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COMBINED STRENGTHENING OF PRE-LOADED BEAMS

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ABSTRACT

Old industrial structures built with compact elements are subject to function changes and require strengthening while under load. A beam panel is analyzed, which under the ultimate load suffers from extensive yield strain at different locations. The study presents a simplified approach to strengthen beams, which have no indications on weak locations, as a "General Basic" procedure. The upper flange is assumed carrying a distributed load, in addition to a negative moment at the support. The strengthening process is based on attaching the minimum combination of different plates at the determined suitable locations. A parameter study gives the strengthening position of each plate and its reasonable extent by using minimum material and welding energy. The proposed procedure describes the crucial effect of the plastic shear on the formation of the plastic moment. Validity is checked by comparing results with previous tests and numerical values. The designer is given a direct method to determine the safe strengthening amount. Simplified formulae are given to estimate the needed material at the required strengthening level. Excessive strengthening is avoided in this method and the expected deflection can be directly predicted. The required precautions and restrictions are given to achieve accurate results for wide scope of application.

KEYWORDS: Combined strengthening, beams, steel plates, welding.

1. INTRODUCTION

Strengthening of structures while under load is important to avoid interruption of production or any other running services. Several reasons indicate the need to strengthen existing elements: increased loads, change of function and/or rehabilitation of old buildings. It is thus expensive and time-consuming to unload and uninstall structural elements to attach required strengthening elements on the ground. Strengthening while under load must, therefore, be done with the minimum amount of

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material and applying the least possible welding heat energy to the existing steel elements. Several techniques have been used for strengthening of beams: by pre-stressing of bridge girders [1], by using Fiber Reinforced Polymers (FRP) and by Carbon Fiber Reinforced Polymers [2-4].

Welding of steel elements is extensively applied to steel beams and columns while under load [5-7]. These studies observed that the welding generated heating had a significant effect on the residual stress redistribution. It rather corrects the effect of the residual stresses and could alone be used for reinforcing purposes. When strengthening columns by using cover plates to the flange and bead weld to the tip of the flange it was noticed that the column strength improved as the residual stresses redistributed favorably [7].

The effect of the welding energy generated heat on the existing beam is numerically as well as experimentally investigated [8]. Upper and lower cover plates are clamped to the flanges and the beam is pre-loaded. A systematic welding process is then applied and the strengthened beam is loaded until failure [8].

Two types of cover plates were used to strengthen beams under different levels of pre-loading [9]. A horizontal plate was welded along the whole beam length, and then the beam is tested until failure. The other type uses two vertical plates to the tips of both flanges. The later has increased the strength remarkably and prevented lateral buckling under the ultimate load. For the cases tested experimentally, numerical studies were carried out [9, 10].

The strengthening of slender I-Beams failed due to web local buckling required two high vertical plates welded along 2/3 the length [11]. The present analysis uses similar beam model and FE as in [11], nevertheless, the beam shape, the goal, and the proposed procedure are quite different. The beam is assumed designed by the ASD-Method as given in old industrial buildings and is therefore compact. The goal is to apply an optimal material consumption and minimal energy. The procedure uses the principles of the plastic analysis of direct design, aiming at increasing the resistance above the plastic limits by using a simplified method.

2. ASSUMPTIONS

The following assumptions are made:

1- At one support the beam carries a negative moment, in addition to a vertical distributed load, which applies on beam top flange. The beam must be capable of developing two plastic hinges**:** one at the left support and another one at the field.

2- Under ultimate load, the beam fails due to excessive yield spread only. No local and\or lateral failure is expected. The top flange is prevented from lateral sway.

3- The best location of the strengthening plates is generally unknown. The extension of each plate is so determined as to achieve the minimum martial and welding requirements. Different types of plates can be used at different locations.

4- Strengthening starts at a maximum pre-load of 0.8 the ultimate load, which should not be exceeded when evaluating the strengthening efficiency.

5- The section is I-shaped and its elements are compact and allow for excessive yield spread until the ultimate load is reached, with no local or lateral failures.

6- The material of the beam is ordinary steel (F_y = 240 MPa), the strengthening plates are of higher steel grade (F_y = 350 MPa), and E = 210 GPa.

3. FINITE ELEMENT MODEL

A finite element model is developed to simulate a flexural member under a vertical uniform load in addition to a negative moment at the left support. The basic structural system is a simple beam as given in [11]. Two vertical strengthening plates, in addition to a horizontal one, are used to adequately strengthen the beam. Since the beam is loaded on the top flange, the horizontal plate can only be attached under the bottom flange. The negative moment at the support is assumed to develop the plastic moment of the cross-section. The two vertical plates are therefore attached starting from the left support, and then extend as needed. All strengthening plates are assumed to be attached at a maximum pre-load of 0.8 the ultimate load of the beam.

Assuming that $\boldsymbol{\sigma}$ is the external stress and $\boldsymbol{\sigma}_R$ is the residual stress, the normal force is then $\int_A (\boldsymbol{\sigma}-\boldsymbol{\sigma}_R)\, dA = \int_A \boldsymbol{\sigma}\, dA - \int_A \boldsymbol{\sigma}_R\, dA = \int_A \boldsymbol{\sigma}\, dA$. Since the second integral equals zero, and for bending: $\int_A (\sigma - \sigma_R)\, y dA = \int_A \sigma\, y\, dA$, because $\int_A \sigma_R\, y\, dA = 0$. The residual stresses do not affect bending, they only increase buckling deformations.

The uniform distributed load is simulated by applying a vertical concentrated load along the whole beam length Fig. 1. The translations in all three directions x, y, and z are prevented at the left lower end, and the translations in y and z directions are prevented at the right one. The entire upper flange centerline is prevented from lateral displacements. The beam elements and the welded plates are represented by using a 4-node shell element (SHELL 181) [12] that is suitable for the analysis of thin to moderately thick plates. Excessive yield strain is avoided to minimize truncation errors in the final result. The material is assumed perfectly elastic-plastic.

