ADAPTIVE FUZZY CONTROL OF ACTIVE SUSPENSION SYSTEMS

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ABSTRACT

Two criteria of good vehicle suspension performance are typically their ability to provide good road-holding ability and increased passenger comfort. The main disturbance affecting these two criteria is terrain irregularities. Active suspension control systems reduce these undesirable effects by isolating car body motion from vibrations at the wheels. The paper describes an adaptive fuzzy control (AFC) schemes for the automobile active suspension system (ASS). The design objective is to provide smooth vertical motion so as to achieve the road-holding (with as small as possible tire deflection) and riding comfort over a wide range of road profiles. Effectiveness of the proposed control scheme is demonstrated via simulations. With respect to the optimal linear control (LQR), it is shown that superior results have been achieved by the AFC.

KEYWORDS: Active suspension system (ASS), linear quadratic regulator (LQR), fuzzy logic control (FLC), adaptive fuzzy control (AFC), road profiles.

1. INTRODUCTION

Poor road-holding capability and decreased passenger comfort are due to excess body vibrations resulting in vehicle speed limitations, reduced vehicle-frame life, biological effects on passengers, and detrimental consequences to cargo. Active suspension control systems aim to ameliorate these undesirable effects by isolating the car body from wheel vibrations induced by uneven terrain. Instead of the passive elements, active suspensions use actuators depending upon operating conditions to create the desired force in the suspension system.

Factors affecting the operating conditions are sprung mass acceleration, suspension deflection and tire deflection. The sprung mass acceleration affects the...
passenger ride comfort. The suspension travel must be limited to a certain range from the design viewpoint. With regards to handling, it maintains the wheels at proper steer and camber attitudes with respect to road surface. That is, the tires should be kept in contact with the road with minimal deflection and load variation. Finally, the control of active force is required to achieve satisfactory performance in relation to road disturbances.

Most of control design methods of ASS are based on the optimal control strategies [1-3]. The suspension system is optimized with respect to sprung mass acceleration, suspension deflection and tire deflection. However, in spite of the optimality, the fixed optimal state feedback cannot be adjusted as the perturbed road conditions occur. As opposed to optimal control theory, fuzzy logic control (FLC) has been considered by many authors [4-6] as an alternative control methodology for the ASS. The kernel of the FLC is a set of linguistic control rules, which captures human thinking and organizes the approximate reasoning to determine control decisions for the process.

While the non-adaptive FLC has proven its value in some applications, the need could arise to tune the rule base parameters if the plant changes. This provides the motivation for adaptive fuzzy control, where the focus is on the automatic on-line synthesis and tuning of the fuzzy parameters (i.e. the use of on-line data to continually “learn” the fuzzy controller). Adaptive fuzzy control (AFC) based on the Lyapunov synthesis approach has been extensively studied [7-9]. With this approach, the fuzzy system’s parameters can be automatically adjusted to achieve satisfactory system response.

In this paper, an AFC is developed for the closed loop system of the ASS. The control goal is to enforce robustness and adaptivity to road irregularities. The actuating force is determined based on the unsprung mass motion characteristics. Computer simulations are performed to verify the validity of the control schemes. The paper is organized as follows. Section 2 presents the quarter car model and the control statement with application to linear quadratic regulators. In Section 3, a FLC is
designed for ASS. In Section 4, the proposed AFC is developed. Simulation results are demonstrated in Section 5. Section 6 offers our concluding remarks.

2. MODELING AND PROBLEM FOUNDATION

A regular quarter-car model with two-degrees of freedom is depicted in Fig. 1, [4]. It is assumed that the tire does not leave the ground and that $z_s$ and $z_u$ are measured from static equilibrium position. In addition, the velocity of sprung mass $\dot{z}_s$ and relative velocity between unsprung mass and sprung mass $\dot{z}_u - \dot{z}_s$ are assumed to be measurable.

![Quarter-car model diagram](image)

Fig. 1. The quarter-car model.

