NEW TYPES OF ELECTRONIC MEASURING INSTRUMENTS (*)

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SUMMARY

Three novel electronic measuring instruments are presented which may be used to measure power, power factor, voltage, current or impedance. They use two tubes, operated as cathode followers, in a push-pull fashion, so that the inherent property of a wide-range linear characteristic is used with advantage. The new principle underlying the operation of the tubes involves the commutation of their currents in alternate half cycles by the help of a square-wave generator. The resulting average plate current is a direct measure of the product I cos ϕ. A rectifier gives a direct current proportional to the alternating voltage, and an electrodynamic instrument multiplies the two direct currents, thus giving a direct reading of the power.

The instruments are characterised by their simplicity, reliablility, high accuracy and stability. They have exceptionally good features and predictable performance over a frequency range extending from the lowest power frequencies to a predicted value of about 30 Mc when using high-frequency tubes; the main limiting factor being the stray capacities across the output of the

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square-wave generator. There are, however, some other factors which tend to lower this operating limit, and 10 Mc is considered a reasonable higher limit.

I.—Introduction

The direct measurement of power at power frequencies and the lower audio frequencies is most conveniently carried out by using an electrodynamic instrument. Alternatively a double-magnet induction instrument may be used at power frequencies. For the measurement of small amounts of power, or for audio- and high-frequency work, these instruments are not suitable, owing to their well-known inherent drawbacks. For such cases, many indirect methods have been devised, but even then, the power factor of the circuit has to be already known.

Many investigators have suggested different instruments which they claimed to be suitable for the direct measurement of power at high frequencies. A number of the early attempts involve the use of two similar square-law elements, which may be thermal (hot-wire or thermocouple elements) or electronic tubes working in the square-law range of their characteristics (1-4). In a later attempt (5) the tubes were operated in their logarithmic range. All these devices have the disadvantages that the two non-linear elements have to be exactly matched which is difficult to perform, replacement of the elements will affect the calibration, and the circuit requires frequent adjustment. As regards the electronic tubes, even when matched, the desired relation of their characteristics holds only over a limited range. Voltage dividers

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(1) Turner H. M. and McNamara F. T., "An electron tube wattmeter and voltmeter and a phase shifting bridge", Proc. I.R.E., Vol. 18, pp. 1743-1747, October 1930.

(2) Lange H., "Power measurements with valves", Arch. f. Elektrot., 26, pp. 570 579, Aug. 3. 1932.

(3) Mallett E., "Valve wattmeter" J.I.E.E., 73, pp. 295-302, Sept. 1933.

(4) Malling L. R., "Electronic wattmeter", Electronics, 18, pp. 133-135, Nov. 1945.

(5) Marconi's Wireless Telegraph Co. Ltd. El-Shishini M. and El-Said M. A. "An Improved Electrical Device for Effecting the Multiplication or Division of Independent Quantities", British Patent 585936.

will thus be unavoidable, and these cause undue insertion loss in the circuit and trouble at high frequencies. Moreover, the dissipation loss of the current circuit has to be high if any reasonable accuracy is to be expected, especially with the thermal devices. Supply voltage variations, besides causing calibration errors, require frequent adjustment of the zero setting. For all the square-law devices, the insertion unit is of T- or π-configuration, with which it is difficult to provide for a satisfactory ground system.

Amongst other attempts, one may distinguish between three distinct classes:—

(*a*) Devices using electrostatic methods. These are not handy and too delicate. Difficulty is encountered in their adjustment and setting. They require high voltages for their operation.

(*b*) Devices using oscillographs. These either measure instantaneous values of power, or give an indirect measure of the average value through tedious and inaccurate operations. They also suffer from lack of sensitivity.

(*c*) Devices using multi-electrode tubes [1-3]. The tubes are arranged to operate on the linear parts of their various grid characteristics, which are too small to avoid rectification, and the instruments were found to give readings with current or voltage alone, even with push-pull operation. Discrepancies in the characteristics of individual tubes of a given type are serious, and the choice of matched tubes is too difficult. Frequent check calibration and zero-setting adjustment is necessary. The use of a voltage divider, with its inherent disadvantages, is unavoidable.

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(1) Wagner T. B., "Thermionic measuring instrument", Elect. Eng., 53, pp. 1621-1623, Dec. 1934.

(2) Pierce J. R., "Proposed wattmeter using multi-electrode valves", Proc. I.R.E., 24, pp. 577-583, April 1936.

(3) Wey R. J., "Thermionic wattmeter", Wireless Eng., 14, pp. 490-495 and 552-555, Sept. and Oct. 1937.

To this class a new instrument has been recently added([1]), having improved features. The tubes operate in the exponential range of their characteristics. However, this device still suffers from the disadvantages of a limited operating range for the control-grid voltage, and that it cannot be used for circuits having a voltage below 20 volts. The requirement of recalibration when replacing tubes, the necessity of stabilised-supply or battery operation, and the unavoidable rectification errors (though claimed to be small) resulting from the discrepancy of the parameters of the tubes themselves from the requisite values under this mode of operation, are all inherent disadvantages.

The instruments described in this paper use two tubes operating in push-pull and working mainly on the linear parts of their characteristics, which usually cover a wide range, and this range can be extended and made as linear as desired by suitable adjustments of the external circuit, thus rectification errors are almost eliminated and voltage dividers are dispensed with. Critical adjustment of the circuit elements is not a necessity in this mode of operation. Negative current feedback is used with advantage, which results in extended linear range, increased stability, less trouble with tube replacements, and insensitivity of calibration and zero setting to supply voltage variations.

The instruments are characterised by their simplicity in theory, circuit design, operation and adjustment. Almost any tube type can be used, and the performance can be readily predicted from knowledge of the normal tube parameters and circuit elements. A unique main feature lies in the fact that individual d-c components which are proportional to the individual a-c components are available in the circuit, so that direct readings of voltage, current, power factor and power can be readily obtained with the same instrument with complete reliability and stability. Furthermore, the instruments are suitable for measurements at

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([1]) El-Said M. A. H., "Novel multiplying circuits with application to electronic wattmeters", Proc. I. R. E., Vol. 37, pp. 1003-1015, Sept. 1949.

very low power factors. They will be, therefore, found useful in measuring impedance/frequency and power-factor/frequency characteristics of a network or apparatus under actual operating conditions.

Amongst the good features of the instruments are the uniform scale of the indicating instrument, the low insetion loss of the series resistor in the current circuit, the high accuracy and the simplicity of adapting the instrument for multirange use.

For unavoidable reasons relating to the lack of certain equipment, stress was given to obtaining the maximum possible output power from the tubes, which forced the use of normal receiving tubes. However, with the use of high-frequency tubes, it is shown that a frequency range comparable with any other type of instrument can be covered. Predicted higher limits of about 30 Mc at unity power factor and 3·5 Mc at zero power factor were obtained. Nevertheless, factors not considered are expected to cause trouble at the higher frequencies, especially with the square-wave generator, and 10 Mc is considered a reasonable higher limit.

The errors resulting from other possible sources are discussed and found remarkably small.

Three types of this instrument have been developed. These are :—

(*a*) The grid-controlled type.

(*b*) The suppressor-controlled type.

(*c*) The cathode-controlled type.

II.—General Outline

The main idea of these commutating-type instruments lies in using an electronic tube having a fairly linear transfer characteristic of control-grid voltage versus plate current, which is readily obtained through the use of negative current feedback. Then a signal is applied to the control grid, and the tube current is commutated only during a proper interval of its cycle. The commutation period is controlled by applying a square-wave signal having the same frequency as the control-grid signal, and of sufficient amplitude, to one of the tube grids, thus allowing the plate current to flow only during one-half of the square-wave cycle. The average value of the plate current follows, linearly, the signal amplitude and the cosine of the phase angle between the signal and the square wave. The square-wave voltage is obtained from a square-wave generator designed such that its output is of the same frequency and in phase with the voltage applied to its input.

