias, screen, and H. T. supply

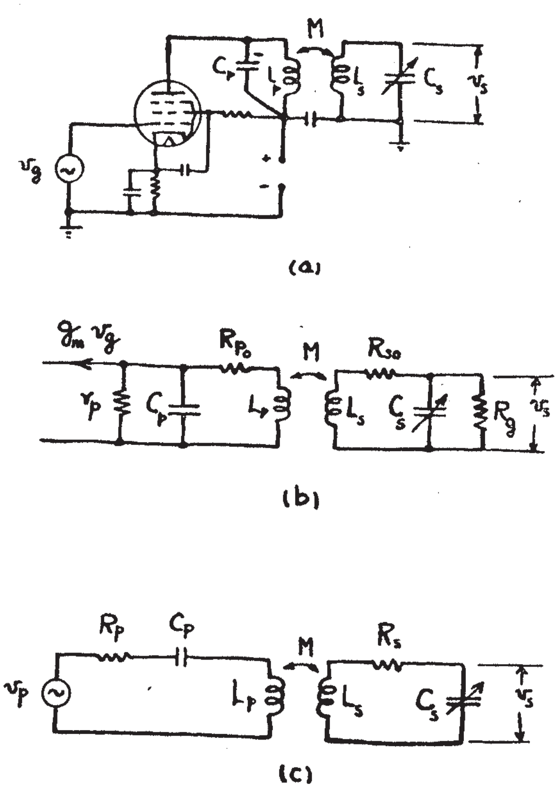

Fig. 6.—Radio frequency amplifier stage and its equivalent circuits

circuits are adequately by-passed. In these circuits, R_{po} and R_{so} and the ac resistances of the coils L_p and L_s respectively, and R_g is the damping resistance of the subsequent stage including any losses is the tuning condenser. If the reactances of the condensers are much smaller than the shunting resistances, then :

$$R_p = R_{po} + X^2_{C_p} / r_p = \beta^2 X_{C_p} / Q_{po} + X^2_{C_p} / r_p \ ,$$

$$R_s = R_{so} + X^2_{C_s} / R_g = X_{L_s} / Q_{so} + X^2_{C_s} / R_g, \ v_p = J \, g_m X_{C_p} v_a ,$$

where Q_{po} and Q_{so} are the magnifications of the coils alone, g_m and r_p are the mutual conductance and internal resistance of the tube respectively. Assuming that the primary circuit is mostly reactive, the maximum voltage transfer ratio is given by :

$$T_{max.} = g_m \ X_{C_p} M / C_s \left[X_s R_p + X_p R_s \right]_{wr}$$
$$= (g_m \ X_{C_{po}}) \sqrt{L_s / L_p} \, k \, Q_{sr} P_r G_r / \beta_r \ . \quad . \quad . \qquad (8)$$

Equation (8) is exactly (2b) multiplied by the quantity (g_m Xc_{pr}) which decreases with frequency. Fig. 7 shows the calculated performance of an amplifier stage in which the transformer particulars are those of Fig. 3 for $k^2 = 4\%$. The curves show that for a constant ratio of circuit magnifications, the gain factor is proportional to the inverse of frequency, provided that this is sufficiently far from primary resonance. The curves show also that it is possible to obtain a sensibly uniform gain throughout a wide tuning range by means of varying the ratio of circuit magnifications properly. Fig. 8 shows similar curves for a transformer with a small primary.

In order to illustrate the above results experimentally, the same transformer used in the circuit of Fig. 5 was associated with a pentode type 6SJ7. The observed results (see Appendix IV) together with those calculated from equation (8) are plotted in Fig. 9. The curves illustrate a considerable descrepancy between the results of the formula neglecting primary losses and those observed experimentally ; the latter check equation (8) closely. The curves marked (1) were taken with the resistance

