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THEORETICAL ENVELOP FOR THE PUNCHING SHEAR FORMULA

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ABSTRACT

This paper is a pioneering step to furnish the theoretical basis behind the semi-empirical punching formula. Principles of the Boundary Element Method BEM and the theory of elasticity are adopted to establish the rationale behind the punching shear stress equation proposed in building codes. It was found that an envelope could be constructed between the theoretical shear stresses in the two directions embracing safe punching values computed from ACI empirical formula. The present research further substantiates this effort through developing a BEM-based checking methodology for punching limit state for columns of any irregular cross-section shape – a task that traditional punching equations fall short of satisfying. This methodology is first verified via regular shape columns then adopted to demonstrate its capability to consider columns with irregular cross sections. Finally, some real-world examples are conducted to demonstrate the accuracy, efficiency, practicality and reliability of the proposed computational "theoretically-based" methodology in checking punching of reinforced concrete columns with arbitrary (either regular or irregular) cross-sections.

KEYWORDS: Boundary element method, punching columns, irregular cross section.

1. INTRODUCTION

Check of safety against punching is an essential part of reinforced concrete flat slab design procedure. The study of unbalanced moment under vertical loading at slab column connections was thoroughly investigated in many research works and then implemented into code provisions for punching safety. Design codes evaluate the influence of shear forces and bending moments acting in slab section within the column support area on the punching shear strength [1-4].

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A number of experimental and numerical investigations were also carried out in the literature in order to improve design codes provisions for punching shear calculations.

They suggested the evaluation of some factors which have an influence on punching shear behavior and accordingly affect punching failure mode. Some researchers carried out experimental investigations [5-7]. They presented their conclusion in a useful format permitting codes to specify some critical section locations for checking punching.

On the other side, many important researches focused on punching analysis using numerical methods. For instance, the effect of a concentrated moment was investgated applied to an uncracked elastic plate at an interior column neglecting the local influence of the column cross-section shape and focused on obtaining the distribution profile/contour for the shear and moments around any closed perimeter [8], this solution for the closed sections was based on the Levi's solution for plates. Later, unbalanced moment transfer parameters was computed namely, γ_v percentage of moment transferred by shear forces and γ_f percentage of moment transferred by bending and twisting moments using Finite Element Method FEM [9] and compared their results against those given by ACI 318-89 building code procedure/equations. Another approach was introduced for computing the punching parameters γ_v and γ_f at any slab-column connection using the Boundary Element Method BEM via the shear deformable plate bending theory [10] and compared their results against those given by ACI 318 equations and [9] results that were based on FEM.

The focus of the present paper is to identify the rationale behind the punching formula without loosing generality in this work, only the ACI 318 formula is considered. Towards this goal, presented research proposes substituting the ad-hoc "single-valued" shear stress v_u at a given point on the critical section for punching by the actual theoretical "two-component" shear stresses as retrieved from the theory of elasticity" q_x and q_y". A BEM-based software is then devised implementing relevant principles of the theory of elasticity in order to draw an envelope for punching strength along the critical section for reinforced concrete columns with irregular cross section shapes – an application that ACI 318 is unable to address except with some user-assumed regularization of the irregular cross section.

2. SHEAR CALCULATION IN THE THEORY OF PLATES

The application of boundary element method BEM in shear deformable plates was firstly studied [11]. Later, the work was extended by considering internal columns terms to be added to the main governing integral equation [12]. The main purpose of employing the BEM with the shear deformable plate theory is its capability to compute value of shear in Mu at the vicinity of columns which usually has high values of stress concentrates.

Consider an arbitrary plate of domain Ω and boundary Γ, loaded by domain loading of intensity q as shown in Fig. 1. Internal columns and walls are modeled using internal supporting cells with the real geometry of their cross sections. Three generalized forces are considered at each internal support: two bending moments in two orthogonal directions as well as shear force in the vertical direction. These generalized forces are considered to vary constantly over the column cross-section. The plate bending theory [13] is used in the formulation of the direct boundary integral equation of the plate. This integral equation can be presented as follows [12-14]:

$$C_{ij}(\xi)u_j(\xi)+\int_{\Gamma(\mathrm{x})}T_{ij}(\xi,\mathrm{x})u_j(\mathrm{x})\,d\Gamma(\mathrm{x})=\int_{\Gamma(\mathrm{x})}U_{ij}(\xi,\mathrm{x})t_j(\mathrm{x})\,d\Gamma(\mathrm{x})$$
$$+\sum_c\left\{\int_{\Omega_c(\mathrm{y})}\left[U_{ik}(\xi,\mathrm{y})-\frac{\nu}{(1-\nu)\lambda^2}U_{i\alpha,\alpha}(\xi,\mathrm{y})\delta_{3k}\right]F_k(y)\,d\Omega_c(\mathrm{y})\right\} \tag{1}$$

where $T_{ij}(\xi,x)$, $U_{ij}(\xi,x)$ are the two-point fundamental solution kernels for tractions and displacements, respectively [11]. The two points ξ and x are the source and the field points, respectively. $u_j(x)$ and $t_j(x)$ denote the boundary generalized displacements and tractions. $C_{ij}(\xi)$ is the jump term. The symbols ν and λ denote the plate Poisson's ratio and shear factor. The symbol c denotes the number of internal columns that have domain Ω_c. F_k represents the column two bending moments and vertical force (M_{ux}, M_{uy}, F). The field point *y* denotes the point of the internal column center.