Figure 1-a shows the circuit arrangement for the measurement of power factor, using a triode. The voltage V across the load is applied to the input of the square-wave generator, whose square-wave output is applied to the control grid of the tube. The voltage drop IR across a small resistance, R, in which the load current passes, is applied to the control grid of the tube, superposed on the square wave. The change in the average value of the output current will be proportional to I cos ϕ, ϕ being the phase angle between V and I. If I is known or measured (which can be done by the same instrument), or if the voltage IR is adjusted to a predetermined value, or if a suitable dividing circuit is added to give the quotient of the two quantities I cos ϕ and I, the device will give a direct reading of cos ϕ.

It is obvious that if the input to the square-wave generator were derived from the load current circuit, cos ϕ will be unity, and the device would be an ammeter with a linear scale. In the same manner if the signal applied to the grid of the tube were derived from the voltage circuit, the device would be a voltmeter with a linear scale.

Now, for the measurement of power, the product I cos ϕ has already been obtained in the form of a direct current, and it now remains to obtain a direct current proportional to V through the use of a conventional rectifying circuit. The two direct currents are multiplied most easily by the use of an ordinary electrodynamic instrument, and the device will then be a direct-reading wattmeter with a linear scale.

In some circuit arrangements, and especially in high-frequency work, and in order to provide a suitable grounding point for the circuit, it may be found more convenient to apply the two signals to different grids of the tube. In Fig. 1-b the square-wave signal is applied to the screen grid of a pentode

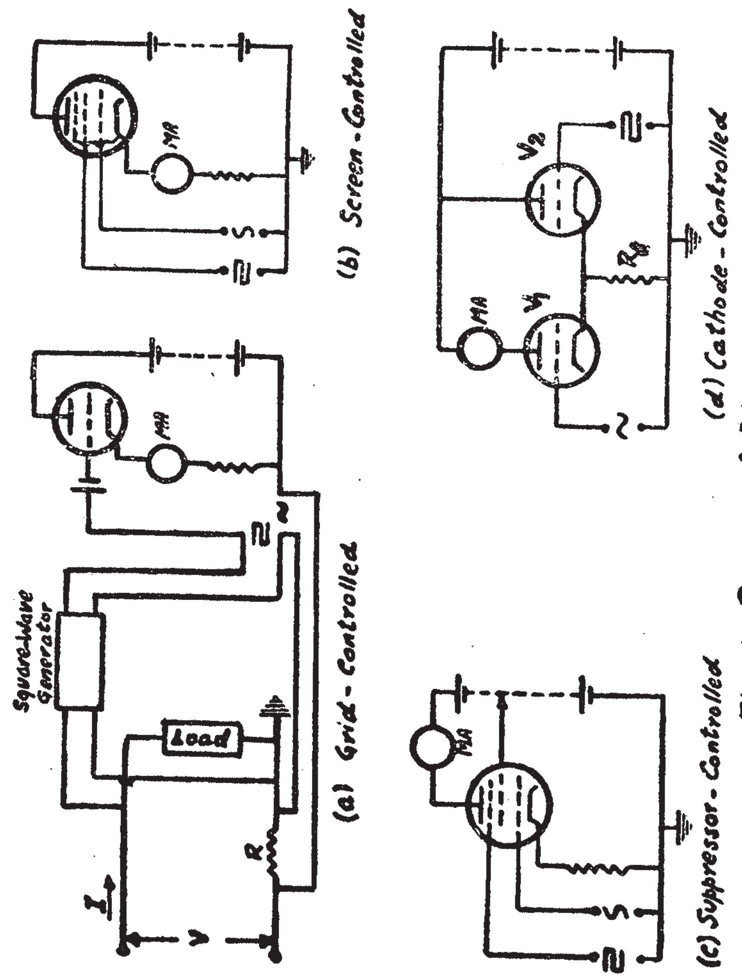


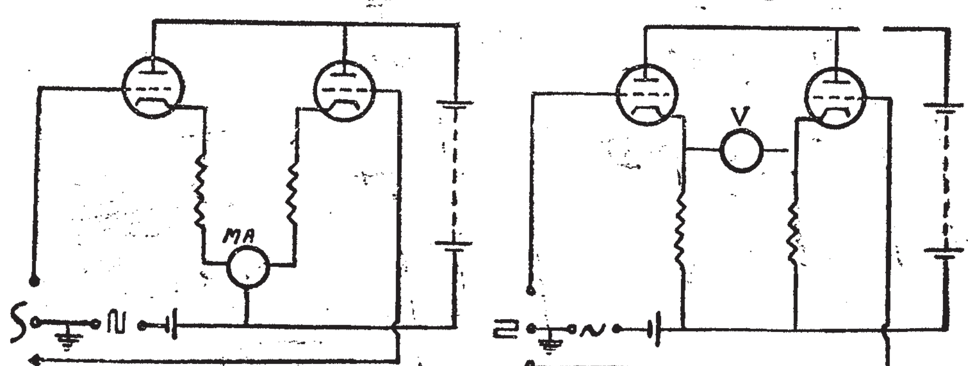
Fig. 1 - Principle of I cos φ Meter.

whose suppressor grid is internally connected to the cathode. With this procedure severe requirements are imposed on the square-wave generator, which has to provide a positive wave of large magnitude and be capable of supplying the screen-grid current of the tube; and moreover, if a push-pull arrangement is used, both sides of its output should give exactly equal amplitudes of the square wave.

In Fig. 1-c the square wave is applied to the suppressor grid of a pentode. It requires a square-wave generator giving a negative signal, and thus precludes all these difficulties.

In Fig. 1-d two tubes having a common cathode resistor are used, which may be triodes or multigrid tubes. Each signal is applied to the control grid of one tube. When V_2 is conducting, the voltage drop across the common cathode resistor R_k will cause V_1 to be biassed beyond cut-off; while when V_2 is cut-off, V_1 will be conducting. This circuit has the advantage that it requires a square wave of small amplitude, and will consequently be suitable for use at much higher frequencies.

All circuit arrangements are well adapted for use in a push-pull fashion. This is shown in Fig. 2 for the grid-controlled type. The arrangement of the input circuits as shown in Fig. 2-b is preferable from the point of view of providing a suitable



(a) Differential Connection (b) Bridge Connection

Fig. 2. Principle of Push-Pull $I \cos \phi$ Meter.

ground point for the circuit. A differential-wound d-c indicating instrument can be used as in Fig. 2-a, or a normal instrument may be used as in Fig. 2-b.

The use of a push-pull circuit has the merits of obviating the necessity of compensating the zero-signal tube current, reducing zero-setting variations with supply voltage variations, reducing the errors due to any possible non-linearity in the tube characteristics, and finally increasing the sensitivity of the instrument, thus reducing the insertion loss of the series resistance in the load current circuit.

The merits of the cathode-follower circuit are well known. Its main advantages relating to the present type of work lie in providing a wide-range linear transfer characteristic, whose slope mainly depends on the value of the cathode resistor and is almost independent of the tube parameters, and reducing tube current variations with supply voltage variations. Therefore, no readjustment or recalibration is necessary when replacing tubes, no readjustment of zero setting is necessary during operation, and the stability of the reading is highly increased.

III.—Grid-Controlled Commutating Power-Factor Meter

1.—*Circuit:*

The actual circuit of this type is shown in Fig. 3. For zero setting a variable resistance and change-over switch are provided in the cathode circuits as shown. The d-c characteristic of one triode section is shown in Fig. 4. It has a linear range beginning from —4 volts upwards. The upper limit of the useful working range is at about + 20 volts, at which point the grid-cathode potential is zero. The operating point is chosen at about the middle of the useful working range, *i.e.* a positive bias of about + 8 volts is required. An a-c signal up to 12 volts peak (8·5 volts RMS) can be applied to the control grid.