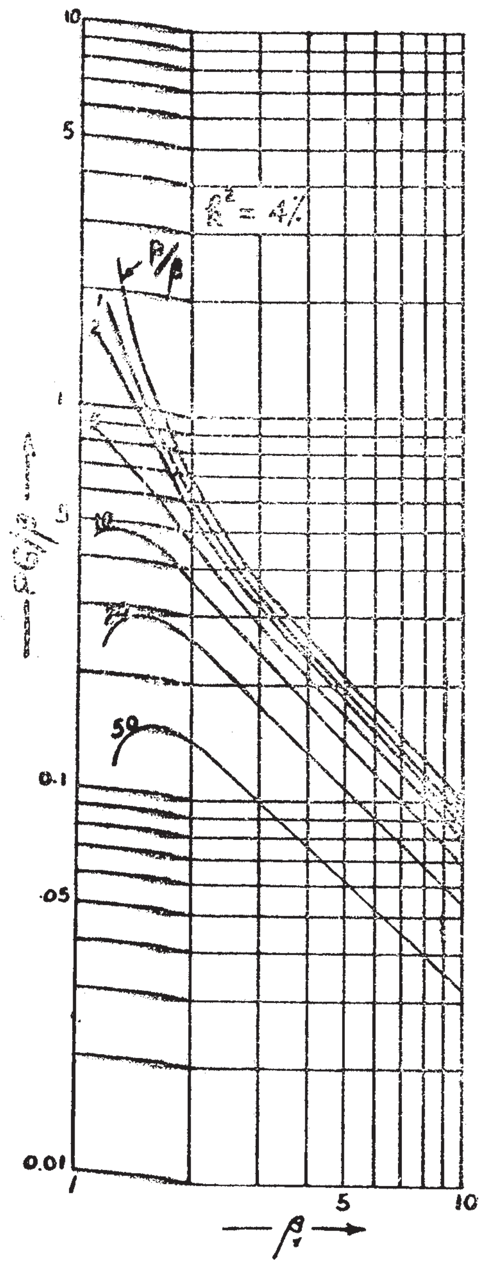

Fig.7.—Calculated performance of amplifier stage employing large primary transformer

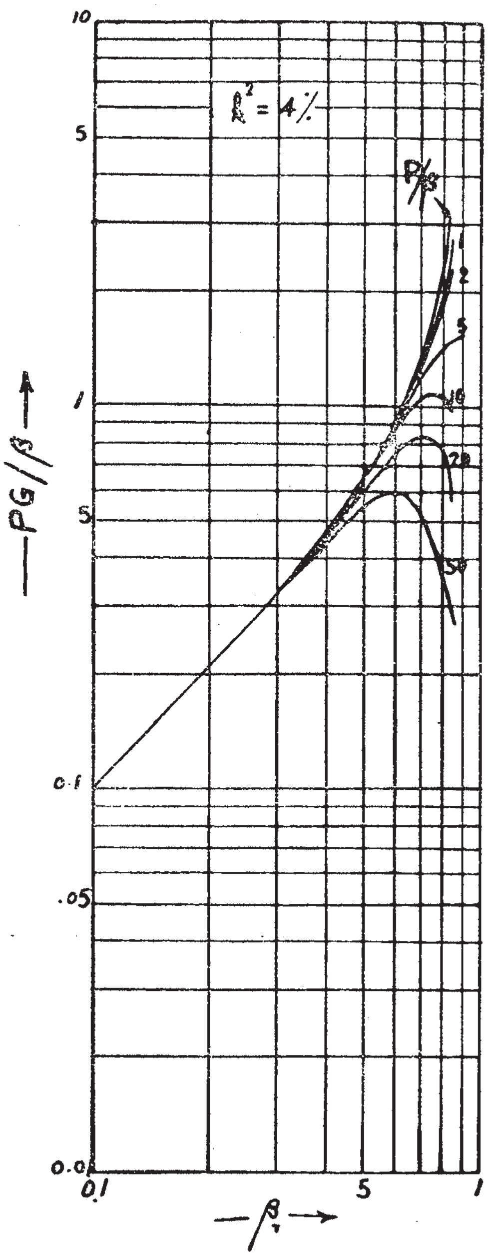

Fig 8.— Calculated performance of amplifier stage employing small primary transformer