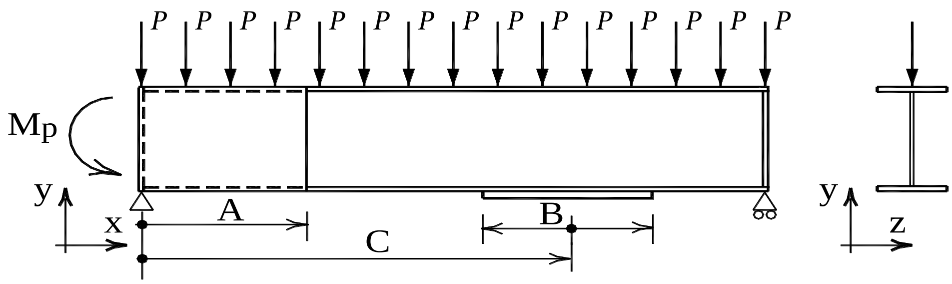


Fig. 1. System, combination, loads and boundary conditions.

4. NUMERICAL ANALYSIS PROCEDURE

The procedure is summarized in two major steps as follows:

In the first step, the ultimate load of the un-strengthened system is determined (ULT1), and then in the second step the un-strengthened beam is usually loaded up to a value of 0.8 ULT1, at this load level the strengthening plates are attached and the loading is then increased until failure (ULT2). In both steps the proposed geometrical imperfections, as well as the material non-linearity, are accurately included as related to the first Eigen-Shape of the structural system.

5. VERIFICATION OF THE FINITE ELEMENT MODEL

The following results are verified at different loading cases and levels. Horizontal plates are welded on top and bottom and loaded with one concentrated load at mid-span [8]. The ultimate load and the local flange failure are verified, but the actual deformations are bigger because of residual stresses and the welding along the whole length [8]. Two plates welded to the tips with loads at 1/3 and 2/3 of the span length and their types of failure are verified and stated in Table 1 [9, 11]. The comparison shows good agreement.

Table 1. Comparison between the proposed model and previous results.

Ref	Result Type	Span (mm)	Pre-load %	Buckl. Type	P_{ULT}-Ref (kN)	P_{ULT} (FE) (kN)	Error %
[8]	Experiment	3000	74	Local	575.86	584.50	+1.50
[9]	Experiment	2400	0	Lateral	188	191	+1.60
[9]	Experiment	2400	69	Lateral	788.5	785.5	- 0.38
[10]	Numerical	3000	0	Lateral	103	104.4	+1.36
[10]	Numerical	3000	31	Lateral	797.3	795.95	- 0.17
[10]	Numerical	3000	62	Lateral	796.4	795.75	- 0.08
[10]	Numerical	3000	93	Lateral	792.1	795.09	+0.38

An enhanced comparison is presented in Fig. 2. Three cases are completely reevaluated and re-plotted using the same assumptions in [11]: Beam length is 9.0 ms, cross-section dimensions h_w, b_f are 750 and 150 mms, h_w /t_w is 100, b_f/2t_f is 10.

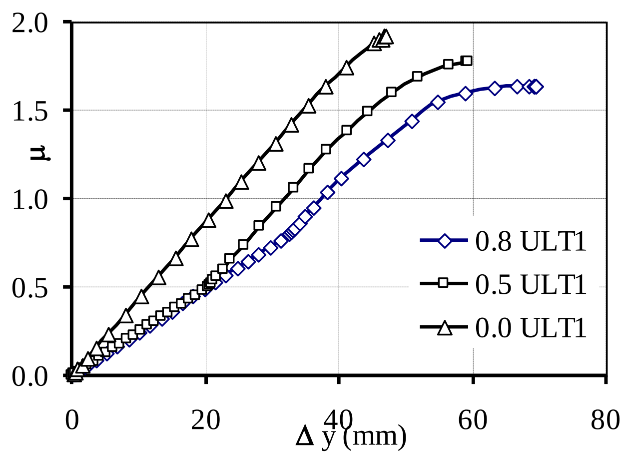


Fig. 2. Comparison with the corresponding plots [11].

Two vertical plates, each is t_w-thick and covers 2/3 of the span length. Steel Grade is 345 MPa. Pre-load is 0.0, 50 and 80 %. The failure at the ULT1 is a local web buckling. The other cases failed due to lateral torsion-buckling in the lower flange. Figure 2 shows the load-deformation curves subject to direct comparison with (Fig.

7.b. in [11]). Both plots are almost similar. The present study though assumes no local failure. It aims at increasing the ultimate load above the plastic limit of the compact beam.

6. PARAMETRIC STUDY

Given a cross-section area *"A"*, where its flanges are concentrated at a height *"H"* and its web area is *"A_w"*, then:

$$M_{PL} = \left[\frac{(A - A_w)}{2}.H + \frac{A_w.H}{4} \right].F_y = \frac{A}{2}H.(1 - \frac{A_w}{2A}).F_y \quad (1)$$

Thus "A_w/A" is the only governing parameter of the plastic moment behavior of a cross-section with constant *A* and *H*. Careful and reasonable selections of the section application limits are stated in Table 2, and designated as the types II and III. They cover most hot-rolled I-sections. Type I is an extreme case, used as a check,

Table 2. Proposed cross-section limits.

Type	B_{Flange}	t_{Flange}	H_{Web}	t_{Web}	A_w/A
I	240	24	900	18	0.58
II	150	16	500	12	0.55
III	250	16	500	8	0.33

The investigated beam lengths L are 4, 6, 8, 10, and 12 ms, the values L/H_{Web} varies between 5 up to 24, and the important ratio A_{Web}/A_{Total} varies from 0.33 to 0.55.

Strengthening follows by combining both vertical and horizontal plates reasonably (V, H or V+H followed by plate thickness in mms). Hence the specimens can be designated by their section type (I, II or III), beam length, plate position, pre-load ratio (i.e. II, 8, 0.9, V10, and an asterisk * denotes a variable field, etc.).

The plastic moment of the original system is denoted by M_{PL}, which under ULT1 is created two times. After strengthening it increases to M'_{PL}, and the second field plastic moment is then $\eta\, M'_{PL}$, as in Fig. 3.