The dynamic equations of the systems are:

$$m_s \ddot{z}_s = k_s (z_u - z_s) + b_s (\dot{z}_u - \dot{z}_s) + u$$  \hspace{1cm} (1)

$$m_u \ddot{z}_u = -k_s (z_u - z_s) - b_s (\dot{z}_u - \dot{z}_s) - u + k_r (z_r - z_u)$$  \hspace{1cm} (2)

where $z_r$ denotes road roughness and is regarded as disturbance, and $u$ is the control input.
To model the road input, we assume that vehicle is moving with constant forward speed. Then the vertical velocity $\dot{z}_v$ can be assumed to be white noise which is true for most real roadways. Let the state variables be defined as

\[ x_1 = z_s - z_u, \text{ body displacement}, \]
\[ x_2 = \dot{z}_s, \text{ absolute velocity of the body}, \]
\[ x_3 = z_u - z_r, \text{ unsprung (wheel) displacement}, \]
\[ x_4 = \dot{z}_u, \text{ unsprung (wheel) absolute velocity}. \]

The state equation of motion can be written as follows:

\[ \dot{x} = Ax + Bu + D\dot{z}_r \tag{3} \]

where

\[ A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -k_s/m_s & -b_s/m_s & 0 & b_s/m_s \\ 0 & 0 & 0 & 1 \\ k_u/m_u & b_u/m_u & -k_u/m_u & -b_u/m_u \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/m_s \\ 0 \\ -1/m_u \end{bmatrix}, \quad D = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \]

and \( x^T = [x_1, x_2, x_3, x_4] \).

Control of ASS is usually classified into linear regulator problem, in which the optimal control criteria are applied to reach an equilibrium operating point. The performance is then optimized with respect to passenger ride quality, suspension travel space, and road-holding ability [1-3,5]. Thus, the performance index is evaluated by the sprung mass acceleration, suspension deflection and tire fluctuation. The control \( u \) is computed to minimize the quadratic performance index

\[ J = \frac{1}{2} \int_0^\infty (x^TQx + u^TRu + \dot{z}_v^2)dt \tag{4} \]

where the matrix \( Q \) and \( R \) are weighting matrices of appropriate dimension corresponding to the state \( x \) and input \( u \). The optimal linear feedback control law is obtained by \( u = -K^Tx \), where \( K = PBR^{-1} \), and \( P \) is the solution of the Riccati equation:

\[ A^TP + PA - PBB^TPR^{-1} + Q = 0 \tag{5} \]
Further improvements have been achieved by the so called preview control [10-13], where the information describing road disturbances is assumed to be valid before the vehicle encounters.

In spite of optimality, there are some problems in practical implementation. First, the fixed feedback gains that are deduced by LQR cannot accommodate the parameter variations of the process. Second, application of the preview control is unrealistic since road irregularities can hardly be known in advance.

3. FUZZY LOGIC CONTROL

In this Section, a model free FLC is designed to overcome the aforementioned disadvantages. First, some basic concepts regarding fuzzy logic systems are recalled.

3.1 Basic Concepts

Fuzzy systems can be represented as linear combination of fuzzy basis functions which can be used as controllers. The most important advantage of the fuzzy basis functions is that a linguistic IF-THEN rule is directly related to a fuzzy basis function expansion providing a natural frame work to combine both numerical information (in the form of input output pairs) and linguistic information (in the form of fuzzy IF-THEN rules) in a uniform fashion.

A FLC consists of a collection of \( L \) fuzzy IF-THEN rules in the following form:

\[
\text{Rule}^l : \text{IF} x_1 \text{ is } A_1^l \text{ and} \ldots \text{and} x_m \text{ is } A_m^l \text{ THEN} u \text{ is } \theta^l
\]

(6)

where \( l = 1,2,\ldots,L \) is the rule number, \( x_j (j = 1,2,\ldots,m) \in U \) and \( u \in R \) are respectively, the input and output variables. \( A_j^l \) are the antecedent linguistic terms in the rule \( l \); and \( \theta^l \)'s are labels of the rule conclusion (control action).