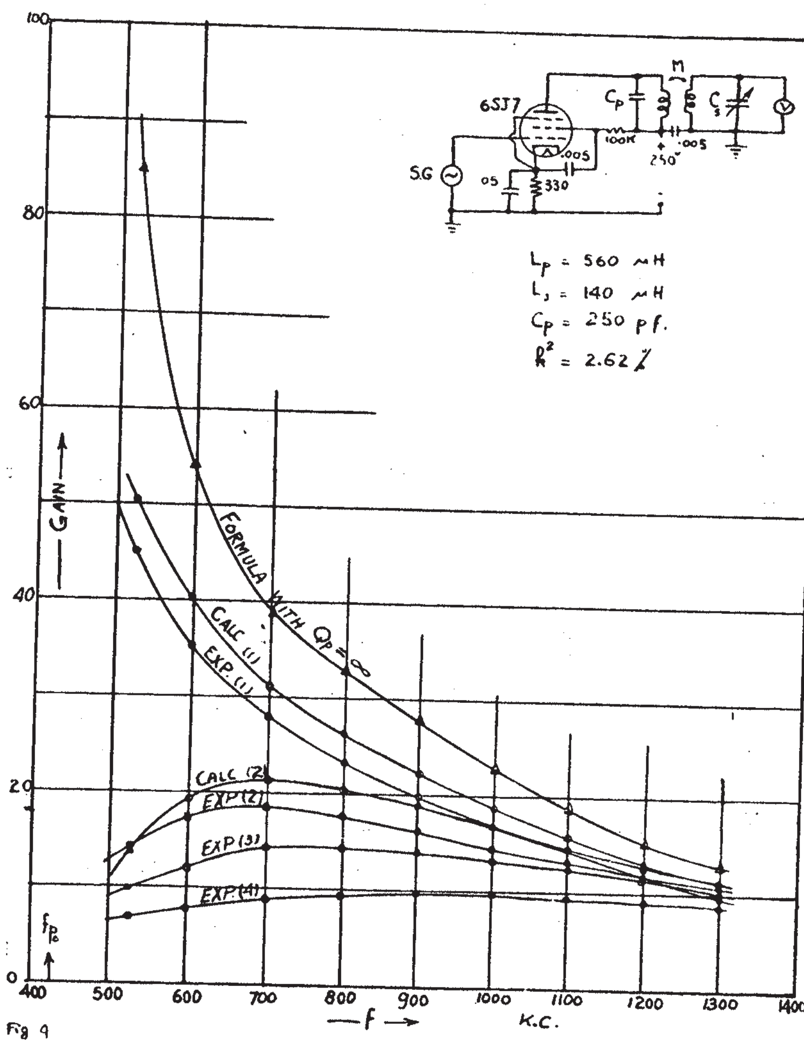

Fig 9

Fig. 9.—Calculated and observed performance of experimental amplifier

of the primary coil and the tube resistance constituting the entire primary losses. The curves marked (2), (3), and (4) were taken with resistors of 6, 2·7, and I kilohms respectively across the primary coil. These added resistors damp the primary coil considerably at the low frequency end of the tuning range, thereby giving a sensibly uniform gain over the band. This method of obtaining a uniform gain is superior to that of using an additional capacitance voltage coupling to level up the gain with frequency, because the latter is critical and requires careful adjustments.

B.—*Selectivity :*

The ratio of the maximum voltage gain to the voltage gain at any frequency is termed the selectivity S of the circuit. For the radio frequency transformer of Fig. 1, this is given by:

$$S = \left[(w^2M^2 + R_sR_p - X_sX_p)^2 + (X_pR_s + X_s R_p)^2\right]^{\frac{1}{2}}_{w} \div \left[X_pR_s + X_sR_p\right]_{wr}$$

which for most practical purposes (see Appendix V) reduces to:

$$S = \pm (\gamma^2 - I) / P \left[(I/P_r Q_{sr}) + k^2 (P_r/Q_{pr})\right] \quad . \quad . \quad (9)$$

For the radio frequency amplifier of Fig. 6 (a), the selectivity S' is given by:

$$S' = \gamma S \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

If the secondary circuit alone is considered, the selectivity S″ is given by:

$$S'' = \pm Q_{sr} (\gamma^2 - I) \quad . \quad . \quad . \quad , \quad . \quad (11)$$

where $\gamma = w/w_r$.