The push-pull square-wave generator has to carry the grid potential in the negative direction so that the tube is cut-off with the maximum value of the a-c signal, and thus the square-wave generator shall give, from each side of its output, zero potential during half a cycle and a negative potential of more than

32 volts during the other half cycle. It is not essential that both sides should give equal negative voltages.

2.—*Square-Wave Generator:*

The circuit is shown in Fig. 5. It is simple and direct. Clipping is done by the double-diode clipper tubes as well as by the amplifier stages. Therefore, the effect of stray capacities, which appears most in the higher portions of the wave, can be neglected in all stages prior to the grids of the output tubes. Stray capacities in the output circuit are the main limiting factor of the frequency range in which the square-wave generator may be used, since no clipping stage follows. It is possible to extend the frequency range through the addition of a clipping stage, the use of high-frequency tubes for the power-factor meter, minimising stray capacities of the connections, reducing the output resistance of the square-wave generator through the use of high plate voltages and more powerful tubes, and adding compensating inductances in the square-wave generator circuit.

The square-wave generator is suitable for use with input voltages covering a very wide range, from a fraction of a volt to several hundred volts, and can therefore be connected directly to the load voltage circuit without the necessity of using a voltage divider. The input circuit has a high impedance which does not cause any appreciable insertion loss. However, if a much higher input impendance is desired, or if it is required to work with very small input voltages, a single-tube conventional amplifier stage can be added at the input. This may use a cathode-follower circuit if the input voltage is not too low.

The square-wave generator must give equal intervals of its two output waves. This can be easily checked by applying the square-wave voltages alone to the power-factor meter, which should then show no indication. Errors in timing may be caused by harmonics existing in the input voltage when these harmonics are not in phase or in antiphase with the fundamental. These errors increase with the harmonic percent and the phase difference

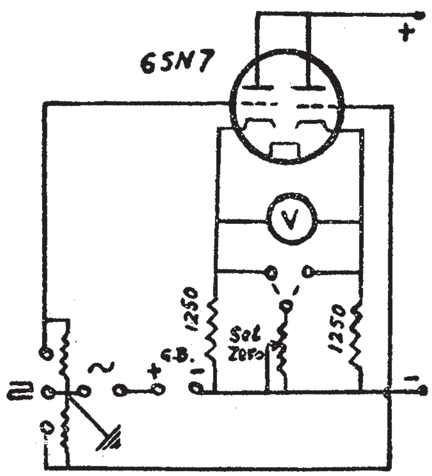


Fig. 3 - Connection Diagram of Grid-Controlled I cos φ Meter

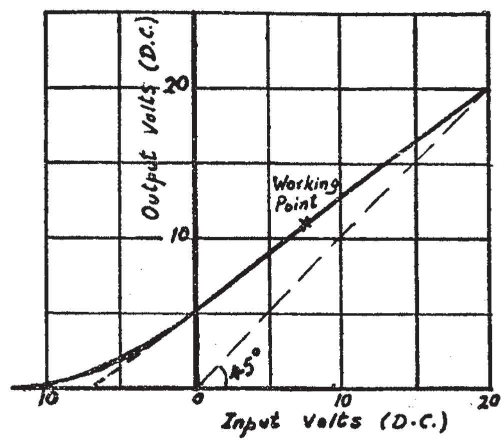


Fig. 4 - Static Characteristics for One Triode Section of Fig. 3.

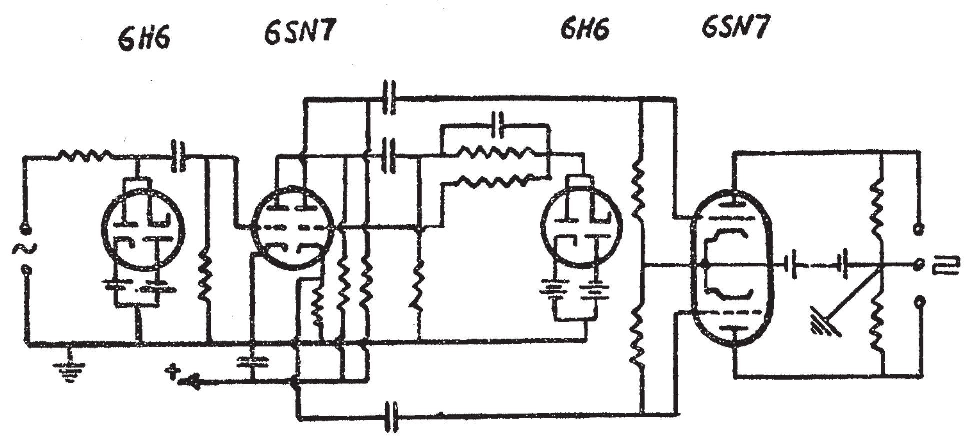


Fig. 5 - Square-Wave Generator for Use with Grid-Controlled I cos φ Meter.

between the harmonic and the fundamental, and decrease for harmonics of higher order. However, the error in timing produced by harmonics that occur in practical cases, even in the worst cases of their operation, can be shown to be negligible, of the order of 1° in 180°. Moreover, the timing of the square-wave generator can always be adjusted by applying a small variable d-c voltage at its input, superposed on the a-c voltage.

3.—*Operation of the Power-Factor Meter:*

(a) *Ideal Operation:*

In this case it is assumed that the two tubes have identical and exactly linear transfer eharacteristics down to the cut-off point. The magnitudes of the applied signals are such that in the half cycle when the square-wave voltage is zero, the a-c signal will not drive the grid of the tube neither more positive than the cathode nor more negative than the cut-off point; and in the other half cycle, when the square-wave voltage is negative the a-c signal will not drive the grid more positive than the cut off point.

Let S denote the slope of the transfer characteristic of either tube, which can be shown to be given by

$$\frac{\mu}{\mu+1}\,\frac{1}{R_k+\frac{\rho+R_a}{\mu+1}},$$ where μ is the amplification factor,

ρ is the plate internal impedance,

R_a is the plate load impedance.

I_o the steady no-signal plate current of either tube,

T the time period of one cycle $= \frac{1}{f} = \frac{2\pi}{\omega}$, f being the frequency,

F sin $(\omega t - \phi)$ the a-c. signal applied to the grid, ϕ being the phase angle between this signal and the square-wave signal.

I_1 and I_2 the average currents of the two tubes when the signls are applied.

then: $$I_1 = \frac{1}{T}\int_0^{\frac{T}{2}} \left[I_o + SF \sin(\omega t\text{-}\phi) \right] dt = \frac{I_o}{2} + \frac{S}{\pi} F \cos\phi$$

$$I_2 = \frac{1}{T}\int_{T/2}^{T} \left[I_o + SF \sin(\omega t\text{-}\phi) \right] dt = \frac{I_o}{2} - \frac{S}{\pi} F \cos\phi$$

and the average current difference is given by:

$$I = I_1 - I_2 = \frac{2S}{\pi} F \cos\phi \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

which is proportional to the product of the magnitude of the applied a-c signal and the power factor.

(b) *Effect of Different Transfer Characteristic Slopes*:

In a cathode-follower circuit, the tube current is almost independent of the tube parameters, and the slope of the transfer characteristic S is mainly defined by the value of R_k. However, if for any reason the two tubes give different transfer characteristic slopes S_1 and S_2, the reading of the indicating instrument will be:

$$I = \frac{S_1+S_2}{\pi} F \cos\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

That is to say, the average value $\frac{S_1+S_2}{2}$ is to be considered instead of S, and the reading is still proportional to the product of the magnitude of the a-c signal and the power factor.

(c) *Effect of Error in Square-Wave Timing*:

It has been stated that the square-wave generator can always be adjusted to give exact timing. However, it is interesting to show that even without such an adjustment, the error produced in the power-factor meter indication is very small.