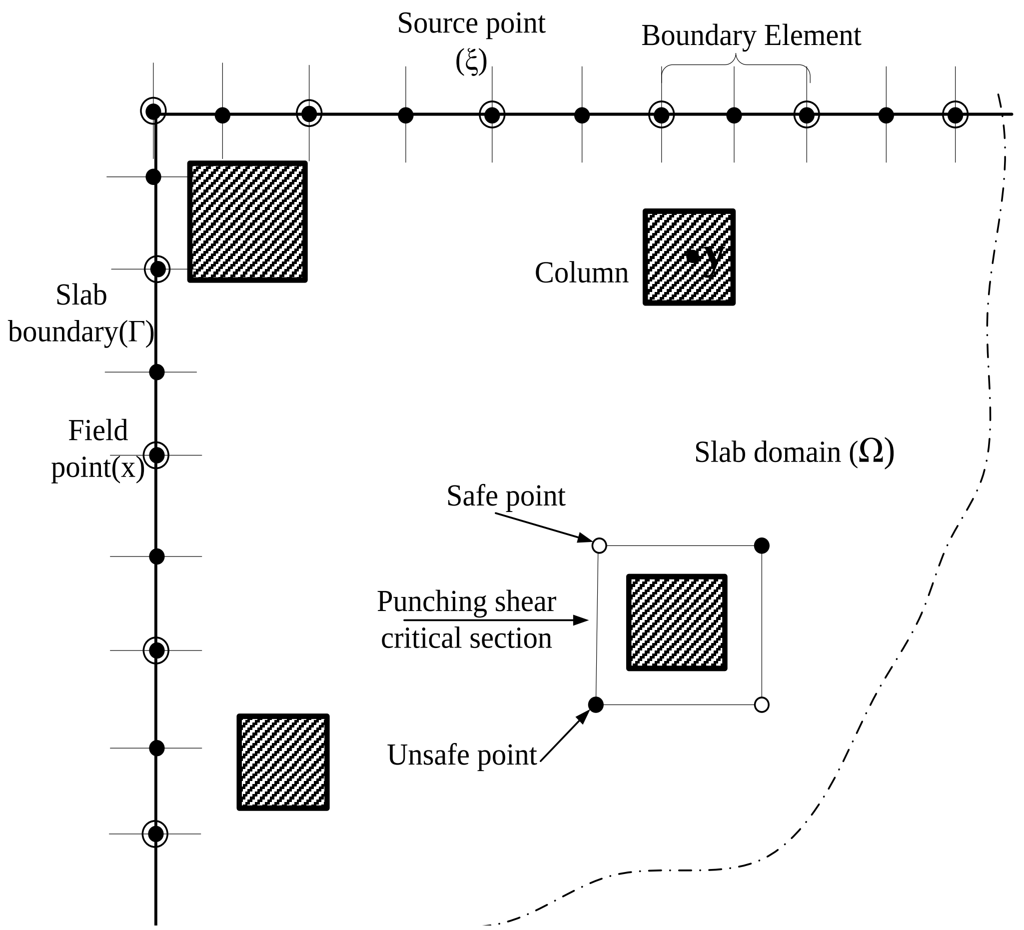


Fig. 1. The boundary element model of general flat plate supported on columns.

Equation (1) represents three integral equations. If the slab boundary is discretized into quadratic elements (three nodes per element), another collocation scheme has to be hence carried out at each "column" center to add more equations as follows:

$$u_i(\mathrm{Y}) + \int_{\Gamma(\mathrm{x})} T_{ij}(\mathrm{Y,x})u_j(\mathrm{x})\, d\Gamma(\mathrm{x}) = \int_{\Gamma(\mathrm{x})} U_{ij}(\mathrm{Y,x})t_j(\mathrm{x})\, d\Gamma(\mathrm{x})$$
$$+ \sum_c \int_{\Omega_c} \left[U_{ik}(\mathrm{Y,y}) - \frac{\nu}{(1-\nu)\lambda^2} U_{i\alpha,\alpha}(\mathrm{Y,y})\delta_{3k} \right] F_k(y)\, d\Omega_c(\mathrm{y}) \tag{2}$$

where, *Y* represents a point at each "column" center as shown in Fig.1.

Both Eqs. (1-2) could be used to solve the problem and obtain the unknown boundary generalized tractions or displacements.