Fig. 10 shows the observed results of selectivity for the experimental transformer described before, together with the values calculated from equations (9) and (11). It will be seen that the selectivity of the transformer is appreciably different from that of the secondary alone. In particular, the selectivity is strongly impaired at frequencies which are on the same side as the primary circuit resonance frequency. This implies that in superheterodyne receivers, since the oscillator frequency is usually higher than the signal frequency, large primary transformers must be used. This precaution is taken to ensure that image rejection may be adequate.

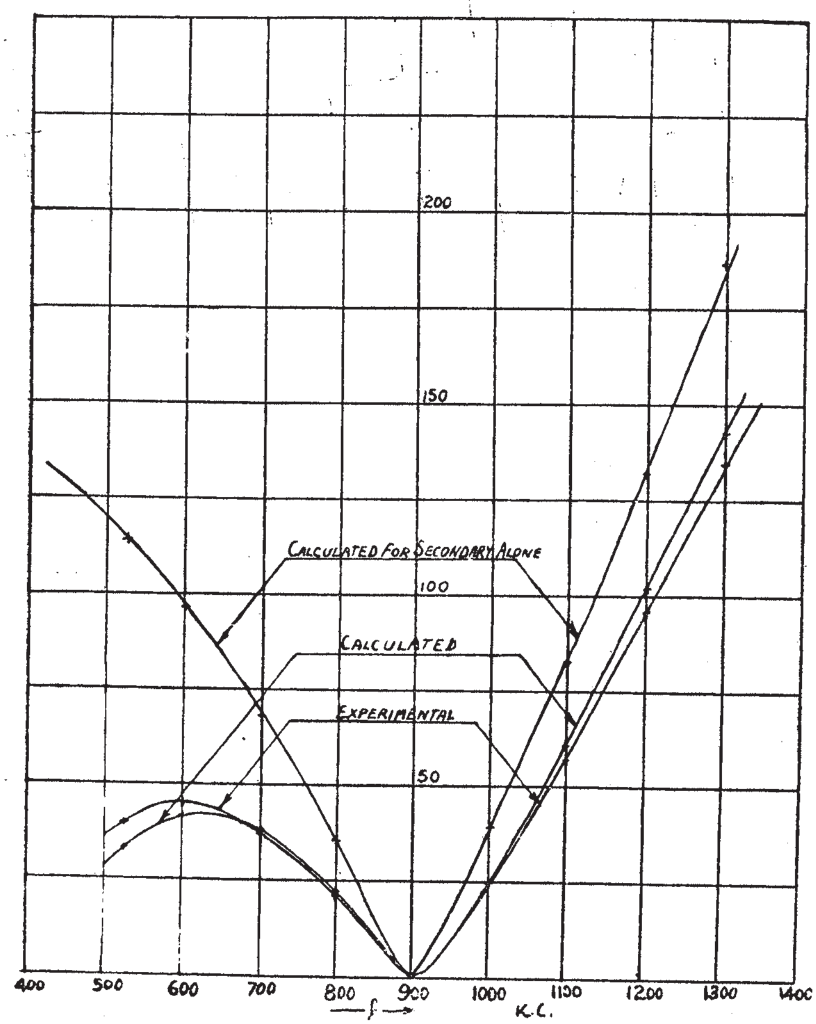

Fig. 10.—Calculated and observed selectivity of experimental amplifier

Equation (9) also shows that if the coefficient of coupling is varied, maximum selectivity is obtained when $k = 0$, and closely approaches that of the secondary alone *i.e.* equation (10).

Furthermore, if the coefficient of coupling is adjusted to the optimum value $k_{op.} = \sqrt{Q_{pr}/Q_{sr}P_r^2}$, the selectivity is halved. For any value of k, the transformer selectivity is less than that of the secondary alone.