If T_1 and T_2 are the time intervals of the square wave, such that $T_1 + T_2 = T$, and writing $\omega T_1 = \pi + \delta$, $\omega T_2 = \pi - \delta$, or $\omega (T_1 - T_2) = 2\delta$, then:

when the square-wave signal only is applied, the indicating instrument will read $I_o \dfrac{T_1 - T_2}{T} = I_o \dfrac{\delta}{\pi}$.

When the signal $F \sin(\omega t - \phi)$ is applied:

$$I = \frac{1}{T_1}\int_0^{T_1}\left[I_o + SF\sin(\omega t - \phi)\right]dt$$

$$-\frac{1}{T}\int_T^{T_1}\left[I_o + SF\sin(\omega t - \phi)\right]dt$$

$$= I_o\frac{\delta}{\pi} + \frac{S}{\pi}F\left[\cos\phi - \cos(\omega T_1 - \phi)\right]$$

and the change in the reading is then given by:

$$\frac{S}{\pi}F\left[\cos\phi - \cos(\omega T_1 - \phi)\right]$$

$$= \frac{S}{\pi}F\left[\cos\phi + \cos(\phi - \delta)\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Comparing this to Eqn. (1), the error in the reading will be:

$$\frac{S}{\pi}F\left[\cos\phi - \cos(\phi - \delta)\right]$$

and the relative error is $\dfrac{\cos\phi - \cos(\phi - \delta)}{2\cos\phi}$

$$= \frac{1}{2}\tan\phi\sin\delta - \sin^2\frac{\delta}{2}$$

and since δ is always very small, then:

$$\text{relative error} \simeq \frac{\delta}{2}\tan\phi - \tfrac{1}{4}\delta^2,$$

and for the estimated value of $\delta = 1°$, this amounts to 0·0076 % at unity power factor and 0·87 % at $\phi = 45°$. This error would naturally be infinite at zero power factor since the true reading should have been zero. However, this is no indication of the error, and the error in indicating instruments is usually and preferably judged by referring to the maximum instrument reading, which is $\dfrac{2S}{\pi}F_{max.}$, hence:

$$\text{Full-scale relative error} = \frac{F}{F_{max.}} \, \frac{\cos\phi - \cos(\phi - \delta)}{2}$$

and for the extreme condition when $F = F_{max.} = 12$ volts,

$$\text{Full-scale relative error} = \frac{\cos\phi - \cos(\phi - \delta)}{2}$$

$$= \sin\frac{\delta}{2}\sin\left(\phi - \frac{\delta}{2}\right)$$

$$\simeq \frac{\delta}{2}\sin\left(\phi - \frac{\delta}{2}\right).$$

For the value of $\delta = 1°$, this error ~~amounts~~ to 0·0076 % at unity power factor, and 0·87 % at zero power factor.

Figure 6 shows this error for different power factors in this extreme case, from which we conclude that this error, if it exists, is always negligible.

(d) *Effect of Non-linearity in Transfer Characteristics:*

(i) Considering that the non-linearity occurs all over the working range of the characteristic, this can be expressed in the form:

$$i = I_o + S\,v + P\,v^2.$$

$$\text{Hence } I_1 = \frac{1}{T}\int_0^{T/2} [I_o + SF\sin(\omega t\text{-}\phi) + PF^2\sin^2(\omega t\text{-}\phi)]\,dt$$

$$= \frac{I_o}{2} + \frac{S}{\pi}F\cos\phi + \frac{PF^2}{4} + \frac{PF^2}{4\pi}\sin 2\phi$$

$$I_2 = \frac{1}{T}\int_{T/2}^{T} [I_o + SF\sin(\omega t\text{-}\phi) + PF^2\sin^2(\omega t\text{-}\phi)]\,dt$$

$$= \frac{I_o}{2} - \frac{S}{\pi}F\cos\phi + \frac{PF^2}{4} + \frac{PF^2}{4\pi}\sin 2\phi$$

and the instrument indication will be: $I = \frac{2S}{\pi} F\cos\phi$ (4)

This is the same as Eqn. (1), and thus no error is produced. This result is to be expected from the push-pull operation of the circuit.

(ii). It is more common that the transfer characteristic is linear along its upper part, and shows non-linearity only in the lower part. Considering an exaggerated case where the non-linearity is assumed to extend up to the static working point of the characteristic, this latter can be divided into two portions, which may be expressed as :

$i = I_o + Sv$ for positive values of the input voltage v,
and $i = I_o + Sv + Pv^2$ for the negative values of v.

Now for the first tube, which is conducting in the time period O to $\frac{\pi}{\omega}$, the applied voltage is $v = F \sin(\omega t - \phi)$, which will then be negative in the time period O to $\frac{\phi}{\omega}$ and positive in the time period $\frac{\phi}{\omega}$ to $\frac{\pi}{\omega}$. The average current of this tube is thus given by :

$$I_1 = \frac{1}{T}\int_{\phi/\omega}^{\pi/\omega}\left[I_o + SF\sin(\omega t - \phi)\right]dt$$
$$+ \frac{1}{T}\int_{0}^{\phi/\omega}\left[I_o + SF\sin(\omega t-\phi) + PF^2\sin^2(\omega t-\phi)\right]dt$$
$$= \frac{I_o}{2} + \frac{S}{\pi}F\cos\phi + \frac{PF^2}{4\pi}\phi + \frac{\sin 2\phi}{2}.$$

For the second tube, which is conducting in the period $\frac{\pi}{\omega}$ to $\frac{2\pi}{\omega}$, the voltage $v = F\sin(\omega t - \phi)$ is positive in the time period $\frac{\pi}{\omega}$ to $(\frac{\pi}{\omega} + \frac{\phi}{\omega})$ and negative in the time period $(\frac{\pi}{\omega} + \frac{\phi}{\omega})$ to $\frac{2\pi}{\omega}$, hence :

$$I_2 = \frac{1}{T}\int_{\pi/\omega}^{\frac{\pi}{\omega}+\frac{\phi}{\omega}}\left[I_o + SF\sin(\omega t - \phi)\right]dt$$
$$+ \frac{1}{T}\int_{\frac{\pi}{\omega}+\frac{\phi}{\omega}}^{\frac{2\pi}{\omega}}\left[I_o + SF\sin(\omega t-\phi) + PF^2\sin^2(\omega t-\phi)\right]dt$$
$$= \frac{I_o}{2} - \frac{S}{\pi}F\cos\phi + \frac{PF^2}{4\pi}(\pi - \phi + \frac{\sin 2\phi}{2}).$$

The instrument indication will then be:

$$I = \frac{2S}{\pi} F \cos\phi \left[1 + \frac{P}{4S\cos\phi} F\left(\phi - \frac{\pi}{2}\right)\right] \qquad (5)$$

Hence the relative error due to non-linearity is given by:

$$\text{Relative error} = -\frac{PF}{4S\cos\phi}\left(\phi - \frac{\pi}{2}\right).$$

This is seen to be proportional to the amplitude of the applied signal, and is not infinite at zero power factor.

Taking actual values from the characteristic curve shown in Fig. 4, the ratio $\frac{P}{S} = 0.008$, and considering the extreme case when $F = 12$ volts, the relative error is shown in Fig. 7 for different power factors. It should be pointed out here that although these values are small, yet they are still too much exaggerated in two ways, the first in considering the non-linearity to extend up to the steady no-signal working point, and the second in taking $F = 12$ volts, while it does not usually exceed a fraction of a volt.

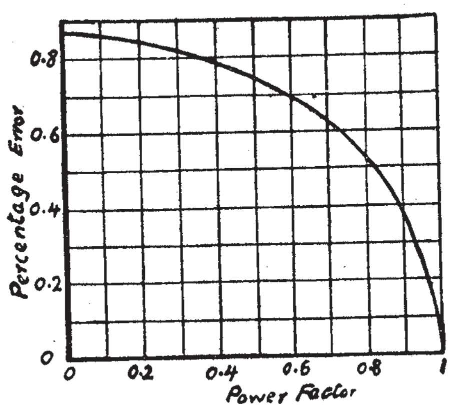


Fig. 6 – Full-Scale Relative Error due to Imperfect Square-Wave Timing for an A.C. Signal Amplitude of 12 Volts.