Internal values at any internal point ξ can be computed as a post-processing stage. For example, displacements at internal points can be computed using Eq. (1) with $C_{ij}(\xi)=\delta_{ij}$ the identity matrix; whereas straining action values (bending and twisting moments $M_{\alpha\beta}$ as well as shear forces $Q_{3\beta}$) can be computed using other integral Eq. [12]:

$$M_{\alpha\beta}(\xi) = \int_{\Gamma(x)} U_{\alpha\beta k}(\xi,x)t_k(x)d\Gamma(x) - \int_{\Gamma(x)} T_{\alpha\beta k}(\xi,x)u_k(x)d\Gamma(x) + \sum_c \left\{ F_k(y) \int_{\Omega_c(y)} \left[U_{\alpha\beta k}(\xi,y) - \frac{\nu}{(1-\nu)\lambda^2} U_{\alpha\beta\theta,\theta}(\xi,y)\delta_{3k} \right] d\Omega_c(y) \right\} \tag{3}$$

The shear stress at the internal point (ξ) are obtained as follows [12]:

$$Q_{3\beta}(\xi) = \int_{\Gamma(x)} U_{3\beta k}(\xi,x)t_k(x)d\Gamma(x) - \int_{\Gamma(x)} T_{3\beta k}(\xi,x)u_k(x)d\Gamma(x) + \sum_c \left\{ F_k(y) \int_{\Omega_c(y)} \left[U_{3\beta k}(\xi,y) - \frac{\nu}{(1-\nu)\lambda^2} U_{3\beta\theta,\theta}(\xi,y)\delta_{3k} \right] d\Omega_c(y) \right\} \tag{4}$$

The relevant new kernel Uijk, Tijk and Wiβ and their relevant derivatives are determined in the literature [12]. It has to be noted that all kernels at internal points are smooth, and could be straightforwardly computed even over column centers. It can be seen from Eq. (4) that at each internal point **(**ξ) there are two values of the shear forces, i.e. $Q_{31}=q_x$ and $Q_{32}=q_y$.

3. CODE-SPECIFIED SHEAR STRESS VERSUS THE THEORY OF ELASTICITY SHEAR STRESSES

This section is dedicated to establish a relationship between the "*two"* components of vertical shear stresses, q_x and q_y obtained from Eq. (4), retrieved at each point from applying the boundary element method and the "*single-valued*" shear stress, υ_u, retrieved through applying the punching Eq. [1].

$$\upsilon_u = \frac{V_u}{A_c} + \frac{M_{ux}\gamma_{vx}}{I_x} y + \frac{M_{uy}\gamma_{vy}}{I_y} \tag{5}$$

where V_u is the value of the direct shear; M_{ux}& M_{uy} are the values of the unbalanced column moments about the center of geometry of the critical section; A_c is the area of concrete of an assumed critical section equal to d multiplied by the perimeter of the assumed section, with d being the effective slab depth; γ_{vx} and γ_{vy} are fractions of moment considered to be transferred by eccentricity of shear;x and y are coordinates of the point at which $\upsilon_u$is calculated; I_x and I_y are second moments of area of the critical section.

The analytical verification procedure illustrated herein entails solving many case-study problems under various loading schemes using the BEM, and obtaining q_x and q_y values by Eq. (4) at the four corners of the critical section around the internal rectangular column under consideration as described in Fig. 2.The process continues by plotting the retrieved pairs of q_x and q_y reported at each corner as single dots such as shown in Fig. 2. Hence, $\upsilon_u$as per Eq. (5) is computed at the same four corners of the critical section of the column considered and is compared to the shear strength of concrete (q_{up}) [1]. If the code-computed value of $\upsilon_u$at any point of the four corners of the critical section is found to be safe from a design perspective i.e., its value is less than the shear strength of concrete the BEM retrieved q_x-q_y pair computed at the same corner of the critical section and represented by a single dot in Fig. 2 is drawn as a white circle. Conversely, if the code-computed value of $\upsilon_u$does not satisfy the shear strength of concrete at any point of the four corners of the critical section, the BEM retrieved q_x-q_y pair computed at the same corner and represented by a single dot in Fig. 2 is drawn as a black dot.

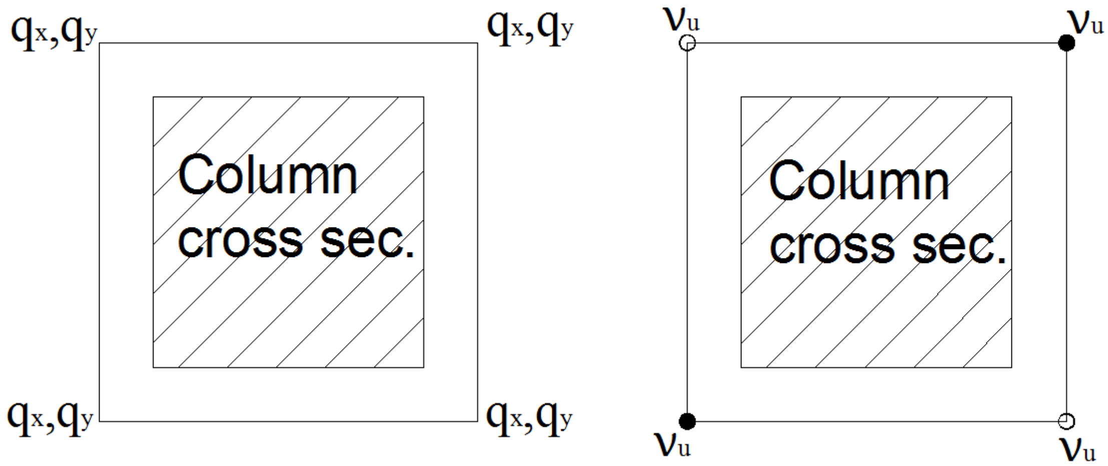


Fig. 2. Computed values of q_x,q_y retrieved using BEM and υ_u determined as per Eq. (5) at corners of critical section of an internal rectangular column.

