JOURNAL OF ENGINEERING AND APPLIED SCIENCE, VOL. 51, NO. 4, AUG. 2004, PP. 795-811
FACULTY OF ENGINEERING, CAIRO UNIVERSITY

A GENETIC ALGORITHM FOR PREVENTIVE MAINTENANCE SCHEDULING IN A MULTI-UNIT MULTI-STATE SYSTEM

M. A. SHALABY[1], A.H. GOMAA[2] AND A.M. MOHIB[3]

ABSTRACT

In this study, a maintenance schedule optimization model is developed for multi-unit, multi-state systems with multi-level of preventive maintenance actions. The decision variable in this model is the sequence of preventive maintenance actions which applied to the system in a finite time horizon. The total maintenance cost includes preventive maintenance, minimal repair, and downtime costs. Moreover, the developed model includes three types of constraints, which are system reliability, minimum interval between maintenance activities, and crew availability. The proposed model is solved using a specialized constrained genetic algorithm technique combined with simulation technique, and are programmed using MATLAB. The presented approach has the potential to solve realistic scale problems.

KEYWORDS: Maintenance Planning, Preventive Maintenance, Multi-unit, Multi-State, Genetic Algorithms, Simulation Model.

1. INTRODUCTION

Models for a multi-unit system are presented and diversified as for their decision variables, objective criteria, constraints, the solution techniques, application and assumptions. The decision variables differ from one model to the other; it could be a sequence of Preventive Maintenance (PM) actions [1 to 4], PM intervals [5, 6], replacement intervals [7, 8], manpower [4, 5], or component importance [9]. The objective in many models is cost-based. Cost items might be total maintenance costs [3, 4, 5], PM cost [2, 3], replacement cost [10, 6], minimal repair (MR) cost [3,5,6], part cost [4], fuel cost [2], labor cost [4,5], economic losses or quality cost [4,6]. Huang [1] considered the production cost and the reserve margin as objectives of a

---

[1] Assoc. Prof. of Ind. Eng., Mech. Design and Prod. Dept. Faculty of Eng., Cairo Univ., Egypt.
[2] Assoc. Prof. of Ind. Eng., Ind. Eng. Dept. Faculty of Eng., Cairo Univ., Fayoum, Egypt.
[3] Graduate student, Dept. of Mech. Design and Prod., Faculty of Eng., Cairo University, Egypt.

generator maintenance scheduling. Other researchers' objective criterion is time based; it is concerned with processing time [11], downtime [6,10] or reliability [2]. Deris et al. [12] applied their model in the Royal Malaysian Navy, where the availability and readiness of a fleet of ships determine the strength of the navy. Few problems are formulated as a multi-criteria decision-making problem. Kralj and Petrovic [2] introduced a multi-objective optimization approach; minimization of fuel costs, maximization of reliability and minimization of constraints violations. Lai et al. [6] applied a model in a company for public transportation where availability of buses as well as the cost are major concerns of the managers. The problem may be modeled as either constraint or unconstraint problem. Some researchers neglected the constraints [6,7,10,11,8,9] while others considered the reliability constraint [3], availability [5,12], number of workers [1,4,5], PM intervals [5, 12] or geographical constraint [12, 2].

Both global and local search approaches were applied to solve maintenance optimization problems. Successful applications of the Genetic Algorithms (GA) to maintenance optimization problem are reported in [3]. Deris et al. [12] mixed GA with constraint-based reasoning. It was proven that a difficult problem faced by the branch and bound could be solved by using a hybrid of the GA and the constraint-based reasoning approach. This is because the GA could find near optimal starting times (global search) and the constraint-based reasoning could search (local search) for feasible solution that satisfies resource and temporal constraints. For multi unit systems, the model is complex; simulation is usually used as a solution technique [7, 10]. Some researchers combined simulation with genetic algorithms [24], with non-linear programming technique [5], with fuzzy algorithms [1], or Microsoft access [9]. When number of units is small, branch and bound technique may be combined with pseudorandom initial solution [2], or column generation approach [11]. Yamashina and Otani [4] used heuristic approach based on local search.

None of the surveyed literature handled directly the problem with multi-unit, multi-state, system with multi levels of PM actions. The objective of this paper is to develop a procedure for scheduling the maintenance actions of this system while trying to minimize total maintenance cost.