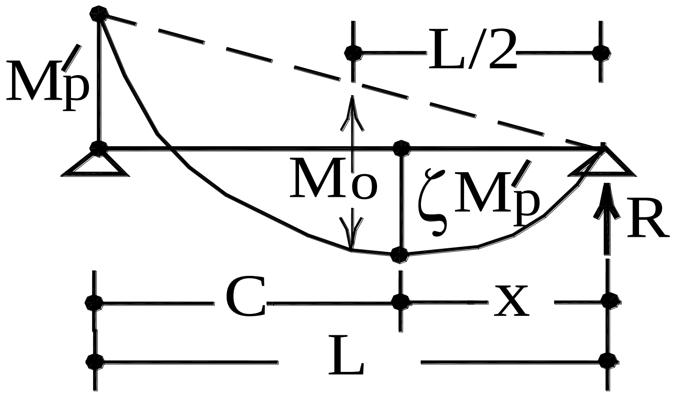


Fig. 3. Ultimate moment diagram of the strengthened beam.

To achieve an analytical solution it is advisable to use dimensionless parameters. In the following analysis, the vertical load is represented by the simple beam maximum moment at its mid-section: $M_O = w.\ L^2 / 8$. Thus the ratio $\mu = M_O/M_{PL}$ is a suitable dimensionless parameter that represents the ultimate load.

Under ultimate load, the un-strengthened beam fails $\mu = M_O/M_{PL} = 1.4571$. At this load level two equal plastic hinges are formed: one at the left support and the second takes place at C > 0.5 L. The ultimate load value of the un-strengthened beam (μ/1.4571) is displayed as a parameter of the section type and the beam length. As shown in Fig. 4 the beam type III, with span lengths 4 and 6 m, the beams failed.

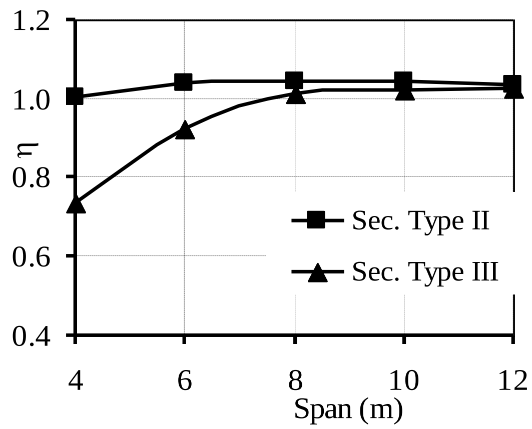


Fig. 4. The ultimate load of the un-strengthened proposed beams (ULT1).

They rather failed due to local web yield failure at left support. The vertical loads necessary to create the plastic moments are high for the small spans or higher web to span ratio. These beams are thus neglected in the following analyses; their ultimate load values are irregular and allow no analytical solution.

The efficiency of the model is demonstrated by the load/deformation relationship in Fig. 5 by strengthening the full length, with two vertical plates. It starts at 0.95 ULT1, i.e. at $\mu = 0.95\times1.4571$ with different plate thicknesses. All cases create two plastic hinges (ULT2 M_O is 1.4571 M_{PL}). Notice that a zero preload makes little or no difference from 0.95 ULT1 in the value of ULT2 (t=10 mm, Fig. 5).

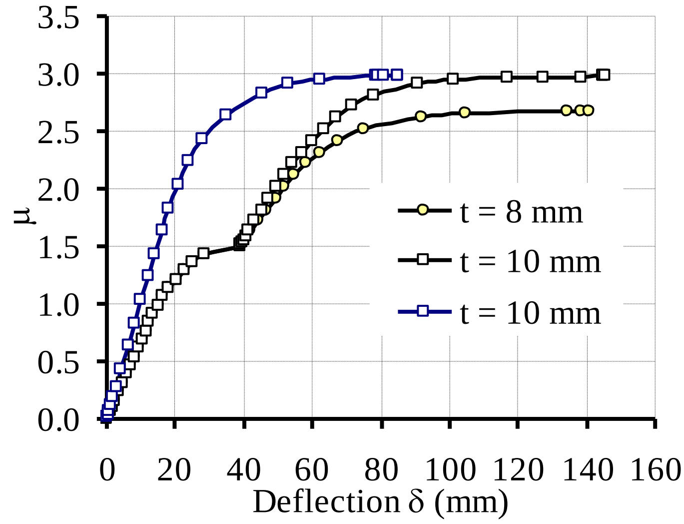


Fig. 5. Ultimate load demonstration of a strengthened beam (II, 8, 0.95, V*).

A usual self-check is given in Table 3**:** All numerical ultimate moments eventually converge at the plastic moment of the beams, with certain error. The values found for curves in Fig. 5 are compared with their analytical mates. Values are in kNm. Error is stated on table. This self-check is repeated in the following 325 cases.

Table 3. Accuracy of demonstration examples ULT2 M_O

$t_{STR-PLATE}$ (mm)	10	8	10
Pre-Load/ULT1	0.0	0.95	0.95
Analytical ULT2	1410.7	1271.1	1410.7
Numerical ULT2	1421.3	1275.4	1420.4
Error %	0.7	0.3	0.7

7. EFFECT OF VERTICAL STRENGTHENING PLATES

If μ_1 is the ultimate ratio (M_O/M_{PL}) of the original beam, and μ_2 is that after strengthening, then the relative increase in strength is $\eta = (\mu_2 - \mu_1)/\mu_1$. By attaching two vertical plates on the cross-section (Type III), the maximum theoretical increase in the section plastic moment due to the vertical plates of thickness 8 mms, considering Fy = 350 MPa, is about 57 %. Yet, its effect on the beam ultimate capacity is different; as shown in Fig. 6. Starting from the left support the vertical plate lengths are increasingly varied towards the right support. The strengthened ultimate load is determined at each interval of 0.1 L, where L is the beam length taken as 4.0-12.0 m.

The variation of the strengthening increase with the length of the plates is determined (section Types II and III). Regular behavior is found for beams complying with the assumptions stated in Section 2. The behavior is summarized as follows: Starting from the left support, two vertical plates only 0.1 L long, one on each side increase rapidly the ultimate load by about 20%. Longer plates add practically no increase up to the "0.6 L-location". An increase occurs between 0.6 L and 0.8 L. Then some further increase is found but with less intensity until 1.0 L.