The diagram shown in Fig. 3, depicting the interaction – if any – between the two components of shear stresses q_x and q_y computed through BEM and their association to the single-valued codified $\upsilon_u$as per Eq. (5) turns out to be a perfect square in shape with its four sides marking the shear strength of concrete. The direct conclusion from Fig. 3, is that there is no interaction between the elasticity-based two components of the shear stresses q_x and q_y at a given point retrieved from BEM, and accordingly, when both q_x and q_y have values less than the codified shear strength of concrete the punching check is satisfied. And conversely, if either q_x or q_y exceeds the concrete shear strength, punching check is violated. The theoretically-based shear state of stresses computed using the BEM and theory of elasticity is hence compatible with the single-valued semi-empirical shear stress resulting from applying Eq. (5).

It is worth noting that along the boundary of the envelop represented in Fig. 3, separating the white circle i.e., satisfying concrete shear strength from the black i.e., not satisfying concrete shear strength dots – the larger of q_x and q_y is equal to the shear strength of concrete. The proposed technique is hence implemented into a BEM-based software in order to check punching directly and accurately for design purposes after proving that the technique is compatible with the results obtained from the semi-empirical Eq. (5) for checking punching. This theoretically-based technique promoted in the present research could further precisely handle columns with irregular cross-sections which Eq. (5) could not satisfactorily consider.

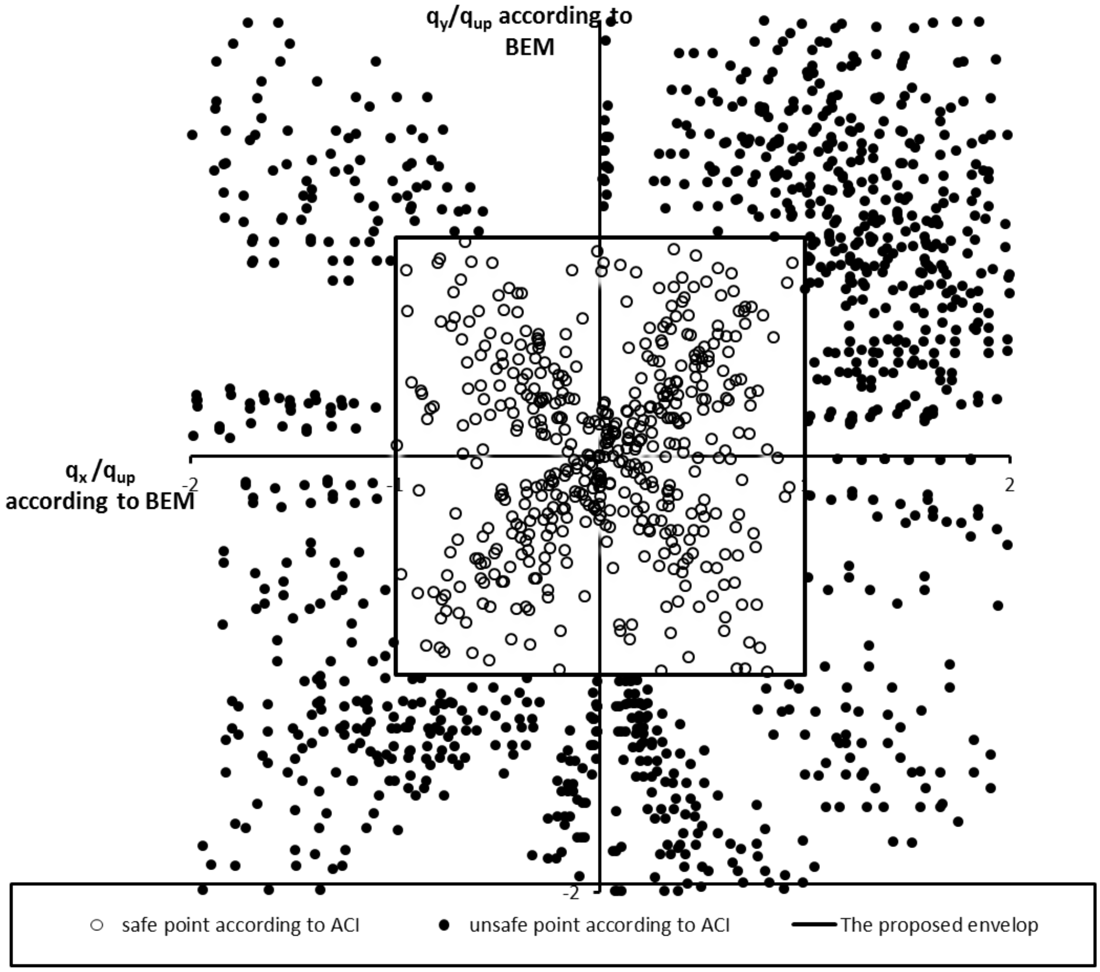


Fig. 3. The relation between q_x and q_y retrieved using BEM and the single-valued shear stress υ_u determined as per Eq. (5).