2. PROPOSED APPROACH

The proposed model is a generalization of PM scheduling optimization problem for multi-unit, multi-state systems, which have a range of performance levels. For multi-state system the reliability is considered to be a measure of the ability of a system to meet demand (required performance level). The objective of the proposed model is to minimize the total maintenance cost. The multi-unit system consists of n units with a parallel/series structure. The unit j has L_j levels of PM actions; where each level l has a predetermined maximum allowable time interval $\delta_{j,l}$ that should never be exceeded. The system also consists of Q crews to perform maintenance activities necessary for a PM action, where each crew is able to perform any level of PM actions for any unit in the system. To model the system, some assumptions are proposed as shown below:

1- Each maintenance crew may be allocated to one maintenance job, at a time.
2- Failure data for the system units are known or estimated.
3- Corrective maintenance in the form of minimal repair is performed when a unit of the system fails.
4- PM actions under considerations are those characterized by their ability to reduce the age of the unit (thus increasing its remaining life).
5- The system structure is reducible.

The developed model contains some realistic constraints. The system reliability is constrained by two predefined levels, the first level is R_{S1} which is the minimum reliability set to start the next PM action, and the other level is R_{S2} which is the minimum allowable system reliability level. Another constraint is the PM interval; for each level of a PM action there is a time limit interval that the unit shouldn't exceed without undergoing another PM action from the same or higher level. Also, a restriction on the number of available maintenance crews in the system is introduced. The system reliability at each time interval is calculated.

Imperfect PM is modeled using the proportional age setback (PAS) age reduction concept. According to this concept the PM action reduces the effective age of the unit that it has immediately before it enters the maintenance action. For each unit *j* the PM action of level *l* has an age reduction coefficient $\varepsilon_{j,l}$ that ranges in the interval [0,1]. For unit *j*, Fig. 1 shows the relation between the unit's age and the chronological time (true time), taking into consideration the downtime of the unit (maintenance duration).

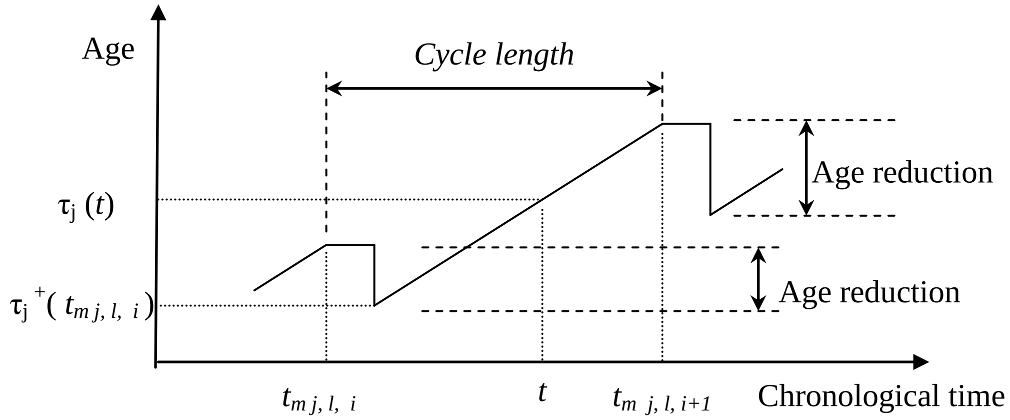

Fig. 1. Evolution of unit's age versus chronological time and downtime.

Often, there is a restriction on the time interval between the current PM action and the previous action of the same level or of higher levels. This constraint limit is called $\delta_{j,l}$ which is determined for each PM level *l* belonging to unit *j*. The other limitation is the number of crews existing in the system. The number of scheduled PM actions at any instance should be less than or equal to the available number of maintenance crews Q. The final form of the proposed model is as follows [refer to table of nomenclature]:

2.1 Mathematical Model

Decision Variable: Sequence of PM action V_j and the corresponding time interval T_{mj} for each unit *j*,

$$V_j = \{v_{j,l,1}, v_{j,l,2}, ..., v_{j,l,i}, ..., v_{j,l,K_j}\} \quad (1)$$

$$T_{m\,j} = \{t_{m\,j,l,1}, t_{m\,j,l,2}, ..., t_{m\,j,l,i}, ..., t_{m\,j,l,K_j}\} \quad (2)$$

for all j = 1, 2, …, n; i = 1,2, …, K_j ; l = 1, 2, …, L_j .

and by definition, $t_{m\ j,l,0}$ = 0 and $t_{m\ j,l,K_j+1}$ > T.