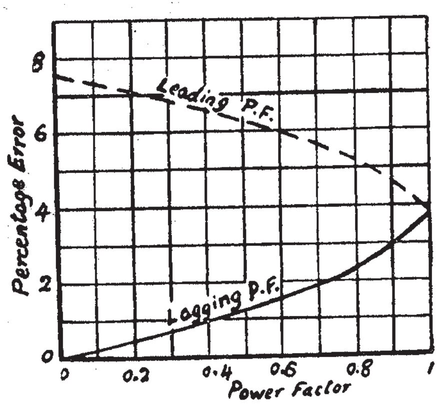


Fig. 7 – Full-Scale Relative Error due to Characteristic Nonlinearity, for an A.C. Signal Amplitude of 12 Volts.

(e) *Effect of Harmonics in the Load Current Waveform:*

Let the a-c signal voltage, in this case, be expressed in the form:

$$v = F_1 \sin(\omega t - \phi) + F_n \sin(n\omega t - \phi_n),$$

where n indicates the order of the harmonic.

Considering perfect square wave and linear characteristic, then:

$$I_1 = \frac{1}{T}\int_0^{\pi/\omega} [I_o + SF_1 \sin(\omega t - \phi) + SF_n \sin(n\omega t - \phi_n)]\, dt$$

$$= \frac{I_o}{2} + \frac{S}{\pi} F_1 \cos\phi \qquad \text{when n is even}$$

or $$\frac{I_o}{2} + \frac{S}{\pi} F_1 \cos\phi + \frac{S}{\pi}\frac{F_n}{n}\cos\phi_n \qquad \text{when n is odd}$$

and $$I_2 = \frac{1}{T}\int_{\pi/\omega}^{2\pi/\omega} [I_o + SF_1 \sin(\omega t - \phi) + SF_n \sin(n\omega t - \phi_n)]\, dt$$

$$= \frac{I_o}{2} - \frac{S}{\pi} F_1 \cos\phi \qquad \text{when n is even}$$

or $$\frac{I_o}{2} - \frac{S}{\pi} F_1 \cos\phi - \frac{S}{\pi}\frac{F_n}{n}\cos\phi_n \qquad \text{when n is odd}$$

and the instrument indication will thus be:

$$I = \frac{2S}{\pi} F_1 \cos\phi \qquad \text{when n is even}$$

or $$\frac{2S}{\pi} F_1 \cos\phi + \frac{2S}{\pi}\frac{F_n}{n}\cos\phi_n \qquad \text{when n is odd.}$$

It is thus seen that the even harmonics produce no effect whatsoever. The odd harmonics, however, produce an error whose relative value is given by $\dfrac{F_n \cos\phi_n}{nF_1 \cos\phi}$.

For a given value of $\cos\phi$, this error is maximum when $\phi_n = 0$. For an exaggerated value of 10% harmonic content, the max. full-scale relative error amounts to 3·33% for the third harmonic, 2% for the fifth harmonic, and so on.

It is, therefore, concluded that errors produced by harmonics, whether in the current or voltage waveforms, are either zero or negligible.

(f) *Effect of Stray Capacities:*

The presence of a stray capacity C across the output, resistance R of the square-wave generator will cause a gradual exponential building up and decay of the voltage instead of an instantaneous one.

With the square-wave voltage applied alone, and no a-c signal, it can be shown that the average tube currents I_1 and I_2 are given by:

$$I_1 = I_2 = \frac{I_o}{2}\left[1 + 2CRf\left(\ln\frac{V_c}{E-V_c} - \frac{E}{V_c}\ln\frac{E}{E-V_c} + \frac{E}{V_c}\,\varepsilon^{-\frac{1}{2CRf}}\right)\right]$$

where f is the frequency,

V_c is the cut-off voltage of the tube,

and E is the amplitude of the square-wave voltage.

If, as usual, E is made $2V_c$, then:

$$I_1 = I_2 = \frac{I_o}{2}\left[1 + 4CRf\left(\varepsilon^{-\frac{1}{2CRf}} - 0{\cdot}6931\right)\right]$$

The zero setting is thus not affected by the increase of frequency or stray capacity, but the average current of each tube decreases. It becomes 86 % of its value at low frequencies when CRf is increased to 0·05.

When a *small* a-c signal of the form F sin (ωt-ϕ) is applied, the instrument indication can be shown to be:

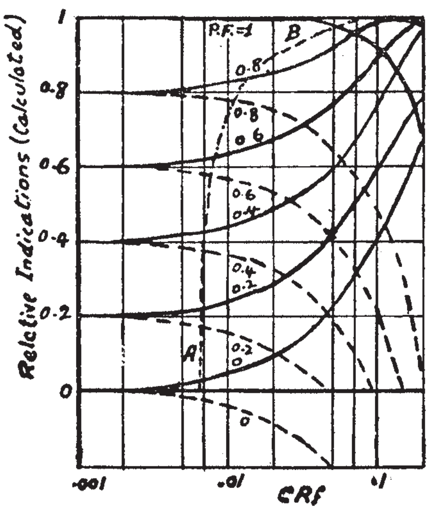
$$I = \frac{S}{\pi}F\left[\cos\left(\phi - 2\pi CRf\ln\frac{E}{E-V_c}\right) + \cos\left(\phi - 2\pi CRf\ln\frac{E}{V_c}\right)\right] \quad . \quad . \quad . \quad . \quad (6)$$

and when $E = 2V_c$, this becomes:

$$I = \frac{2S}{\pi}F\cos(\phi - 4{\cdot}354\ CRf) \quad . \quad . \quad . \quad . \quad . \quad (7)$$

hence Relative error $= 1 - \dfrac{\cos(\phi - 4{\cdot}354\ CRf)}{\cos\phi}$

It is seen that the error is independent of the signal amplitude, but depends on the value of the product CRf. Fig. 8 gives the instrument indication at different power factors as CRf increases, calculated from Eqn. (7). The curve AB shown gives the limiting values of CRf if the error is not to exceed 3% of full scale. This gives a value of CRf=0·0565 at unity power factor and 0·007 at zero power factor. With acorn triodes in a cathode-follower circuit, and with proper layout, the value of C can be kept as low as 10 μμF. Since R is of the order 1000 ohms, therefore the instrument is suitable for use at frequencies as high as 5·65 Mc at unity power factor and 0·7 Mc at zero power factor. However, Eqn (6) shows that these values can be increased by increasing E. Moreover, since the errors at higher frequencies are predictable either by calculation or from curves similar to those of Fig. 8, the instrument can be used for obtaining approximate values at much higher frequencies, provided that f and ϕ are roughly known.



— Lagging P.F. --- Leading P.F.

Fig. 8 - Effect of Stray Capacities on Meter Readings (Theoretical)

IV.—Suppressor-controlled Commutating Power-Factor Meter

1.—*Circuit:*

In this version of the instrument, the square wave is impressed on the suppressor grid of a pentode. When the suppressor grid is driven negative, the anode current falls and the screen-grid current rises, and may attain values in excess of the safe limit. Therefore the developed circuit has in view two purposes: to limit the maximum value of the screen-grid current by lowering the screen potential when the plate current is suppressed, and to make use of the rise of the screen current in increasing the sensitivity. The circuit developed to achieve these two functions is shown in Fig. 9. The screen grid of each tube is cross-connected to the plate of the other tube. Now when one tube is commutating, the other is suppressed. The plate current of the commutating tube, as well as the relatively-high screen current of the suppressed tube, both flowing in the plate-load resistor of the commutating tube, will cause a large voltage drop, so that the voltage at the screen grid of the suppressed tube is largely reduced. The commutating tube, however, has its screen grid connected to the plate of the suppressed tube. Only the relatively-small screen current of the commutating tube flows in the plate resistor of the suppressed tube, so that the voltage at the screen grid of the commutating tube is only slightly less than the full d-c supply voltage of the circuit, and thus almost the normal plate current of the commutating tube is obtained.