4. PROPOSED PUNCHING CHECK TECHNIQUE AND VERIFICATIONS

The solution procedures in Eqs. (1-4) have been implemented into computer code that allows computing the q_x & q_y according to Eq. (4) at series of points in the vicinity of the punching area around columns.

The user suggests the critical section according to any specific building code guidelines [2-4].

Hence values of shear stress results q_x & q_y are computed from Eq. (4).

The problem shown in Fig. 4, is considered. It consists of a square flat slab of two continuous spans [10m] each. The slab thickness (t) is [0.22m] with an effective depth (d) assumed as 0.65 [0.20m]. Young's modulus for concrete (E) is [2.1×10^6 t/m^2]. The cross section dimensions of the column under consideration (C_1) are [1m$\times$ 1m] and its height is [3m]. The cross section dimensions of the other (corner and edge) columns are [0.6m $\times$0.6m].

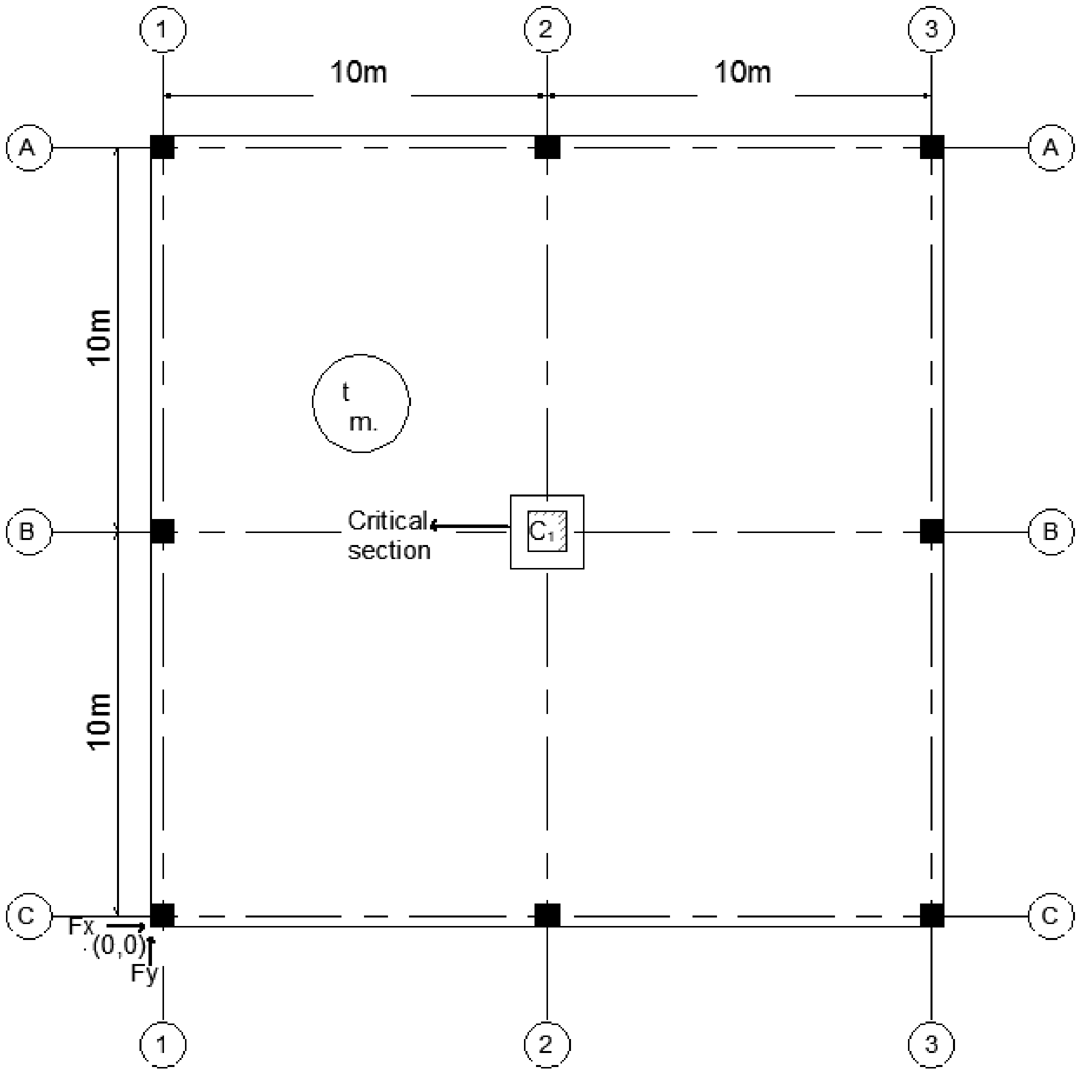


Fig. 4. A typical generic flat slab scheme supported on columns.

The building consists of 10 stories and the check of punching is conducted at the first story level. The building is subjected to eccentric lateral forces (F_x and F_y) applied at the lower left corner of the floor plan/footprint at the tenth floor i.e., at the roof level.

The critical section for punching is assumed at a distance *d*/2 from the face of the column. The punching check has been conducted for column (C1) Fig. 4, for a series of various permutations of magnitudes and directions of loading. Results are demonstrated in the Table 1 for the different loading schemes.