Objective: Minimization of total maintenance cost.

$$Min.\ C_M(V) = \sum_{j=1}^{n}\left[\sum_{l=1}^{L_j}\left[K_{j,l}\ C_{PM\ j,l}\right] + C_{Dtr\ j}\sum_{l=1}^{L_j}\left[K_{j,l}\ D_{PM\ j,l}\right] + C_{mrr\ j}\sum_{i=0}^{K_j}\left[H\left(\tau_j\left(t_{m\ j,l,i+1}\right)\right) - H\left(\tau_j^+\left(t_{m\ j,l,i}\right)\right)\right]\right] \quad (3)$$

where, $K_{j,l}$ is the number of repetitions of PM action level l for unit j,

$$\tau_j^+(t_{m\ j,l,i}) = \tau_j(t_{m\ j,l,i})\ \varepsilon_{j,l}, \quad (4)$$

$$\tau_j(t_{m\ j,l,i+1}) = \tau_j^+(t_{m\ j,l,i}) + (t_{m\ j,l,i+1} - (t_{m\ j,l,i} + D_{PM\ j,l}))\ . \quad (5)$$

at t = 0, $\tau_j^+(t) = \tau_j(t) = 0$

Constraints:

- Reliability Constraint

$$R_S(t,W^*) \geq R_{S2},\quad t_{m\ j,l,i} < t < t_{m\ j,l,i+1},\quad i = 1,2,\ldots,K_j,\ l = 1,2,\ldots,L_j\ . \quad (6)$$

Where
$$R_S(t,W^*) = \sum_{e\in E} p_e(t) \quad (7)$$

E is the set of all possible system output performance levels G_{Se}.

$$\sum_{all\ e} p_e(t)\ Z^{G_{Se}} = U_{sys}(t,z),\ \ p_e(t) = p_r(G_s(t) = G_{Se}) \quad (8)$$

This U-function of the system $U_{sys}(t,Z)$ can be obtained by applying the parallel/series composition operators to the individual U-function of each unit j, see [14], i.e. $U_j(t,Z)$, where

$$U_j(t,Z) = (1 - r_j(t))Z^0 + (r_j(t))Z^{G_j}\ , \quad (9)$$

where, G_j is the output performance rate of unit j, $\forall\ j$ = 1, 2, …, n, and

$$r_j(t) = e^{\left(H_j\left(\tau_j^+\left(t_{m\ j,l,i}\right)\right) - H_j\left(\tau_j(t)\right)\right)}, \quad (10)$$

where, $$\tau_j(t) = \tau_j^{+}(t_{m\ j,l,i}) + (t - (t_{m\ j,l,i} + D_{PM\ j,l})),\tag{11}$$

- PM intervals constraint

$$\min_{k \geq l}\left\{\left(t_{m\ j,l,i} - t_{m\ j,k,i} - D_{PM\ j,k}\right) X_{l,i}\right\} \leq \delta_{j,l}, \text{ for each unit j of action level l.}\tag{12}$$

where, $$X_{l,i} = \begin{cases} 1 & \text{level } l \text{ is in action } i \\ 0 & \text{otherwize} \end{cases}\tag{13}$$

- Maintenance crews availability constraint

$$\sum_{j=1}^{n} U_{PM\ j,i}(t) \leq \mathrm{Q}\tag{14}$$

where, $U_{PM\ j,i}(t)$ is the counter representing the number of units undergoing a scheduled PM action for unit j at a certain time interval t, t=1,2,….

3. SOLUTION TECHNIQUE

The presented mathematical model includes highly non-linear constraints. A simpler unconstrained mathematical model introduced by Levitin et al. [3] was solved using GENITOR. This model neglected the downtime cost item and assumed the PM duration equal zero, and didn't consider the levels of PM actions and the interval constraint. A specialized constrained GA technique is used to solve the proposed mathematical model. Figure 2 shows a block diagram describing the modules and the interrelation between them. Three modules are built to solve this problem; which are the main, GA, and simulation modules. A data file describing the system under study is linked to the simulation module. GA operators are linked to the GA module. A final report that includes the best-obtained solution is produced from the main module.

The main module is the interface between the user and the program, the user inputs all the data required for the optimization and the simulation. This data is the number of variables (N_{Var}, i.e. length of the string or the number of PM actions which will be initiated in the string), the maximum values for those variables ($V_{\max}$, i.e. the highest number in the PM list), population size (N_{Pop}, i.e. the number of strings "Chromosomes" used in the optimization procedure), and the number of generations

($N_{Gen,}$ i.e. the number of iterations from one population to the next). The user also selects the frequency of using each operator from the list of GA operators. (i.e. mutation (N_{UM}, N_{WUM}, N_{NUM}, N_{WNUM}), cross-over (N_{SC}, N_{UC}), renewal number (N_{new}), stopping criteria (N_{Stop})). The main module is also organizing the link between other modules.