The fuzzy rules (6) can be reduced into the following fuzzy logic system:
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\[ u(x) = h_0 \sum_{l=1}^{L} \theta^l \prod_{j=1}^{m} \mu_{A_j^l}(x_j) \]

(7)

where \( h_0 > 0 \) is a tuning gain, \( \mu_{A_j^l}(x_j) \) is the membership grade of the input \( x_j \) in the membership functions \( A_j^l \) of rule \( l \) and \( \theta^l \)'s are free (adjustable) parameters. This form of fuzzy system represents static mapping between the input variables and the output and is called standard fuzzy systems [14,15]. In Section 4, it is implemented it to generate the control input to the ASS. Equation (7) can be rewritten as:

\[ u(x|\theta) = h_0 \theta^T \psi(x) \]

(8)

where \( \theta = (\theta^1, \ldots, \theta^L) \) is the parameter vector and \( \psi(x) = [\psi^1(x), \ldots, \psi^L(x)]^T \) is a regression vector with the regressor given by

\[ \psi^l(x) = \prod_{j=1}^{m} \mu_{A_j^l}(x_j) / \sum_{l=1}^{L} \prod_{j=1}^{m} \mu_{A_j^l}(x_j) \]

(9)

Through out this work, Gaussian membership functions have been selected for the input variables. A Gaussian membership function is specified by two parameters \( \{c, \sigma\}: \)

\[ \mu_{A_j^l}(x_j) = \text{gaussian}(x_j; c, \sigma) = \exp \left[ -\frac{1}{2} \left( \frac{x_j - c}{\sigma} \right)^2 \right] \]

where \( c \) represents the membership function’s center and \( \sigma \) determines its width.

3.2 Fuzzy Logic Control Design

As universal approximators, fuzzy systems can be implemented in varieties of ways for any control problem. Essentially, the designer should make sure that the controller will have the proper information available to make good designs. Therefore, there are many choices for the proper control inputs so that the controller is able to steer the system in the direction needed to be able to achieve high performance. For
example, the authors in [4] have chosen three inputs and one output fuzzy control system. The inputs were the sprung mass velocity \( \dot{z}_s \) and the unsprung mass velocity \( \dot{z}_u \) and the actuating force is the output. Their selection, which ignores the other state variables, was based on some characteristics of the system dynamics, like frequency response which cannot be generalized to all road irregularities. Furthermore, the rule base is complex and cannot be easily interpreted by the designer. It is the author's point of view that a good fuzzy system incorporates all the state variables stated in Section 2, and is as simple as possible in order to facilitate the design.

The fuzzy controller proposed in this paper has two inputs: suspension deflection (or suspension stroke) \( z_s - z_u \) and suspension velocity \( \dot{z}_s - \dot{z}_u \) and one output; i.e. the actuating force \( u \). With this structure, the type of FLC is called PD fuzzy controller [14]. The input variables are assumed to be available feedback signals. The rule base consists of 25 rules. They are listed in Table 1, in which memberships of the inputs and output are negative large (NL), negative small (NS), zero (Z), positive small (PS) or positive large (PL). This rule set represents the expert knowledge on how to control the ASS given the suspension deflection and velocity as inputs. The linguistic rules of the fuzzy controller can be read as follows:

\[
R^1: \text{IF} (z_s - z_u = \text{NL}) \text{AND} (\dot{z}_s - \dot{z}_u = \text{NL}) \text{THEN} u = \text{NL}
\]

\[
R^{17}: \text{IF} (z_s - z_u = \text{PS}) \text{AND} (\dot{z}_s - \dot{z}_u = \text{NS}) \text{THEN} u = \text{Z}
\]

<table>
<thead>
<tr>
<th>Force ( u )</th>
<th>Suspension Deflection ( ( \dot{z}_s - \dot{z}_u ) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NL</td>
</tr>
<tr>
<td>Suspension Deflection ( ( z_s - z_u ) )</td>
<td>NL</td>
</tr>
<tr>
<td>NS</td>
<td>NL</td>
</tr>
<tr>
<td>Z</td>
<td>NL</td>
</tr>
<tr>
<td>PS</td>
<td>NS</td>
</tr>
<tr>
<td>PL</td>
<td>Z</td>
</tr>
</tbody>
</table>
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The body of Table 1 can be viewed as a matrix with a diagonal of zeros where a certain kind of symmetry can be noticed. This symmetry is no accident and is actually a representation of abstract knowledge about how to control the ASS. Indeed, it arises due to symmetry in the system dynamics.