C.—*Ganging and Tracking:*

The amount of detuning of the secondary necessary to obtain maximum gain is given by $(F\beta_r^2 - 1)$ which varies along the tuning range according to whether the primary is large or small. When there are more than one transformer, the tuning condensers are usually ganged. Perfect alignment is achieved when the overall gain of the ganged circuits is equal to the product of the gains of the constituent circuits when separately adjusted to maximum voltage transfer ratios. This does not usually occur at all points of the tuning range unless the circuits are perfectly identical.

Fig 11 shows the amount of detuning $(F\beta_r^2 - 1)$ against β_r for various values of k. In a large primary transformer, the amount of detuning tends to be sensibly constant over a tuning range of more than 3 : 1, provided that β_r is kept greater than a certain limiting value depending on the coefficient of coupling. In a small primary transformer, the amount of detuning varies widely along the tuning range, but its value is neglegibly small if β_r is kept smaller than a cartain limiting value which depends also on the coefficient of coupling. The so-called limiting values of β_r may be calculated on the basis that the variation in the amount of detuning does not exceed $\pm 1\%$ over a tuning range of say 3 : 1. This means that if the secondary inductance L_s is calculated from the relation :

$$w_r^2 L_s C_s = F \beta^2 \qquad (12)$$

then the maximum deviation ΔC_s of the tuning condenser from the correct value C_s which gives maximum gain, will correspond to a ratio of $\Delta C_s/C_s = 1\%$. In other words, if Q_s does not exceed 100, the loss of gain due to ganging does not exceed

3 dbs. The frequency at which this loss may occur is close to the end of the band which is nearest to primary circuit resonance.

In Fig. 11, the points marked B are the selected limiting values of β_r, and those marked A are the best ganging points at which equation (12) may be applied to determine L_s. The limiting values of β_r are also plotted against k^2 on the same figure.

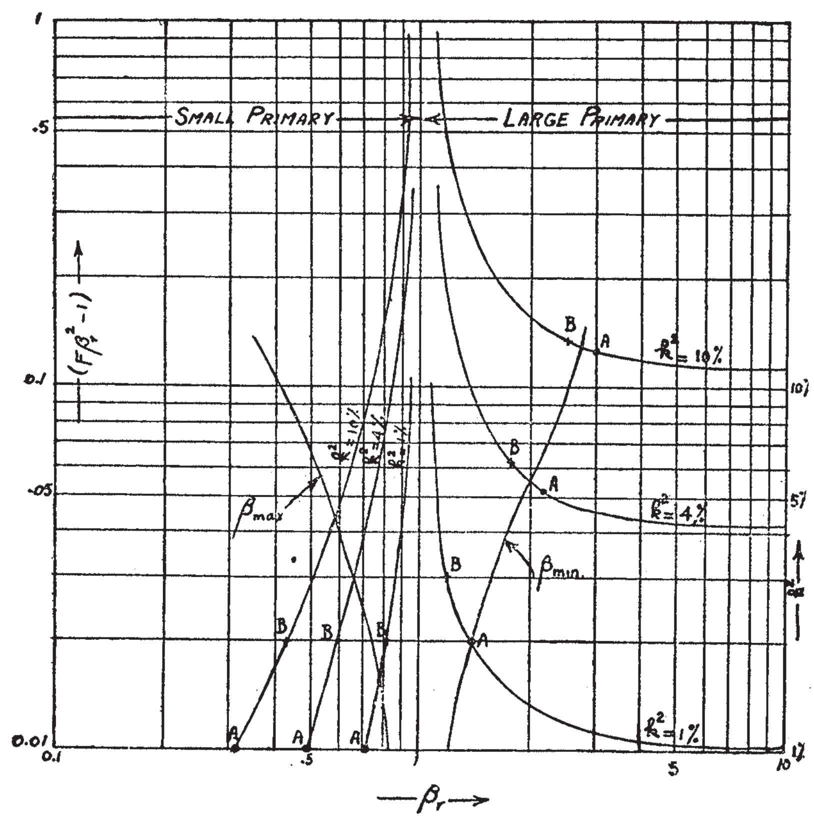

Fig. 11-Amount of detuning necessary for maximum gain

In practice, to simplify the process of alignment and tracking, it is advantageous to use the same type of radio frequency transformer throughout the signal circuits. Preference is usually given to the large primary type of signal transformer. For calculations of the oscillator tracking, it will be necessary to determine the total effective capacity in the secondary circuit of each signal transformer at the frequency of maximum voltage transfer ratio. This capacity is given by :

$$C_{s_{eff.}} = I / w_r^2 \, L_s = C_s / F \, \beta_r^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