It can be seen from Table 1 that results retrieved from the proposed technique Eq.(4) for theoretically computed shear stress components q_x & q_y are compatible to those determined from Eq. (5) which verifies the developed code and the technique as well.

Table1. Results for checking punching stresses at critical section of the internal column C_1 black dots mean punching takes place at this location while white circles mean no punching risk.

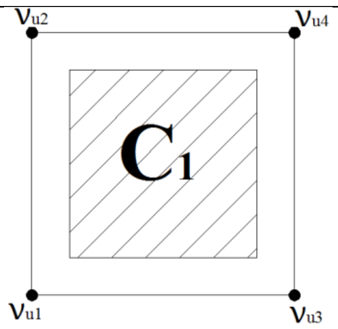
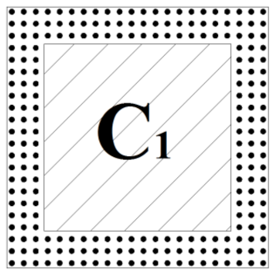
Case no.	Type of load	Building code Eq. (5)	Proposed technique
1	F_x=1000 t F_y= 1500 t	 Unsafe	 Unsafe

Table1, Continued

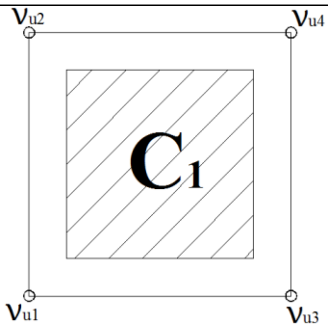
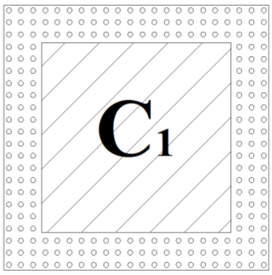
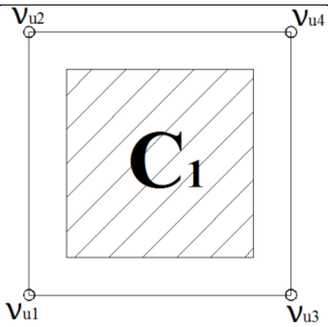
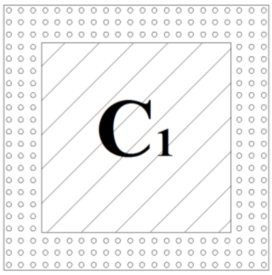
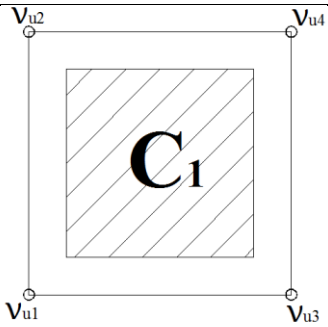
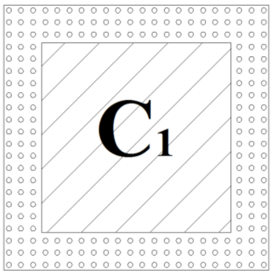
Case no.	Type of load	Building code Eq. (5)	Proposed technique
2	Fx=500 t Fy = 500 t	 Safe	 Safe
3	Fx=200 t Fy=200 t	 Safe	 Safe
4	Fx=1000 t Fy=1000 t	 Safe	 Safe