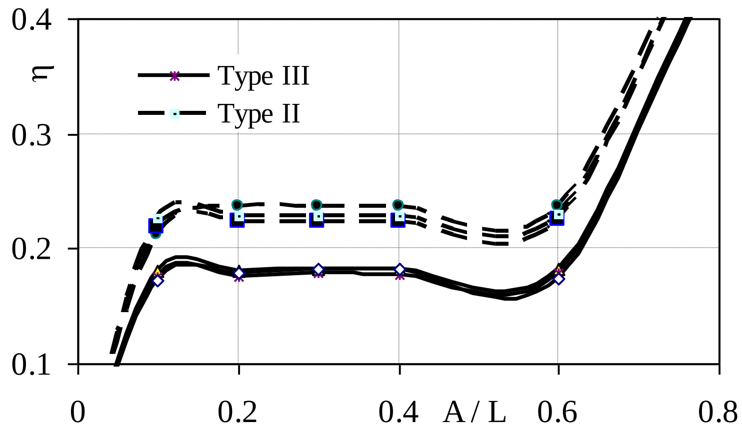


Fig. 6. Ultimate load increase (II, III, *, 0.8, V8).

Web shear failure gives wrong and high results for they are related to the reduced ULT1. Only cases, complying with the assumptions, are displayed together in Fig. 6 for all their lengths. It is found that the cross-section type and the web area ratio A_w/A are relevant parameters, not the beam length. It has practically no effect. This confirms the generic validity of plastic theory.

The vertical plates are not quite effective at a length of 0.6 L as shown in Fig. 7. They must extend up to 0.8 L to achieve the required action. It requires about 0.2 L on both sides of the location of maximum positive moment. The further increase in length, until the span ends, increases the shear resistance at the right support.

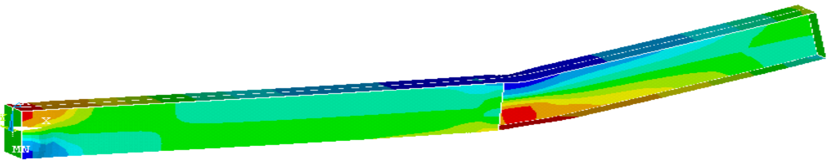


Fig. 7. Ultimate normal stresses (III, 6, 0.8, V8).

Completely elastic beams, under the given loads, indicate a maximum bending moment at C = 0.625 L (5/8 L) measured from the left support. Nevertheless, when two equal plastic moments are formed ($M'_{PL} = M_{PL}$), the maximum positive moment is found at 0.5858 L (4.6863/8 L) from left. It is easy to determine such a distance at any ratio between the two different strengthened plastic moments: given $\mu_1 = M_O / M_{PL}$, and knowing that "*w*" is the load per unit length = $8\,M_o/L^2$, and the maximum positive moment $\zeta M'_{PL} = R^2/(2w)$, then:

$$R = \frac{wL}{2} - \frac{M'_{PL}}{L} = \frac{1}{L}(4M_0 - M'_{PL})$$

$$\zeta M'_{PL} = \frac{1}{2wL^2}(4M_0 - M'_{PL})^2$$

$$= \frac{1}{16M_0}(16M_0^2 + M'^2_{PL} - 8M_0 M'_{PL})$$

$$\left(\frac{M_0}{M'_{PL}}\right)^2 - (\zeta + \frac{1}{2})\frac{M_0}{M'_{PL}} + \frac{1}{16} = 0$$

By replacing M_0/M'_{PL} by μ:

$$16\mu^2 - (16\zeta + 8)\mu + 1 = 0. \qquad (2)$$

And

$$C = 1 - \frac{R}{w} = L\left[\frac{1}{2} + \frac{1}{8\mu}\right]. \qquad (3)$$

The above equations are generic, but only applicable to beams that create two plastic hinges and conform to the requirement of failure under extended yield spread only. Note that conforming cases make regular behavior and are generic according to the fundamentals of the plastic theory.

8. EFFECT OF A HORIZONTAL STRENGTHENING PLATE

By repeating the previous study using one horizontal plate the relationship is shown in Fig. 8. Since the beam is top-loaded, the plate can be attached only on the bottom flange (Type II). The horizontal plate thickness is taken 10 mms. It is effective when extended by 0.2 L on each side of the location 0.6 L, i.e. it should extend from 0.4 to 0.8 L. If so, it increases the total ultimate load by about 15 %, which is reasonable compared to the small length (0.4L) of the attached one-side-horizontal-plate compared to the double vertical ones. No effect is found if it extends above 0.8L. The case for L = 4 m is non-complying due to shear failure and is excluded.

For complying cases, the parameter beam length is also here ineffective. Two plastic hinges are created in every single case represented as a point in Fig. 6. Therefore, the plastic theory prevails here as well. Every similitude beam with the same web area ratio A_w/A fails at the same dimensionless values.

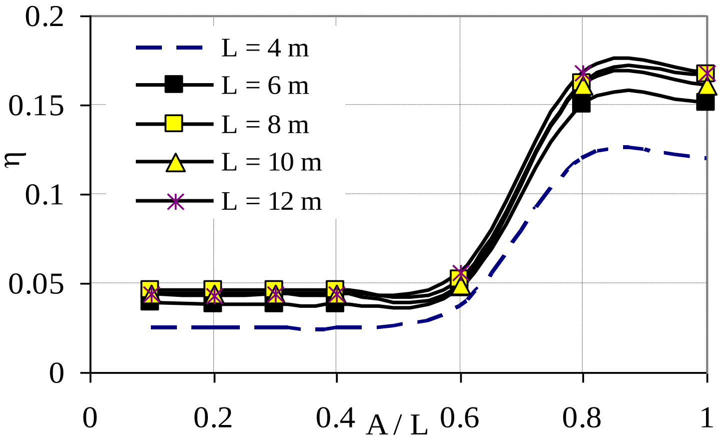


Fig. 8. Ultimate load increase (II, *, 0.8, H10).