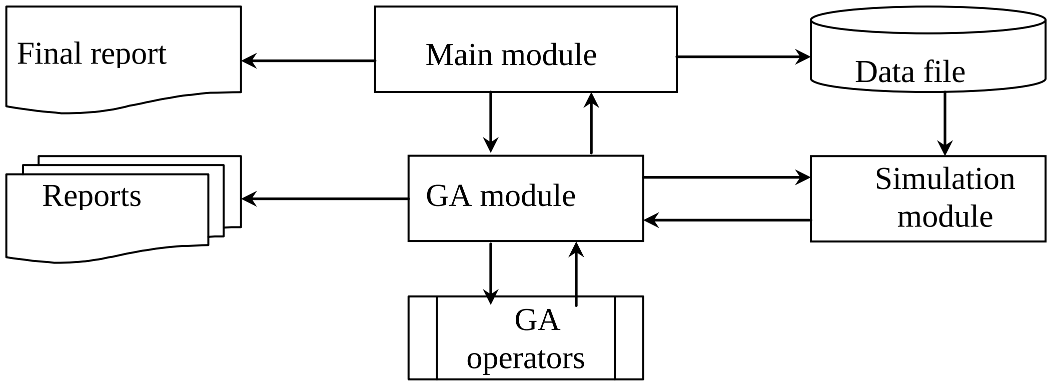

Fig. 2. Model Block Diagram.

The GA module is concerned with generating a population of integer strings, communicating with the simulation module to simulate and evaluate each string, and determining the best string. The GA module applies the operators (mutations and crossovers) to the population for N_{Gen} generations to find the best-obtained solution. The GA algorithm may be described in the following steps.

Step 1: Receive the data from the main module

Step 2: Initialize the population randomly

Generate N_{Pop} strings of N_{Var} variables each chosen randomly from the interval [1 - V_{max}].

Step 3: Repair each string by applying the PM interval constraint

Each value in a string is chosen from a PM list (Table 3); each value in the PM list is associated with a calculated number of repetitions. The constraint satisfaction operators check for the presence of each value in the interval [1 - V_{max}] with a certain number of repetitions.

Step 4: Evaluate the cost corresponding to each string

Send each string to the simulation module and receive the total maintenance cost $C_M(V)$ and the number of performed PM actions "K".

Step 5: Sort the strings in an ascending order of the cost value.

Step 6: Begin the generation loop

Store the string, which has the least cost so far in [V_{best}, C_{best}]. The generation loop consists of repeating the next steps (up to step 10) for N_{Gen} times.

Step 7: Apply GA's operators

Switch between GA's operators in a random sequence. A temporary population is formed from applying the GA's operators (Uniform Mutation (UM), Whole Uniform Mutation (WUM), Non Uniform Mutation (NUM), Whole Non Uniform Mutation (WNUM), Simple Cross-over (SC), Uniform Cross-over (UC)). To have an unbiased population, the numbers chosen to repeat the operators defined by the user (N_{UM}, N_{WUM}, N_{NUM}, N_{WNUM}, N_{SC}, and N_{UC}) are reordered randomly. The new strings are repaired and evaluated as in step 3 and 4.

Step 8: Switch the populations

Copy the best-obtained value so far ([V_{best}, C_{best}]) and the rest of the population, which didn't undergo mutation or cross-over to the temporary population. Sort the temporary population strings in an ascending order and copy them back to the original population.

Step 9: Renew the bad solutions of the population

To make sure of the existence of new blood in the new generation, i.e. to apply the randomness through all generations an operator called "Renewal operator" is applied. In which the worst strings (known as criminals) in the population are executed and replaced by a totally new randomly generated strings. The new strings are repaired, evaluated and sorted as in steps 3, 4 and 5.

Step 10: Eliminate the redundant strings in the population

Step 11: Terminate the generation loop

Generate loop ends whenever the number of generations reaches "N_{Gen}" or the least cost values "C_{best}" are almost the same for the last "N_{Stop}" generations. The best solution V is sent back to the main module to be presented in the Final output report.

The simulation module imitates the application of the maintenance schedule on the system, satisfies the predefined constraints and calculates the total maintenance cost. For more details, the reader may refer to Mohib 2003, [13].

4. CASE STUDY

The proposed model is applied to an existing case study, which is carried out in a major power station. The system structure and the output performance are presented in Fig. 3. The system contains four units in parallel. The PM actions' parameters are presented in Table 1. The historical data for the last few years was analyzed to obtain those parameters. The Weibull distribution parameters for the four units are presented in Table 2. Parameters of PM actions and relative cost units are expressed in Table 3. The number of maintenance crews Q = 1.