Generally speaking, adjusting parameters of the fuzzy system is a tedious process which usually involves some trial and error procedure. In the coming Section, an adaptive law is derived to on-line tune the consequent parts (control actions) of the rule base. The antecedent parts are fixed with appropriate parameter values.

4. ADAPTIVE LAW SYNTHESIS

There are two main reasons for using adaptive fuzzy systems (8) as building blocks for adaptive fuzzy controllers. Firstly, it has been proved that they are universal approximators. Secondly, all the parameters in $\Psi(x)$ can be fixed at the beginning of adaptive fuzzy systems expansion design procedure so that the only free design parameter vector is $\theta$. In this case, $u(x|\theta)$ is linear in parameters. This approach is adopted in synthesizing the adaptive control law in this paper. The advantage of this approach is that simple linear parameter estimation methods can be used to analyze and synthesize the performance and robustness of adaptive fuzzy systems. If no linguistic rules are available, the adaptive fuzzy system reduces to a standard nonlinear adaptive controller.

The suspension mathematical model given by equation (3) can be expressed as:

$$\dot{x} = Ax + Bu + E_1$$

where $A$ is Hurwitz and $E_1 = D\ddot{r}$. Therefore, there exists a unique positive definite matrix $P$ that satisfies the Lyapunov equation:

$$A^T P + PA = -Q$$

If the control input $u$ is approximated by an adaptable fuzzy system (8), then equation (10) becomes,

$$\dot{x} = Ax + h_u B \theta^T \psi(x) + E$$

where $E = E_1 + E_2$ and $E_2$ is the approximation error.
The control objective is to determine a feedback control \( u = u(x|\theta) \) based on fuzzy logic systems and an adaptive law for adjusting the parameter vector \( \theta \) such that:

(a) The closed loop system must be globally stable in the sense that all variables, \( x(t), \theta(t) \) and \( u(x|\theta) \) must be uniformly bounded; that is \( |x(t)| \leq M_x < \infty \), \( |	heta(t)| \leq M_\theta < \infty \) and \( |u(x|\theta)| \leq M_u < \infty \) for all \( t \geq 0 \), where \( M_x, M_\theta \) and \( M_u \) are designed parameters specified by the designer.

(b) The tracking errors \( e = x - x_d \) should be as small as possible under the constraints in the previous objective. To this end, let

\[
\dot{x} = Ax + h_xB\phi^T\psi(x) \tag{13}
\]

be the ideal system model (identification model), where \( \phi^* \) denotes the optimal \( \theta \) defined as

\[
\phi^* = \arg \min_{\theta \in \mathcal{U}} \left[ \sup_{|x| \leq M_u} u(x|\theta) - u(x|\theta) \right] \tag{14}
\]

Therefore

\[
\dot{e} = A\epsilon + h_xB\phi^T\psi(x) + \hat{E} \tag{15}
\]

where \( \phi = \theta - \phi^* \) and \( \hat{E} \) is an estimate of \( E \) to be defined. To derive a control law that ensures that \( e \to 0 \) as \( t \to \infty \) a candidate Lyapunov function can be chosen as:

\[
V = \frac{1}{2} \left[ e^TPe + \phi^T\phi \right] \tag{16}
\]

where \( \gamma > 0 \) is a design parameter. The time derivative of \( V \) is

\[
\dot{V} = -e^TPe + e^TPB(\dot{\hat{E}} + h_x\phi^T\psi(x)) + \frac{\phi^T\dot{\phi}}{\gamma} \tag{17}
\]

Rearranging equation (17) yields

\[
\dot{V} \leq -e^TPe + e^TP\dot{\hat{E}} + \frac{\phi^T}{\gamma} \left( h_x\gamma\|\dot{\hat{E}}\|e^TPB\psi(x) + \dot{\phi} \right) \tag{18}
\]

Now choosing the adaptive law (recalling \( \dot{\phi} = \dot{\theta} \))
\[ \dot{\theta} = -h_0 \gamma \| \hat{E} \| e^T PB \psi(x) \]  \hspace{1cm} (19)