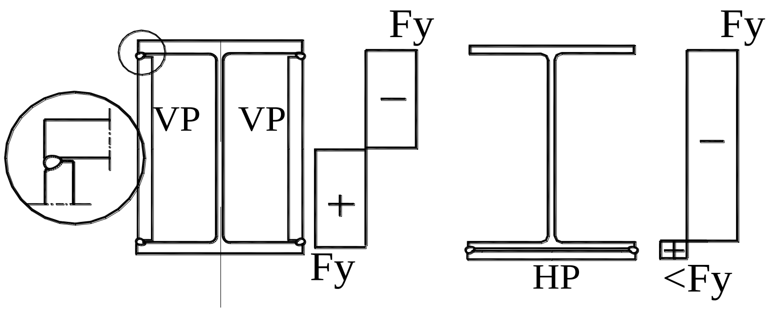
9. STRENGTHENING THE CROSS-SECTION

By strengthening the cross-section to a certain degree the system resistance does not develop the same increase in strength. The relative increase in section resistance η_S is rather the upper bound of the system strength η. Nevertheless, the increase in cross-section relative strength η_S, where $M'_{PL} = (1+\eta_S)\, M_{PL}$ is a useful parameter when estimating the generic plastic behavior under two plastic hinges.

Two types of cover plates are investigated: Double vertical plates attached to the flange lips at the left support as in Fig. 9a and a single cover plate to bottom flange at the maximum positive moment accurate location as in Fig 9b.

The double vertical plates must develop their own plastic moment. They should sustain yield strain with no signs of local failure. Their plastic moment contributes directly to increasing ULT1 if attached at the left support. Note that their welding is an expensive issue [11]. It often requires skilled manpower, laser survey of the beam lips, precise cutting and chamfering of plates, and applying the least possible welding energy; Fig. 9a. A butt weld is mandatory (Half V), root welded if possible.

The horizontal plate increases indirectly the cross-section resistance. If the area of the horizontal plate is equal or greater than the cross-section web area then the redistribution of the stresses of plastic moment $\zeta M'_{PL}$ is as shown in Fig. 9b.



(a) (b)

Fig. 9. Plastic moment stress distribution.

If the added plate sectional area is ΔA, the ratio to the web area $(\Delta A/A_w)$ is plotted versus the relative section plastic moment increase "η_S", where A_w represents the section web area. By applying a horizontal plate on the lower flange of each section type, the increase η_S is shown in Fig. 10. Noting that Type II has a stronger web, yet all types have compact elements and can sustain extended yield strain.

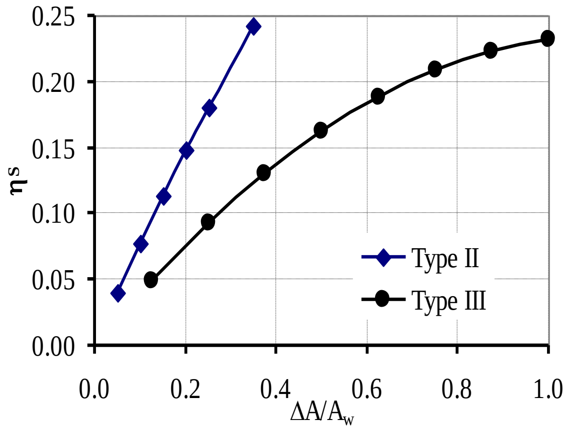


Fig. 10. Plastic moment increase due to horizontal plate.

The added plate thickness is taken 0.6 t_{Flange} in both cases and has the same flange breadth with steel grade 240 MPa. The value of ($\Delta A/A_w$) for the cross-section types II and III are 0.25 and 0.625 and their plastic moment increase is 0.181M_{PL} and 0.188 M_{PL}. The horizontal plate is therefore effective in both cases. By taking the thickness of all plates as 8 mms - steel grade 350. Then the strengthened plastic moment at the support is 1.733 M_{PL} and 1.569 M_{PL} for section types II and III respectively.

Since the values of all M'_{PL} are determined, the accurate location of the strengthening horizontal plate in a given beam can be determined by the following steps: First, we determine the increased plastic moments at both sections. The second step is to calculate the ratio between the plastic moments at the two sections $\zeta = M'_{PL}/M_{PL}$. Substituting by this value in Eq. (2) and solving, the found μ value in Eq. (3) gives the location C/L.

In our case, the values of ζ_{II} = 1.181/1.733 and ζ_{III} = 1.188/1.569 are 0.681 and 0.757 respectively. The accurate location of the horizontal plate can easily be determined by applying the Eqs. (2) and (3). By substituting with ζ = 0.681 and 0.757 in Eq. (2) μ is given = 1.1258, 1.2054. Substituting these values in Eq. (3) C is found at 0.611 L and 0.604 L respectively. These two values confirm the fact that an approximate C-value at 0.6 L provides a reasonable and simplified approach as stated in Section 8. The maximum C / L – value is the unity at ζ = zero, and its minimum value is asymptotic to 0.5 at $\zeta = \infty$.

Figure 11 displays the actual behavior of cross-sections, at field and at support under ultimate load of a strengthened system while under load. It shows the accurate yield redistribution that is triggered by each strengthening plate.

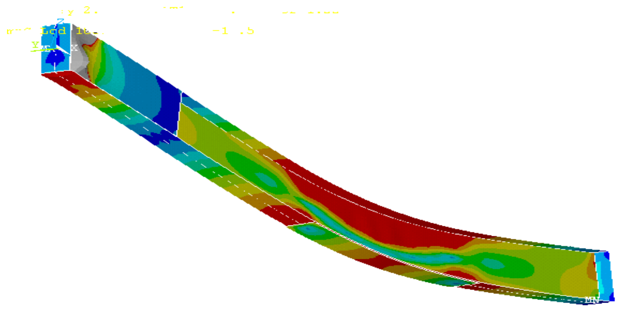


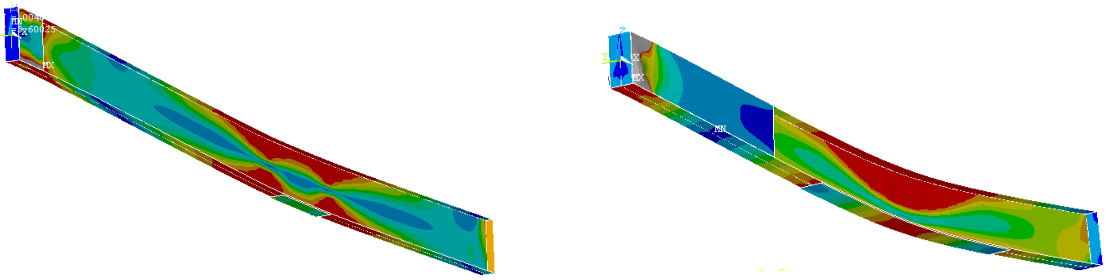
Fig. 11. The yield spread due strengthening plates at ULT2.
(III, 8, 0.85, V8, H10, S3).