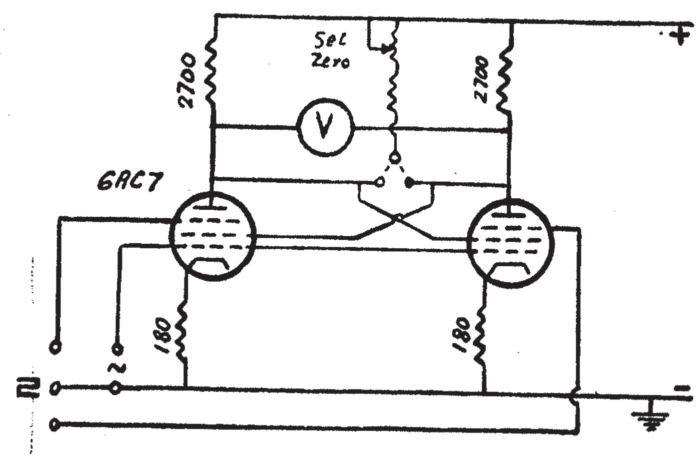


Fig. 9 - Connection Diagram of Suppressor-Controlled I cos φ Meter.

These operations are reversible in each half cycle of the square wave, so that the suppressed tube will always have a low screen-grid potential and the commutating tube a high screen-grid potential.

The square-wave generator used with this type is essentially the same as that shown in Fig. 5, with minor modifications, which mainly imply the addition of a push-pull amplifier stage at the output to cope with the required higher amplitude of the square wave.

The static characteristics shown in Fig. 10 were obtained with the same circuit arrangement of Fig. 9. In Fig. 10-a the suppressor-grid potential of one tube is varied, while that of the other is kept at zero potential. It shows that the amplitude of the square wave has to be not less than 80 volts, so that the currents of the two tubes are independent of the square-wave amplitude. Fig. 10-b was obtained by keeping the suppressor-

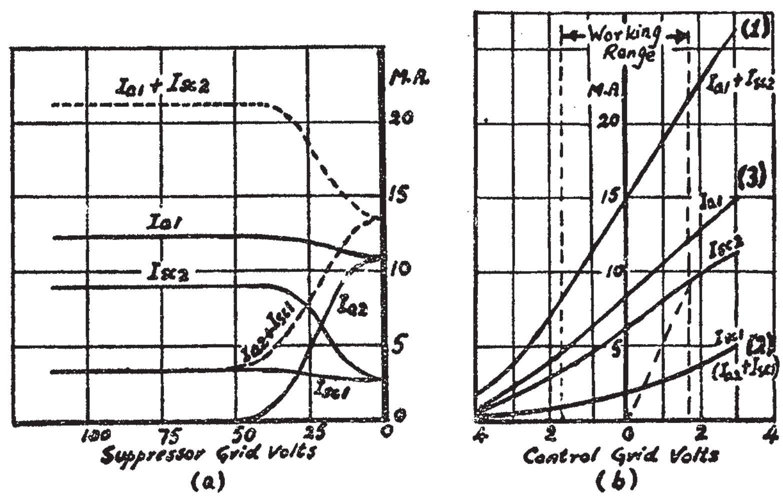


Fig. 10- Static Characteristics of One Tube of Fig. 9.

grid potential of one tube sufficiently negative, while that of the other was kept at zero. It shows that the linear useful range of the characteristic lies between $\pm$ 1·7 volts peak, or an a-c signal of not more than 1·2 volts RMS can be used, and in this case the control grids are never driven positive with respect to the cathodes.

2.—*Operation of the Power-Factor Meter:*

Curves (1) and (2) of Fig. 10-b give the total currents in the plate loads of the commutating and suppressed tubes respectively. They may be represented by the expressions:

$$i_1 = I_{1o} + Av$$

and $$i_2 = I_{2o} + av + bv^2$$

When an a-c signal $F \sin(\omega t\text{-}\phi)$ is applied to the control grids, then:

In the period 0 to $\frac{T}{2}$, tube (1) is commutating and tube (2) is suppressed, thus:

$$i_{1\,av.} = \frac{1}{T}\int_0^{T/2} [\,I_{1o} + AF \sin(\omega t\text{-}\phi)\,]\,dt$$

$$= \frac{I_{1o}}{2} + \frac{A}{\pi} F \cos\phi$$

$$i_{2\,av.} = \frac{1}{T}\int_0^{T/2} [I_{2o} + aF \sin(\omega t\text{-}\phi) + bF^2 \sin^2(\omega t\text{-}\phi)]\,dt$$

$$= \frac{I_{2o}}{2} + \frac{a}{\pi} F \cos\phi + \frac{bF^2}{4} + \frac{bF^2}{4\pi} \sin 2\phi$$

For the remaining period $\frac{T}{2}$ to T, tube (1) is suppressed and tube (2) is commutating, thus:

$$i'_{2av} = \frac{1}{T}\int_{T/2}^{T} [I_{1o} + AF \sin(\omega t - \phi)]\,dt$$

$$= \frac{I_{1o}}{2} - \frac{A}{\pi} F \cos\phi$$

and $$i'_{1av.} = \frac{1}{T}\int_{T/2}^{T} [I_{2o} + aF \sin(\omega t - \phi) + bF^2 \sin^2(\omega t - \phi)]\,dt$$

$$= \frac{I_{2o}}{2} - \frac{a}{\pi} F \cos\phi + \frac{bF^2}{4} + \frac{bF^2}{4\pi} \sin 2\phi$$

and the indication of the meter will be:

$$I = (i_{1av.} + i'_{1av.}) - (i_{2av.} + i'_{2av.})$$
$$= \frac{2(A-a)}{\pi} F \cos \phi \quad . \quad . \quad . \quad . \quad . \quad (8)$$

which is proportional to the product of the signal magnitude and the power factor.

Equation (8) shows that by making use of the screen-grid currents, the sensitivity of the instrument has increased in the ratio $\frac{A-a}{S}$, where S is the slope of the plate current characteristic (curve 3 Fig. 10-b), i.e. an increase in sensitivity of more than 50% has been attained.

3.—*Effect of Non-Identical Tube Characteristics:*

The use of the cathode-follower circuit will minimise the effect of any difference in tube characteristics. However, it may be useful to show that even if such differences exist, the errors produced are negligible.

Let the two expressions given above apply for the characteristics of one tube, while for the other tube the coefficients A, a and b are replaced by A', a' and b', then the instrument indication will be:

$$I = [(A+A') - (a+a')] \frac{F}{\pi} \cos \phi + (b-b') \frac{F^2}{4} \left(1 + \frac{\sin 2\phi}{\pi}\right) \quad (9)$$

In the first term, the constant $[(A+A') - (a+a')]$ replaces $2(A-a)$ in Eqn. (8), and means that it takes the average values of the slopes of the characteristics of the two tubes. This does not mean an error, since it will be taken care of in the calibration of the instrument. The second term, however, represents an error whose relative value is

$$\frac{F(b-b')\left(1 + \frac{\sin 2\phi}{\pi}\right)}{4\pi[(A+A') - (a+a')] \cos \phi}.$$

Using the curves of Fig. 10-b to obtain numerical values of the coefficients A, a and b, and taking an exaggerated case where $b'=0$, $A=A'$ and $a=a'$, then for the maximum possible value of F the relative error will be 0·24% at unity power factor. If the error is expressed as usual in terms of the maximum indication at unity power factor, the maximum error will also be 0·24% at zero power factor.