Therefore, equation (18) can be reduced to

\[ \dot{V} \leq -e^T Q e + e^T PB \dot{E} \]  \hspace{1cm} (20)

Equation (20) can be recast using vector norms;

\[ \dot{V} \leq -\lambda_{\text{min}}(Q) \| e \|^2 + \| e^T PB \| \| \dot{E} \| \]  \hspace{1cm} (21)

Let \( \| \dot{E} \| \) be bounded such that

\[ \| \dot{E} \| \leq \frac{\lambda_{\text{min}}(Q) \| e \|^2 - \alpha \| e \|}{e^T PB} \]  \hspace{1cm} (22)

where \( \alpha > 0 \), substituting for \( \| \dot{E} \| \) in equation (21) gives

\[ \dot{V} \leq -\alpha \| e \| \]  \hspace{1cm} (23)

Hence, the control law in (19) will ensure that the state \( e \) converges and the closed loop system is stable in the Lyapunov sense. The overall control system is demonstrated in Fig. 2.

![Diagram](Fig. 2. The adaptation mechanism.)
5. SIMULATIONS AND RESULTS

The model parameters selected for this study are similar to those in [4]. They are listed in Table 2. To implement the adaptive control law (19), singletons of the controller \( \theta \), are initialized within the interval \([-60,60]\); i.e. \(-60,-30,0,30,60\) for NL,NS,Z,PS,PL, respectively. The remaining parameters are set as follows: \( h_o = 2 \), \( \gamma = 1.0 \), and \( \hat{E} = 50 \). \( \psi(x) \) is formulated using the IF part of the fuzzy rule Table 1, and matrix \( P \) has been computed according to (11); the matrix \( Q \) is assumed as \( Q = 3I \) where \( I \) is the \( 4 \times 4 \) identity matrix.

Gaussian membership functions with width \( \sigma = 1.0 \), have been chosen as the membership functions of the input variables. Membership functions of the suspension deflection are shown in Fig. 3. They are equally spaced in the universe of discourse; \([-0.5,0.5]\). Similarly, equally spaced functions have been used for the suspension velocity, expect that the universe of discourse is \([-3.0,3.0]\).

In the following numerical tests, we consider two types of road profiles. Their inputs on the wheel are step and frequency disturbances. The performance of the proposed AFC against road irregularities is compared with LQR and the passive case (no feedback control).

![Fig. 3. Membership functions of the suspension deflection.](image-url)
Table 2. Parameter values of the quarter-car model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_s$</td>
<td>250 (kg)</td>
</tr>
<tr>
<td>$m_u$</td>
<td>30 (kg)</td>
</tr>
<tr>
<td>$k_s$</td>
<td>15000 (N/m)</td>
</tr>
<tr>
<td>$k_t$</td>
<td>150000 (N/m)</td>
</tr>
<tr>
<td>$b_s$</td>
<td>1000 (N.s/m)</td>
</tr>
</tbody>
</table>

Step disturbance: Let the road disturbances on the wheel $z_r$ take place in two consequent step inputs. The first is a step variation in which the wheel is exposed to a positive $0.1m$ step. Then, after 2 sec., the wheel is exposed to a negative $0.2m$ step. From Fig. 4, it can be seen that significant improvement has been achieved using the AFC, where the sprung mass deflection is well damped compared with passive suspension and LQR. Higher control effort (force) was needed by the AFC to achieve this improvement.

Sinusoidal disturbance: To synthesize road disturbances $z_r(t)$, the following function may be used (also followed in [4,6]):

$$z_r(t) = \frac{C \sin(\omega t_i)}{\left[ \frac{\omega_c}{2Pv} \right]^{23}}$$

which includes the driving parameters like sprung mass natural frequency $\omega_c$, vehicle speed $v$, sampling time $t_r$, iteration number $i$, road quality $P$ and coefficient $C$. For the sake of simplicity, the above formula may be rewritten as:

$$z_r(t) = \mu_r \sin \omega t$$

The authors in [4] selected numerical values resulted in a sinusoidal wave with frequency $\omega_c = 7.7 \text{ rad/sec}$ and amplitude $\mu_r$ smaller than $0.05m$. In this work, the simulation parameters are selected so that the road amplitude $\mu_r = 0.1m$ with excitation frequencies ranging from $\omega_c = 4$ to $16 \text{ rad/s}$ which can be regarded as
severe testing conditions. Fig. 5 (a), shows that significant improvement has been achieved by the AFC with respect LQR, $\omega_s = 4 \text{rad/s}$. In comparison with LQR, AFC is able to reduce the amplitude of the sprung mass to almost 50%. At higher frequency, i.e. $\omega_s = 16 \text{rad/s}$; Fig. 5 (b), the amplitude has been reduced 25% with respect to LQR.

![Graph showing vertical motion of the sprung mass and the control input for step disturbance using LQR, passive and AFC.]

Fig. 4. Vertical motion of the sprung mass and the control input for step disturbance using LQR, passive and AFC.
Fig. 5. Vertical motion of the sprung mass and the control input for sinusoidal disturbance with (a) \( \omega_r = 4 \text{ rad/s} \) and (b) \( \omega_r = 16 \text{ rad/s} \).

As a measure for the ride comfort, Fig. 6 demonstrates the sprung mass acceleration when \( \omega_r = 4, 8, 12, 16 \text{ rad/sec} \). It shows superiority of the AFC, where the sprung mass has been greatly reduced, i.e. the best ride comfort has been achieved. At \( \omega_r = 4 \text{ rad/s} \), performance of the LQR is similar to passive suspension. At higher road frequency \( (\omega_r = 12, 16 \text{ rad/s}) \), the performance of the LQR has deteriorated and no improvement can be noticed with respect to passive suspension.

With regard to road-holding ability, Fig. 7 shows that the proposed AFC exhibits the minimum tire deflection. At high road frequencies \( (\omega_r = 12, 16 \text{ rad/s}) \), similar performance can be noticed for the LQR and passive suspension, however the proposed AFC still exhibits relatively low tire deflection.
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Fig. 6. Sprung mass acceleration \( (m/s^2) \) at sinusoidal road profiles with (a) \( \omega_r = 4 \text{ rad/s} \), (b) \( \omega_r = 8 \text{ rad/s} \), (c) \( \omega_r = 12 \text{ rad/s} \) and (d) \( \omega_r = 16 \text{ rad/s} \).

Fig. 7. Tire deflection \( (m) \) at sinusoidal road profiles with (a) \( \omega_r = 4 \text{ rad/s} \), (b) \( \omega_r = 8 \text{ rad/s} \), (c) \( \omega_r = 12 \text{ rad/s} \) and (d) \( \omega_r = 16 \text{ rad/s} \).
6. CONCLUSIONS

In this paper, an AFC has been developed for ASS. The adaptive controller is derived based on Lyapunov direct method and has the following advantages:
(a) does not require system model,
(b) guarantees the stability of the closed loop system, and
(c) control rule base and membership functions are simple yet generic.
The control law ensures fast convergence and enforces robustness against road irregularities. With respect to LQR, computer simulations demonstrate that the proposed AFC achieves superior performance. Furthermore, the proposed adaptive closed-loop control system is applicable to a large class of linear and nonlinear systems.
Like LQR, the proposed AFC is a state feedback controller. Inserting four transducers in the car’s ASS seems unreasonable, and a state observer is needed. An adaptive (self-tuning) fuzzy state estimator for the ASS is still an open question.

REFERENCES

التحكم ضمني متآقل لمنظومه التعلق بالسيارات

التعامل الجيد مع تضاريس الطريق وراحة الراكب هما العاملان الأساسيان لتقييم أداء نظم التحكم بالسيارات، ونظام التعلق الفعال (باستعمال تغذية خلفية) يهدف إلى عزل السيارة عن التأثر بتضاريس الطريق ويهدف البحث إلى تصميم نظام للتحكم ضمني المتآقل لتحقيق أفضل أداء ممكن للسيارة بالأضافه الى أفضل راحة ممكنه للراكب، بالمقارنة مع نظام التحكم المثالي (LQR) حيث ظهر أن نظام التحكم ضمني المتآقل والموترج يحقق أداء أفضل.