Table1, Continued

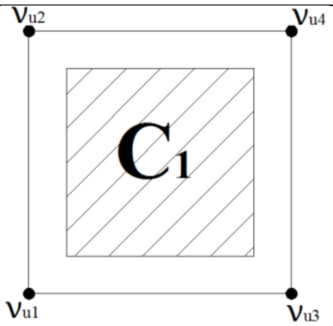
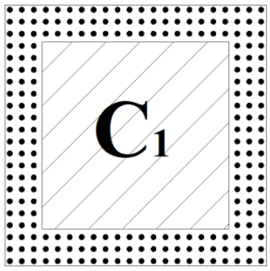
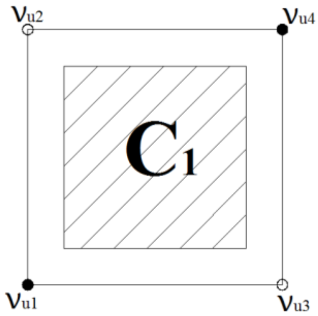
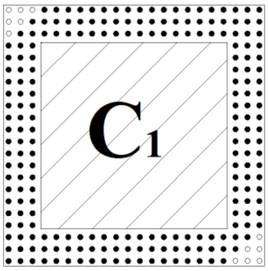
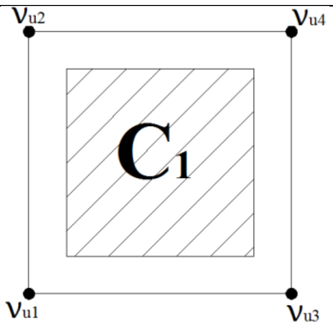
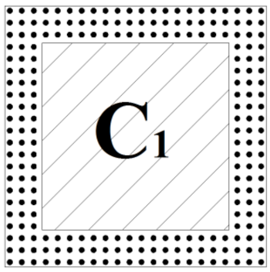
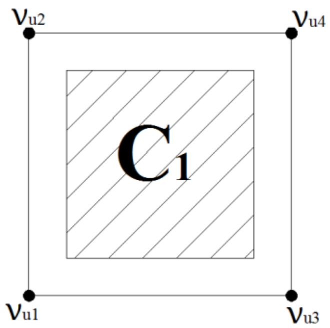
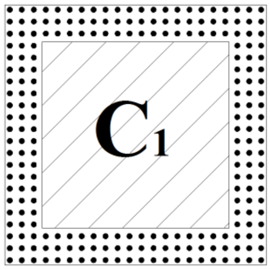
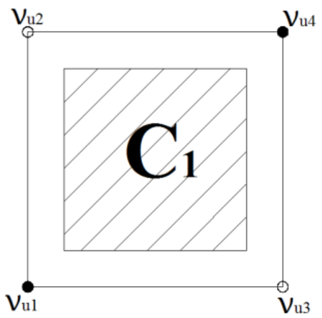
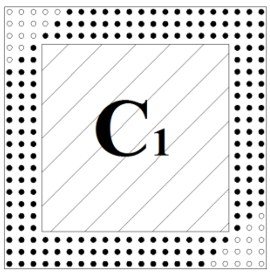
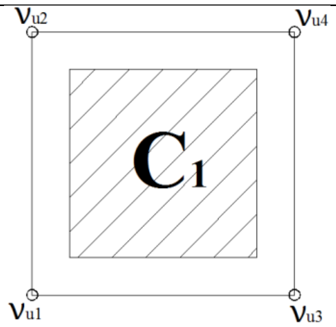
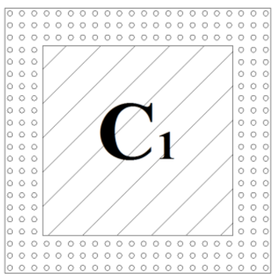
Case no.	Type of load	Building code Eq. (5)	Proposed technique
5	Fx=3000 t	 Unsafe	 Unsafe
6	Fx=3000 t Fy=3000 t	 Unsafe	 Unsafe
7	Fy=3000 t	 Unsafe	 Unsafe

Table1, Continued

Case no.	Type of load	Building code Eq. (5)	Proposed technique
8	Fx=1000 t Fy= 3000 t	 Unsafe	 Unsafe
9	Fx=2000 t Fy=2000 t	 Unsafe	 Unsafe
10	Fx=700 t Fy= 700 t	 Safe	 Safe

5. PUNCHING CALCULATION FOR COLUMN WITH IRREGULAR CROSS SEC.

The main advantage of the developed software is its versatility in dealing with columns with arbitrary/irregular cross sections and its capability to accurately and efficiently draw any shape of the critical section for punching; a task that Eq. (5) cannot straightforwardly address except with some assumptions and/or approximations. In order to illustrate the software competence let us consider the problem shown in Fig. 5. The slab thickness (t) is 0.22m with an effective depth (d) assumed as 0.20m. Young's modulus for concrete (E) is 2.1×10^6 t/m^2. The column under study is as shown in Fig. 5. The slab is subjected to 2 t/m^2 distributed load, and the primary critical section for punching is assumed at a distance *d*/2 from the face of the column while the secondary critical section is assumed at a distance 2d from the face of the column. Fig.6. demonstrates the BEM mesh of the problem. It can be seen from Fig.7, that punching is unsafe around the irregular column under consideration; a conclusion that is accurately achieved through applying the proposed technique and the developed software.

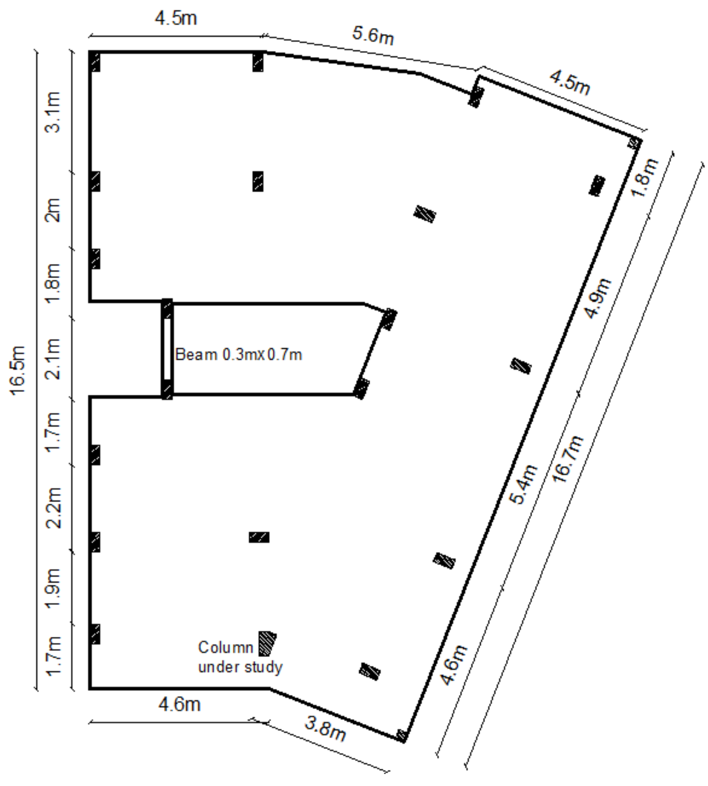


Fig. 5. Flat slab scheme supported on columns.