4.— *Other Effects:*

In the above, non-linearity of curve (2) of Fig. 10-b has already been dealt with and found of no consequence. However, non-linearity in curve (1) will cause errors identical to those discussed for the triodes of the grid-controlled power-factor meter, and will also be negligible.

In the same manner it can be shown that inaccuracy of square-wave timing, and the presence of harmonics in the load-current wave-form (i.e. in the a-c grid signal) will not produce any appreciable error.

Stray capacities at the output of the square-wave generator will have effects which are almost identical with those discussed above, and actually this has been verified experimentally, as will be mentioned later.

V.—Cathode-Controlled Commutating Power-Factor Meter

The main feature of this type is that it allows the use of small square-wave and a-c signals, while these are applied to different tube electrodes to allow a suitable grounding point of the system.

Fig 11 shows the circuit developed for push-pull operation. The value of the common cathode feedback resistor is chosen such that the linear part of the static characteristic extends equally on both sides of the zero-voltage axis, thus eliminating the necessity of using a grid bias.

However, it can be shown that for proper operation, this resistance should be more than $\frac{2}{g}$, g being the mutual conductance of the tube V_2. The square-wave signal applied to V_2 is a positive one. The bias batteries will bring this tube to cut-off when the square-wave voltage is zero. The circuit can also be modified so that V_2 is self-biassed. When the square-wave signal is positive, the voltage drop across the common cathode resistor should be sufficient to bring V_1 to cut-off, with the maximum a-c signal applied.

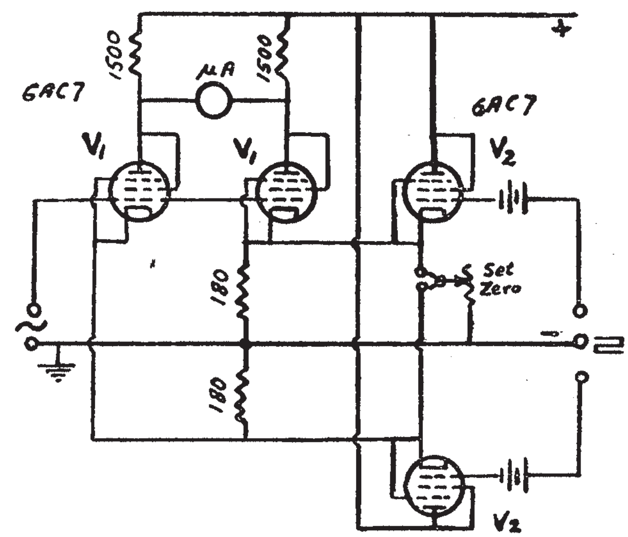


Fig. 11 - Connection Diagram of Cathode - Controlled I cos φ Meter

The square-wave generator used with this type of power-factor meter is essentially the same as that described before, with some modifications incorporating major simplifications. Since the required square wave is a positive one, the high-voltage supply of the output stage can be rounded at its negative side, which means considerable simplification in the power-supply circuit. Owing to the required small amplitude of the square wave, the output stage was changed from an amplifier circuit to a cathode-follower circuit. This will reduce the effect of stray capacities, and moreover, since a small output resistance is used, the frequency range of the meter is much increased. If we consider, as mentioned before, that the upper limit of the value of CRf is 0·0565 at unity power factor and 0·007 at zero power factor (for a maximum error of 3%), and taking R as 200 ohms, and C about 10 μμF (for a high-frequency pentode in a cathode-follower circuit), then the predicted higher limits of the frquency range will be about 30 Mc at unity power factor and 3·5 Mc at zero power factor. However, other factors not considered will cause trouble at such a high frequency as 30 Mc, especially as regards the operation of the square-wave generator, and 10 Mc can be considered as the probable higher limit.

As far as the different sources of error are concerned, this type of power-factor meter will behave the same as the other two types, and the errors will be also negligible.

VI.—Commutating Wattemeter

By means of any of the three aforementioned devices, a direct current proportional to I cos φ could be produced. If now the voltage across the load is rectified so that a direct-current output proportional to the voltage is obtained, the product VI cos φ is most easily secured through the use of an ordinary electro-dynamic instrumsnt, and the whole arrangement takes the form shown in Fig. 12.

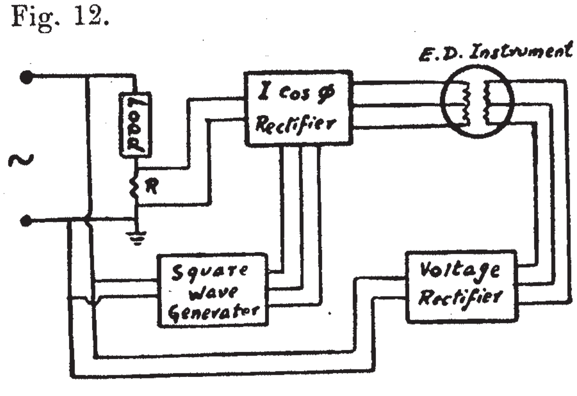


Fig. 12 - Principle of Commutating Wattmeter

1.—*The Electrodynamic Instrument:*

A survey of a number of typical electrodynamic instruments of the laboratory type has shown that in order to obtain full-scale deflection, a relatively large power, of the order of a few hundred milliwatts, is required for the current as well as for the voltage coils.

It should be pointed out that some of the modern types of instruments have better sensitivity, and much more sensitive electrodynamic instruments could be obtained by abandoning the classical technique of using air-core instruments, and using iron-core instruments instead, and consequently a sensitivity as good as that of the moving-coil d-c instrument could be obtained. Iron has many defects, especially when used as a core for a coil carrying direct current to produce a magnetic flux. However, several papers ([1]—[3]) have described iron-core instruments which overcome these defects.

In the present work, it was not possible to obtain nor to construct such a new type of instrument, and therefore stress was given to obtaining as much power from the circuits as possible through the use of normal powerful receiving-type tubes and comparatively large a-c signals, and sacrificing the use of acorn tubes to extend the working frequency range.

The usually-available electrodynamic instruments have their coils designed for operation in ordinary power-supply circuits, and will thus not be suitable for use in conjunction with the circuits under consideration. Therefore, the coils of one of these instruments had to be rewound with the proper number of turns and wire diameter and to the proper value of their resistance,

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(1) Geyger W.," Tests by the compensation method of an a-c wattmeter containing a closed iron circuit", Arch. f. Elektrot., 22, pp. 119-140, June 15, 1929.

(2) Zwierina O., "Measurements with the iron-cored dynamometer", E. u. M. 53, pp. 589-597, Dec. 1935.

(3) MacGahan P., "Improved type of d-c wattmeter of the shunted type", Trans. Am. I. E. E., 60, pp. 4-7, Jan. 1941.

but at the same time having almost the same physical shape and size as the original coils. Moreover, the use of push-pull circuits necessitates that each coil should consist of two sections which should be electrically and physically identical, and therefore the bifilar construction was adopted.

The rewound moving coils of the instrument had 1400 turns and about 1500 ohms each, while the field coils had 4000 turns and 900 ohms each. The instrument was provided with a uniform scale.

2.—*The Voltage Rectifier:*

The voltage rectifier circuit is shown in Fig. 13. A type 6H6 tube is used for rectification. The d-c voltage appearing across condenser C_1 will be equal to the peak value of the applied

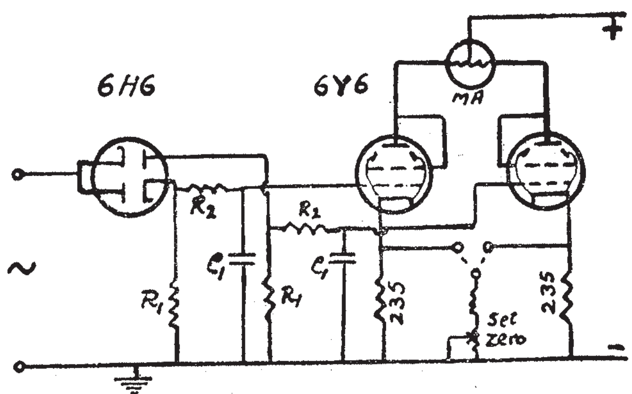


Fig. 13 - Connection Diagram of Voltage Rectifier

a-c voltage. If this voltage contains appreciable harmonics, it would be preferable to obtain the average value, which is less sensitive to harmonics, across C_1. This can be effected through reducing R_1 and increasing R_2.