APPENDIX I

Condition for Maximum Voltage Transfer Ratio.

$$T = -M / C_s \left[(w^2 M^2 + R_s R_p - X_s X_p)^2 + (X_p R_s + X_s R_p)^2 \right]^{\frac{1}{2}}$$

$$= -w \, M / \left[(w^2 M^2 + R_s R_p - X_s X_p)^2 / (w \, L_s - X_s)^2 + (X_p R_s + X_s R_p)^2 / (wL_s - X_s)^2 \right]^{\frac{1}{2}}$$

In order that this may be a maximum w. r. t. X_s, the differential of the denominator must be equated to zero, thus :

$$w \, L_s \, X \;\; (R_p^2 + X_p^2) + R_s^2 \; (R_p^2 + X_p^2) = w^2 M^2 \; (w \, L_s \, X_p + X_s X_p - 2 \, R_s R_p - w^2 M^2)$$

Since the operating conditions are such that the primary circuit is mostly reactive, then R_p^2 may be neglected compared to X_p^2. Also, since the secondary magnification factor $Q_s \gg I$, then :

$2 \, R_s R_p \ll w \, L_s X_p$, and $R_s^2 \ll w \, L_s \, X_s$, hence :

$w \, L_s X_s X_p^2 \simeq w^2 M^2 [w \, L_s X_p + X_s X_p - w^2 M^2]$ from which :

$X_s X_p (w \, L_s \, X_p - w^2 M^2) = w^2 M^2 (w \, L_s \, X_p - w^2 M^2)$, giving :

$$w^2 M^2 = X_s X_p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In the above analysis, it was assumed that the primary is mostly reactive, *i. e.* $X_p \gg R_p$. In order that the formulae in this analysis may be used with good accuracy, the value of Q_p must exceed a certain limiting value determined by the type of primary and the coefficient of coupling. Using Fig. 11 for the limiting values of β, and the relation that:

$X_p / R_p = Q_p \left(I - \frac{1}{\beta^2} \right)$, the minimum values of Q_p may be determined by assuming a reasonable value of the ratio X_p / R_p. Should the primary losses be so exceptionally high that the actual values of Q_p are less than those calculated from above relation, the value of β_r should be modified accordingly, *i.e.* use larger $\beta_{min.}$ for a large primary and a smaller $\beta_{max.}$ for a small primary. Usually, values of X_p / R_p greater than 2 make the above formulae sufficiently accurate.

Appendix II

Maximum Voltage Transfer Ratio

$$T_{max.} = M/C_s \left[R_s (R_p + JX_p) + JX_s R_p \right]$$

Assuming that $R_p \leqslant X_p$, then:

$$T_{max.} = \left[M/C_s X_p \left(R_s + \frac{w^2 M^2}{X_p^2} R_p \right) \right]_{wr}$$

$$= w_r M w_{po}^2 w_r L_s / w_r^2 F X_{C_{pr}} (\beta_r^2 - I) R_s \left[I + (w_r^2 k^2 L_s L_p R_p w_r^2 C_p^2 L_p) / R_s (\beta_r^2 - I)^2 L_p \right]$$

$$= w_r M w_r C_p Q_{sr} (I - P_r k^2) / \left[I + k^2 P_r^2 (Q_{sr}/Q_{pr}) \right] (\beta_r^2 - I)$$

$$= k Q_{sr} \sqrt{L_s / L_p} P_r G_r \quad . \quad . \quad . \quad . \qquad (2\text{ b})$$