10. STRENGTHENING PLATE COMBINATION

At ULT2, given η_S and ζ, each of the Combination Systems S1 to S4 creates two plastic hinges, and therefore is generic according to the plastic theory. Despite the variable resistance along the beam length, it is still statically determinate. They are stated relative to the beam length "L" in Table 4. The von-Mises mode shape distribution of (S1 and S4) is illustrated in Fig. 12. Each shape, with its two plastic hinges, remains the same for the tested relative lengths, for the same S-combination, ζ, "η_S" and if A_w/A is within the two tested values 0.33-0.55 that covers most hot rolled sections as indicated in Table 2.

Table 4. Relative lengths of the combination system (Fig. 1).

Combination System "S"	A/L	B\L	C\L
S 1	0.075	0.10	0.60
S 2	0.150	0.20	0.60
S 3	0.225	0.30	0.60
S 4	0.300	0.40	0.60



(a) S = 1, (II,8,0.8,V8,H10,S1) (b) S = 4, (III,8,0.8,V8,H10,S4)

Fig. 12. Examples of ULT2 - plate combination and distribution.

11. EVALUATION AND RESULT PRESENTATION

The selected cross-sections, lengths, and beams must satisfy the assumptions stated in Sections 2 and 13. Most important is the adequate web shear resistance under ultimate load. By taking μ=1.4571 (Section 6), then at ULT1 Mo gives:

$$1.4571\,M_{PL} = \frac{wL^2}{8}\,. \tag{4}$$

Due formation of two plastic hinges, the reaction at the right support for ULT1 is equaled to wL * 0.4142, and the left one is given as:

$$Shear\ force = (1-0.4142)\times 8\times 1.4571\frac{M_{PL}}{L} = 6.83\frac{M_{PL}}{L} \tag{5}$$

Thus the web area is:

$$A_w > \frac{12\,M_{PL}}{F_y\ L}\,. \tag{6}$$

The web area must sustain the shear force at ULT1, if not satisfied as shown in Fig. 13, the beam behavior is irregular and the Eqs. (2-6) should not be applied.

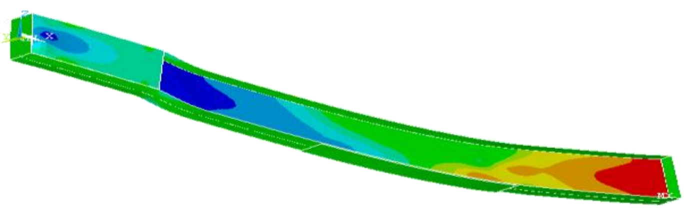
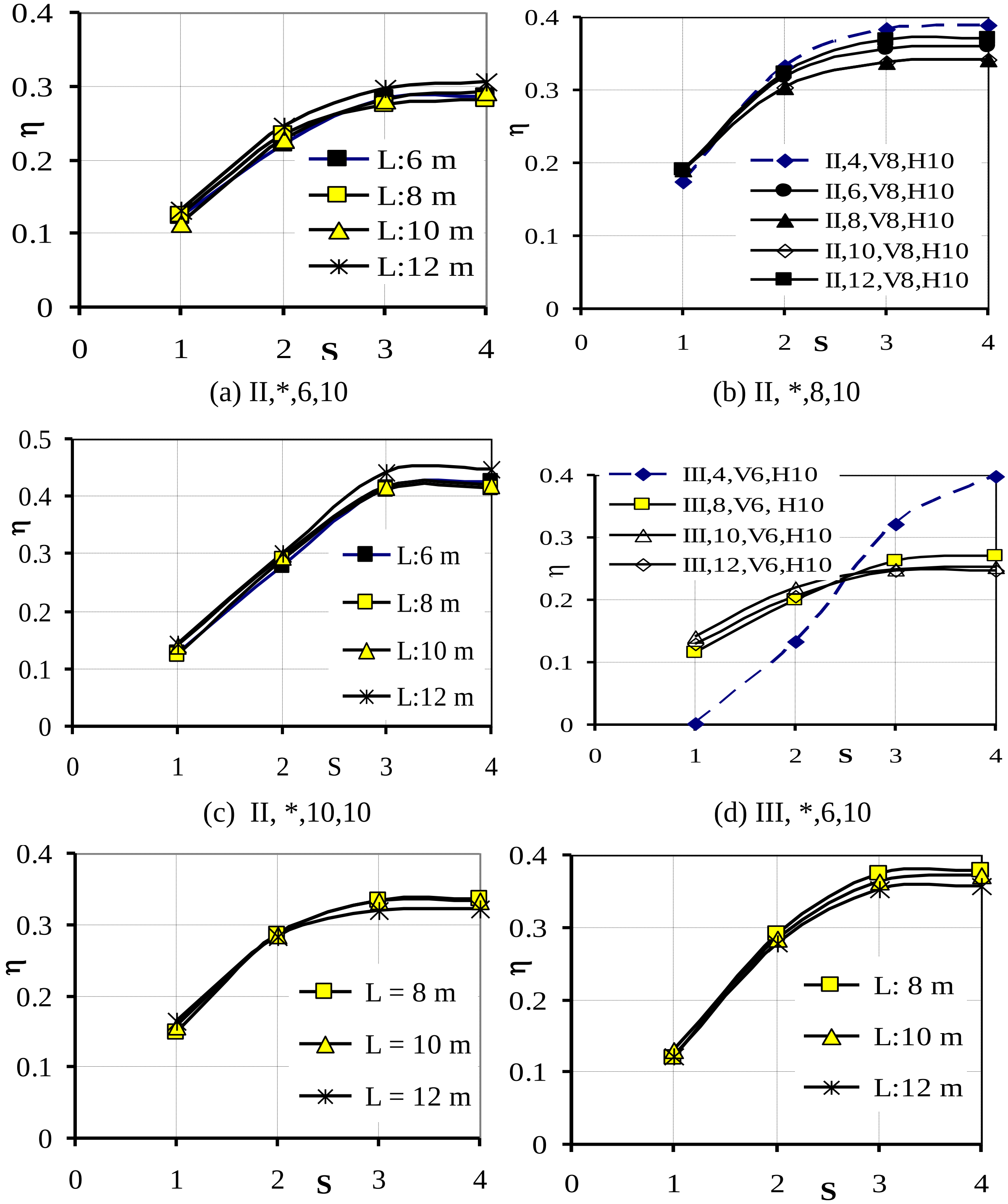