Two type 6Y6 beam tetrodes, used as cathode followers, are operated in push-pull in order to eliminate errors due to non-linearity that may exist in the working range of their characteristics. These tubes are chosen because of their large current ratings and large mutual conductance, in order to provide the large current necessary for the operation of the electrodynamic indicating instrument. The value of the cathode resistor was chosen in view to obtaining as large a linear range of characteristic as possible, as maximum output power as possible, and the linear part of the characteristic extending equally on both sides of the zero-input-voltage axis so that the tubes are operated without grid-bias supply.

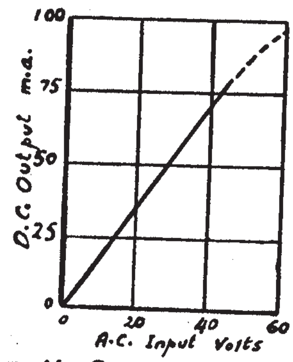


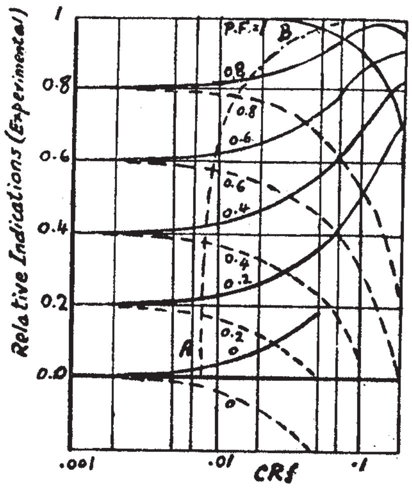
Fig. 14 - Results of Test on Voltage Rectifier of Fig. 13

The circuit was tested for varying a-c input voltages, and the results are shown in Fig. 14, which shows good linearity from zero up to 45 volts a-c with these tubes. It is obvious that this range can be easily extended as desired by applying only a known fraction of the rectified voltage to the grids of the type 6Y6 tubes.

VII.—Experimental Verification

Each type of the power-factor meter was tested by comparing its readings with those of a dynamometer wattmeter at 50 cycles. Line voltage was applied to the voltage coils of the dynamometer wattmeter as well as to the input of the square-wave generator. This same voltage was made to supply current to a resistive load, and the phase angle of this current was varied by means of a phase shifter introduced between the supply and the load. The same current was passed through the current coils of the electrodynamic wattmeter, while a small fraction of the voltage across the resistive load was applied to the input of the power-factor meter. Tests were carried out for different values of supply voltage, load current, a-c input signal and for power factors from unity to zero. In all cases, results showed close linear relation between the readings of the power factor meter and the readings of the electrodynamic wattmeter (divided by the supply voltage). The accuracy as well as the sensitivity were quite satisfactory. Full-scale reading could be obtained by an a-c signal of only a small fraction of a volt, at unity power factor, depending on the sensitivity of the indicating instrument. The sensitivity can still be increased by using tubes of higher transfer characteristic slopes, or using a more sensitive indicating instrument.

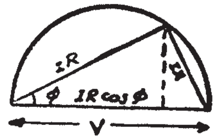
The frequency characteristics of the power-factor meter, predicted in Fig. 8, were simply checked by increasing the value of C instead of increasing f, so as to be able to compare the readings to those of the electro-dynamic wattmeter. Two similar decade condensers were connected across the square-wave generator output. The readings obtained by the grid-controlled power-factor meter are shown in Fig. 15 for different power factors. A large input signal (7·5 volts) was used. It is interesting to note that these curves agree quite well in shape and in magnitude with the curves of Fig. 8. The same test was repeated for different input signals, and the error was found to be almost independent of the signal amplitude.



— Lagging P.F. --- Leading P.F.

Fig. 15 — Effect of Stray Capacities on Meter Readings (Experimental)

Some tests were also carried out on the power-factor meters at frequencies other than the power-line frequency. In one of these tests an oscillator was connected to a simple series circuit consisting of a non-inductive resistance and a calibrated decade condenser. The frequency and output voltage of the oscillator were kept constant, while the condenser was varied, and the meter was connected to give readings of IR cos ϕ and IR, from which R, X and cos φ were calculated, and the results plotted in vector form as shown in Fig. 16, which shows also the theoretical semicircle to be expected, for comparison. The figure shows close accuracy of the readings, as the error was always less than 1%. The fact that the experimental points tend to lie outside the circle indicates that this small error is mainly due to stray capacities. Actually this test is a rather severe test, since a small error in the readings appears exaggerated on the vector diagram.



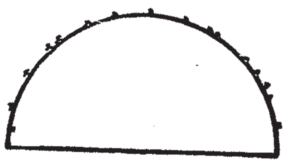


Fig. 16 - Test on I cos φ Meter of Fig. 11, Using R-C Circuit.

The tests on the complete wattmeter were carried out on similar lines to those done on the power-factor meters, that is to say, by comparison to the readings of an ordinary electrodynamic wattmeter at power-line frequency. In each test, two of the three variable quantities, viz. a-c voltage to power-factor meter, a-c voltage to rectifier circuit, and power factor, were kept constant and the third varied over its full range. The power factor was varied from unity down to zero. The instrument indications are plotted against the electrodynamic wattmeter readings in Fig. 17, using the suppressor-controlled power factor meter. The figure shows very good linearity and accuracy.

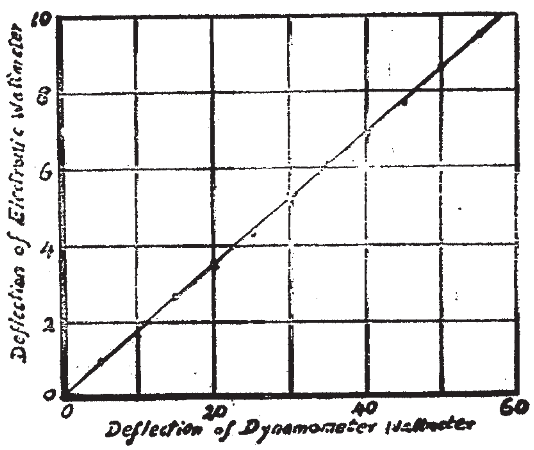


Fig. 17 - Results of Tests on Commutating Wattmeter, Compared to Readings of Dynamometer Wattmeter.

VIII.—Applications of the Instruments

It is to be gathered from the above, that one instrument can be used for the measurement of voltage, current, power factor or power. This can be readily accomplished through the addition of simple switching arrangements to switch over one or the other of the two a-c signals to the inputs of the square-wave generator, I cos ϕ rectifier or the voltage rectifier, and to take readings either on a d-c instrument or on the electrodynamic indicating instrument.

The instrument may also be used as a multirange one. The a-c signal derived from the load-current circuit can be changed by making the load-current resistor vary in known steps. The a-c signal derived from the load-voltage circuit can be changed either by using a voltage divider to take known fractions of this voltage (though this may cause some difficulties at the higher frequencies), or preferably using a condenser voltage divider circuit for the rectified output of the type 6H6 tube in the voltage rectifying circuit.

The instrument can also be used for impedance measurements under actual operating conditions of voltage and frequency.

Finally, the instrument can be made as one self-contained a-c operated unit incorporating its power supply circuit.