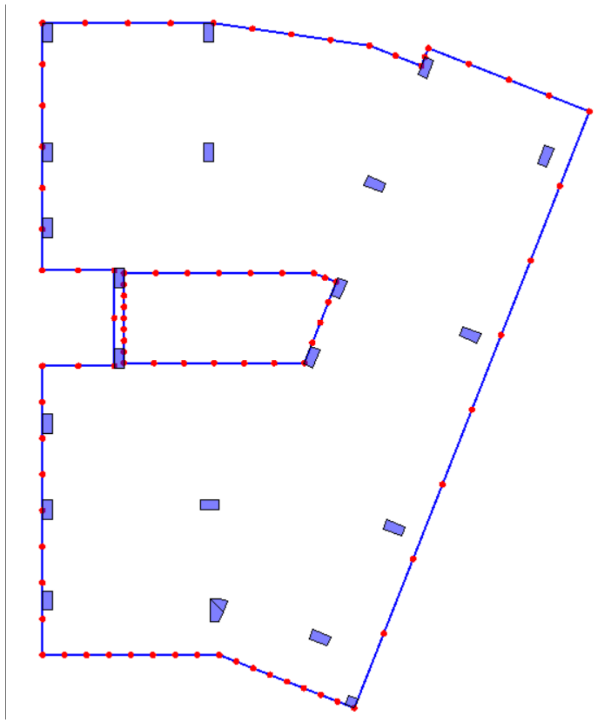


Fig. 6. BEM mesh of the flat slab scheme supported on columns.

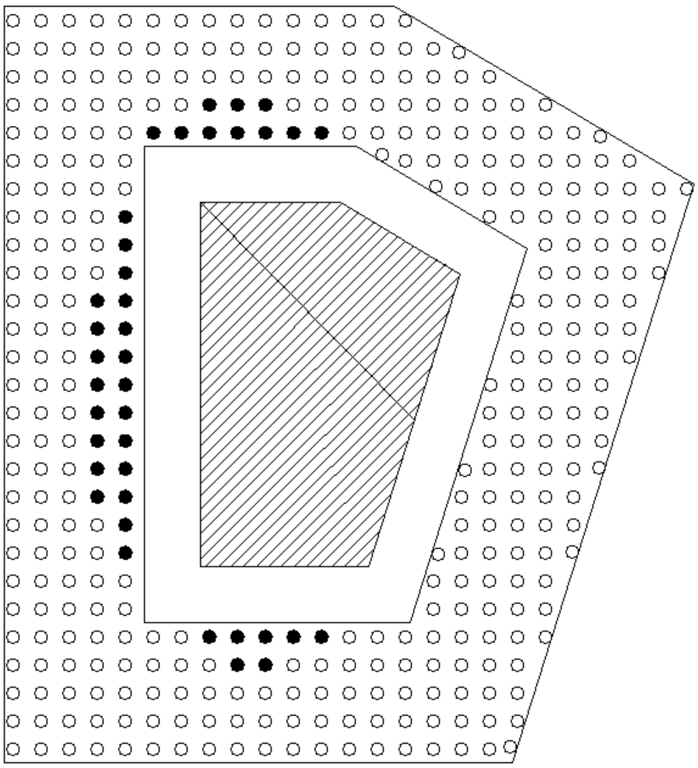


Fig. 7. The check of punching around an irregular column using the proposed technique.

6. CONCLUSIONS

The paper illustrates the link between the shear stresses retrieved per the theory of elasticity principles and the single-valued shear stress referred to in the semi-empirical punching shear formula adopted in ACI 318 as an example. It further proposes a computational/numerical robust technique based on the BEM formulation to check punching stresses of RC columns with arbitrary cross section (regular or irregular). The proposed method is implemented into a versatile BEM-based software with a smart graphical interface. The software has been tested for rectangular as well as real-world irregular shape columns that building code formula cannot straightforwardly address. Satisfactory results have been obtained as illustrated in the present manuscript when applying the proposed technique proving the reliability, efficiency, robustness and practicality of the software that shall be hence promoted for the research and design community.

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الاساس النظرى لمعادلة القص

تم فى البحث توقيع العلاقة بين اجهادى القص فى نظرية المرونة لعدة اعمدة وتحديد هل العمود امن ام لا بواسطة معادلة القص المقترحة فى الاكواد، ومن هذه العلاقة تم ايجاد علاقة أخرى بين الاساس النظرى المعتمد على نظرية المرونة والاساس المعملى المعتمد على معادلة القص، وتم تطوير الاساس النظرى بواسطة برنامج حاسب لفحص القص الثاقب لاى عمود غير منتظم القطاع حيث ان هذه الجزئية غير مغطاة فى الاكواد العالمية، وتم تطبيق البرنامج اولا على الاعمدة ذات المقطع المنتظم ومقارنة النتائج ثم تطبيقه على الاعمدة ذات المقطع غير المنتظم.