Fig. 13. Ultimate shear stresses (III, 6, 0.8, V8, H10, S3)

All regular cases are displayed in Fig. 14 a-f: The increase in strength η is plotted vs. the system combinations S1-S4 indicated in Table 4. All curves have the same shape. The maximum η is at S3 and S4. The case (III, 4, 0.8, V6, H10) is not complying. The value of η_{max} increases mainly with the increase of vertical plate strength.



(a) II,*,6,10

(b) II, *,8,10

(c) II, *,10,10

(d) III, *,6,10

(e) III, *,8,10

(f) III, *,10,10

Fig. 14. Relative strength increase vs. Combination Systems (S1-S4).

Equations (2-5) are exact and generic because they are found by using the plastic analysis. A close inspection of Figs. 6, 8 and 14 proves that the parameters: beam length and A_w/A can be ignored if A_w/A is within the limits indicated in Table 2.

If the relative section plastic moment increase η_S is the plastic moment of the vertical plates divided by the cross-section plastic moment M_{PL}**.** The vertical plates plastic moment is = F_y 2 t H^2 / 4, where H and t are the plate dimensions. For beams complying with all conditions on Section 13, the maximum increase in ULT1 of the strengthened beam can approximately be given by the following formula:

$$\eta_{max} \approx 0.38\sqrt{\eta_S} \qquad (7)$$

The validity of Eq. (7) is checked in the stated domain (Section 13) and the result is given in Fig. 15 by plotting $\eta_{max,Approx}$ / η_{FE} for all investigated valid cases, compared to unity, where η_{FE} is numerically determined.

The working load on the strengthened beam must not exceed the value of 0.8 ULT1. The maximum deflection can thus be estimated using the following formula:

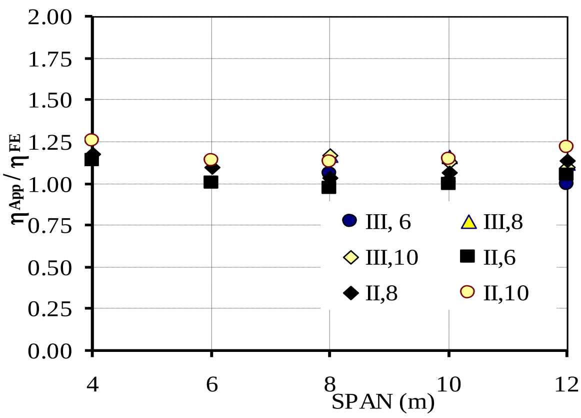


Fig. 15. Accuracy of Eq. (7) (*, *, S3)

$$\delta_{0.8} \approx 0.16\,\frac{L^2}{H}, \qquad (8)$$

Where δ is in mms, L and H are in meters. The accuracy of the approximate Eq. (8) is displayed in Fig. 16. When the working load is lower than 0.8 ULT1, the required deflection can be estimated by linear interpolation relative to $\boldsymbol{\delta_{0.8}}$.

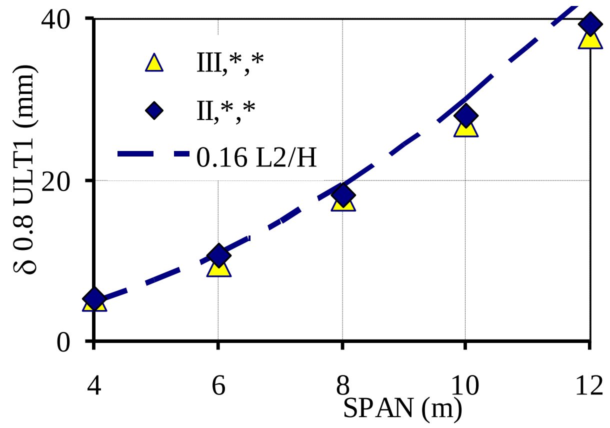


Fig. 16. Deflection comparison Eq. (8) with FE.

The rules for applying Eq. (7) are as follows: a) Check compact cross-section.; b) The original system must develop two plastic hinges without shear failure; Eq. (6); c) Estimate the maximum possible increase in ultimate load; Eq. (7); d) If the required increase is smaller then interpolate the plate lengths by relative to the Combination System S3 (Table 4); e) Estimate the deflection under working load; Eq. (8) by interpolating relative to $\delta_{\mathbf{0.8}}$ (ULT1). Satisfy all conditions (Section 13).

12. CROSS SECTION SIMILITUDE

Together with the dimensionless Plastic Theory, the cross-section Similitude makes Eq. (7) valid for other section series (Table 5). All samples are at validity limits.

Table 5. Examples on section Type I: (*, *, 0.8, V:2/3 t_w, H:0.8 t_F, S3).