Appendix III

Experimental Data on Large Primary Transformer

$L_p = 560$ uH, $C_p = 250$ pf, $f_{po} = 425$ KC, $L_s = 140$ uH, $k^2 = 2{\cdot}62\%$, $k = 16{\cdot}2\%$

Common Data				Added Resn[istance in] Series with Primary						Calculated T_{max} For $Q_p = \infty$
				Zero			100 Ohms			
f KC	β_r	P_r	Q_{sr}	[Q]po	Observed $T_{max.}$	Calculated $T_{max.}$	Q_p	Observed $T_{max.}$	Calculated $T_{max.}$	
525	1·23	3	125	57·5	16·2	18·5	14	9·3	9·04	30·4
800	1·88	1·4	163	41	13·4	14·8	16·6	10·8	11·8	18·5
1000	2·35	1·22	168	32	12·3	13·3	16·7	10·7	11·6	16·6
1300	3·06	1·12	131		11	10	18·1	10·4	9·2	11·8

The above tabulated results give samples of the observed and calculated data on the experimental transformer. During the experiment the values of Q_s and Q_p were measured on the circuit magnification meter before the transformer was assembled. The measured values were corrected for the effects of self capacity. A further correction was made to include the tuning condenser losses. The self capacity of the primary coil was found 10 pf, that of the secondary 12·5 pf. The coefficient of coupling was measured by the usual open and short secondary method employing the circuit magnification meter at a fixed frequency. The observed results of measurements at several fixed frequencies in the range 400 to 500 KC gave a value of $k^2 = 0{\cdot}0262$.

Appendix IV

Performance of Radio Frequency Amplifier Stage

In this experiment, the transformer described in Appendix III was used with a pentode Type 6SJ7. The added capacity across the primary coil together with the strays, tube, and coil self capacity were adjusted to 250 pf so that the primary circuit resonated at 425 KC. The operating condition of the tube was such that the measured value of mutual conductance was $g_m = 2{\cdot}2$ ma/v. Both screen grid and cathode bias were adequately by-passed, and the input signal to the grid of the amplifier tube was kept constant at 0·5 volt rms.

The following tabulated results give samples of the observed and calculated data on the amplifier stage:—

Common Data		Added Resistance across Primary Coil								Calculated T_{max} For $Q_s = \infty$
		∞			K 6			2. 7K	IK	
f KC	β_r	Q_p	Observed $T_{max.}$	Calculated $T_{max.}$	Q_p	Observed $T_{max.}$	Calculated $T_{max.}$	Observed $T_{max.}$	Observed $T_{max.}$	
525	1·23	55	45	50	6·45	14·2	13·6	10	6·94	82·5
800	1·88	41	23·4	26·4	15·9	17·8	20·6	14·4	9·5	33
1000	2·35	32	16·9	19	19·5	14·8	17·1	13·3	9·8	23·6
1300	3·06	30	11·24	10·9	23·8	10·7	9·65	10·1	8·5	12·9

Appendix V

Selectivity

$$\left[\left\{(w^2 M^2 + R_s R_p - X_s X_p)^2 + (X_p R_s + X_s R_p)^2\right\}_w\right]^{\frac{1}{2}} \cong \left\{X_s X_p\right\}_w$$

The above approximation is justified at all frequencies sufficiently far from the natural resonance frequencies of either primary or secondary circuits alone. Thus, the selectivity is given by:

$$S = [X_s X_p]_w / [X_p R_s + X_s R_p]_{w_r}$$

But $X_s = wL_s - (1/wC_s) = wL_s\left[1 - (1 - P_r k^2)/\gamma^2\right]$

$$\cong wL_s[1 - (1/\gamma)]$$

and $X_p = wL_p - (1/wC_p) = wL_p/P$, hence:

$$[X_p R_s + X_s R_p]_{w_r} = w_r L_p w_r L_s[(1/P_r Q_{sr}) + k^2 P_r/Q_{pr}],$$

which gives:

$$S = (\gamma^2 - 1) / P\left[(1/P_r Q_{sr}) + (k^2 P_r / Q_{pr})\right] \quad . \quad (9)$$

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