Sec. Prop.	Sample	H/t_w mm	B/t_F mm	L m	A_w/A	M_{PL} kNm	Anal. η_{max}	ULT2 x M_{PL}	FEM η_{FE}
	I-a	900/18	240/24	7.2	0.584	2152	0.343	2.048	0.384
	I-b	750/15	200/20	6.0	0.584	1245	0.343	2.048	0.384
	I-c	600/12	160/16	4.8	0.584	638	0.343	2.042	0.380
	I-d	450/09	120/12	3.6	0.584	269	0.343	2.044	0.381
	I-a-1	924/18	300/19	7.2	0.591	2199	0.348	2.041	0.379
	I-a-2	924/18	320/18	7.2	0.591	2199	0.348	2.039	0.378
	IPE 600	581/12	220/19	7.2	0.455	826	0.292	1.951	0.318
	HEB 650	619/16	300/31	7.8	0.347	1749	0.246	1.886	0.274
	HEA1000	959/16.5	300/31	10.6	0.460	3051	0.274	1.919	0.297

By changing a single dimension, and incrementing, a new series is created, etc. If A_w/A is the same, the results of the analytical "η_{max}" are identical. The huge number of numerical iterations gives practically constant "η_{FE}" values. Thus one sample can represent an endless series with different cross-sections. Cases Ia-1 and 2 are from I-a, but calculated, as usual as if they were thin-walled. The error is small. Most hot rolled cross-sections are within the limits of A_w/A.

13. SUMMARY AND CONCLUSIONS

An integrated procedure is presented that uses the Similitude, the Plastic Theory, analytical and numerical analyses to design strengthening combinations of beams with minimal material and energy. The beam must develop two plastic hinges at ULT1. The procedure can be used for further studies on different beams.

The estimated increase in ULT1 η_{max} ; Eq. (7), above the plastic limit; Eq. (4) only applies on I-sections between $0.33 < A_w/A > 0.55$ for the given loading (Fig. 1), and only when the following conditions are completely satisfied: Elements are compact. Shear resistance is > 6.83 M_{PL}/L; Eq. (6). Beam span to depth from 8 to 24. Beam grade is F_y: 240 MPa and plates 350 MPa. The section plastic moment increase η_S is between 0.40 and 1.00. The lower plate t = 0.8 t_{fl}. The combination system is S3 (Table 4). Local and lateral buckling are prevented. Preload < 0.8 ULT1.

The sample I-a Table 5 reaches, for example, a final strengthened total load according to Eq. (4): ULT2 = (1+0.343)* 8*1.4571*M_{PL}/L = 4679 kN. This strength increase is high compared with the limited usage of material. A simplified formula; Eq. (8) is given to estimate the expected deflection at 0.8 ULT1.

It is recommended to use the given procedure to strengthen other beams with other loading cases.

DECLARATION OF CONFLICT OF INTERESTS

The authors have declared no conflict of interests.

REFERENCES

1. Bradford, M. A., "Stability of a Steel Bridge Girder Strengthened by Pre-stress", IABSE Reports, 1995.
2. Katrizadeh, E., and Narmashiri, K., "Experimental Study on Failure Modes of MF-CFRP Strengthened Steel Beams", Journal of Constructional Steel Research, Vol. 158, pp. 120–129, 2019.
3. Elkhabeery, O. H., Safar, S. S., and Mourad, S. A., "Flexural Strength of Steel I-Beams Reinforced with CFRP Sheets at Tension Flange", Journal of Constructional Steel Research, Vol. 148, pp. 572–588, 2018.
4. Yousefi, O., Narmashiri, K., and Ghods, A., "Investigation of Flexural Deficient Steel Beams Strengthened by CFRP", Indian Journal of Fundamental and Applied Life Sciences, Vol. 4, pp. 372-380, 2014.
5. Wu, B., Cao, J. L., Kang, L. "Influence of Local Corrosion on Behavior of Steel I-Beams Subjected to End Patch Loading: Experiments", Journal of Constructional Steel Research, Vol.135, pp. 150-161, 2017.
6. Nagaraja, R. N., and Lambert, T., "Columns Reinforced Under Loads", Fritz Laboratory Report No. 286.1. Bethlehem, Pennsylvania: Lehigh University; 1962.
7. Lambert, T., "The Reinforcement of Steel Columns", Engineering Journal, AISC, Vol. 26, No. 1, pp. 33-39, 1989.
8. Yuan-qing, W., Liang, Z., Rui-Xiang, Z., Xi-Yue, L., and Yong-jiu, S., "Behavior of I-section Steel Beam Welding Reinforced while Under Load", Journal of Constructional Steel Research, Vol. 106, pp. 278- 288, 2015.
9. Lui, Y., and Gannon, L., "Experimental Behavior and Strength of Steel Beams Strengthened While Under Load", Journal of Constructional Steel Research, Vol. 65, pp. 1346-1354, 2009.
10. Lui, Y., and Gannon, L., "Finite Element Study of Steel Beams Reinforced While Under Load", Engineering Structures, Vol. 31, pp. 2630-2642, 2009.
11. Gendy, B., and El Dib, F., "Strengthening of Slender I-Beams during Loading", Journal of Engineering and Applied Science, Vol. 62, No. 5, pp.447-463, 2015.
12. ANSYS, "User's Guide Manual", Release 12.0.1. Canonsburg, PA., 2009.

التقوية المجمعة للكمرات تحت الأحمال

يتناول البحث تحليل الكمرات تحت الأحمال التي تعاني الإنفعال اللدن في مواضع عدة عند الحمل الأقصى. ويقدم البحث مقاربة مبسطة أساسية لتقوية الكمرات التي لا تعطي مؤشرات على مواطن الضعف. ولما كانت الشفة العليا تحت الحمل الموزع إضافة إلى عزم سلبي عند الركيزة فإن البحث تأسس على استخدام الحد الأدنى من الألواح المجمعة وإلحاقها بأماكن سبق تحديدها بطرق مبسطة مع حساب الحد الأدنى لإنتشارها وفرا للمواد والطاقة. وقد تم التحقق من صلاحية طريقة المقاربة بالمقارنة مع الأبحاث السابقة. ويقدم البحث للمصمم طريقة مباشرة لتحديد مقدار التقوية المطلوبة للوصول لمستوى الحمل الأقصى وتحاشى البحث التقوية المفرطة مع تحديد الإنحراف المتوقع بطريقة مبسطة. ويمكن تطبيق الطريقة المقترحة على كمرات مختلفة الأبعاد ولكن مستوفية الشروط. ويعطي البحث الإحتياطات والشروط المطلوبة للوصول إلى نتائج دقيقة.