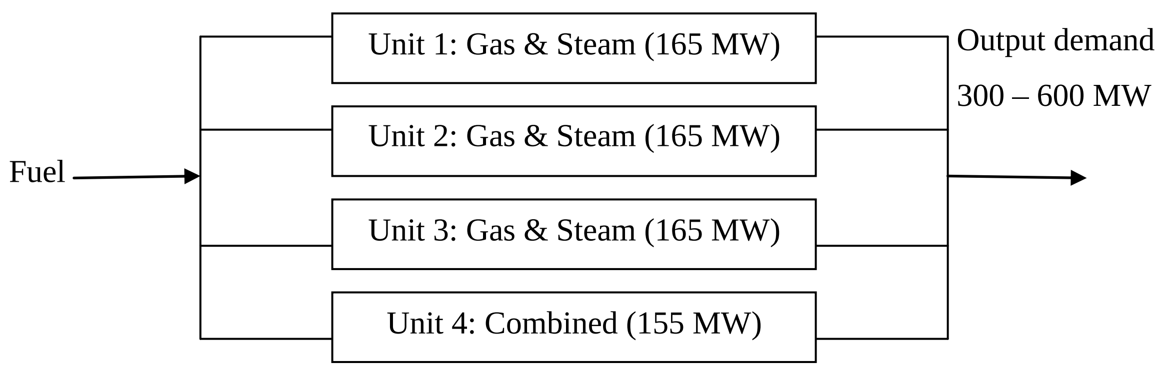

Fig. 3. Simplified System Structure and Performance Rates of the SCPS.

Table 1. PM interval constraints for Gas Turbines.

PM Action	Description	Maximum allowable time Interval "δ" (hours)
CI	Combustion Inspection	8000
HI	Hot Gas Path Inspection	24000
MI	Major Inspection	48000

Table 2. Failure Data Analysis.

Unit	Weibul Distribution Parameters		
	Scale λ	Shape γ	Location t_o
1	9200	1.3	974
2			
3			
4	5400	0.75	3813

Table 3. Parameters of PM Actions and Cost Elements.

Unit #	PM action #	PM type	Age reduction coefficient	PM cost (cost unit)	PM duration (hours)	MR cost (cost unit)	Downtime cost (cost unit/hour)
1	1	CI	0.5	40	720	15	1
	2	HI	0.3	700	1440		
	3	MI	0.1	1900	2160		
2	4	CI	0.5	40	720		
	5	HI	0.3	700	1440		
	6	MI	0.1	1900	2160		
3	7	CI	0.5	40	720		
	8	HI	0.3	700	1440		
	9	MI	0.1	1900	2160		
4	10	CI	0.5	50	720	13	
	11	HI	0.3	750	1440		
	12	MI	0.1	2000	2160		

Unit 1, 2 and 3 are similar units.

4.1 Existing Maintenance Plan

In this power station, a preventive maintenance plan is set every five years. The plan objective is to generate a predefined power capacity, and due to the manpower limitation, it is not to interfere the PM actions of one unit with another. The existing maintenance plan of the system for the years 1996 till 2000 is presented in the bar chart of Figure 4. Using the introduced mathematical relations, the resulting total maintenance cost of this plan was 32407 cost units and the associated system reliability curve is presented in Figure 5.

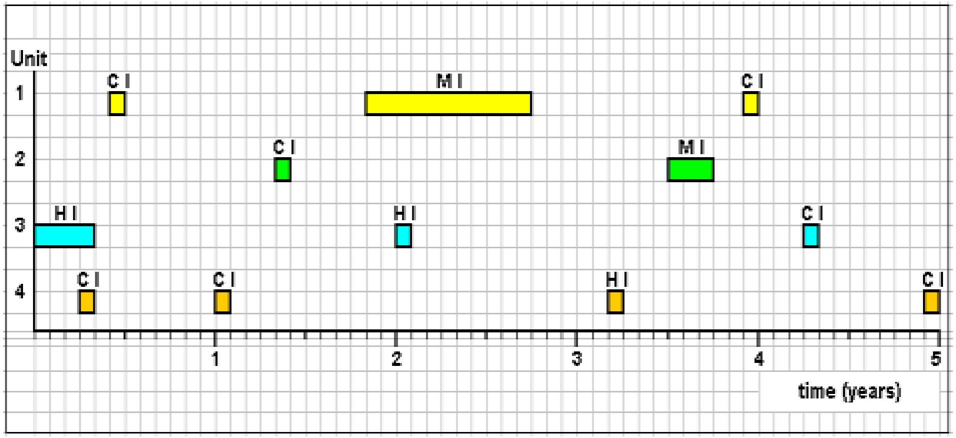

Fig. 4. Existing Maintenance Schedule (1996-2000).

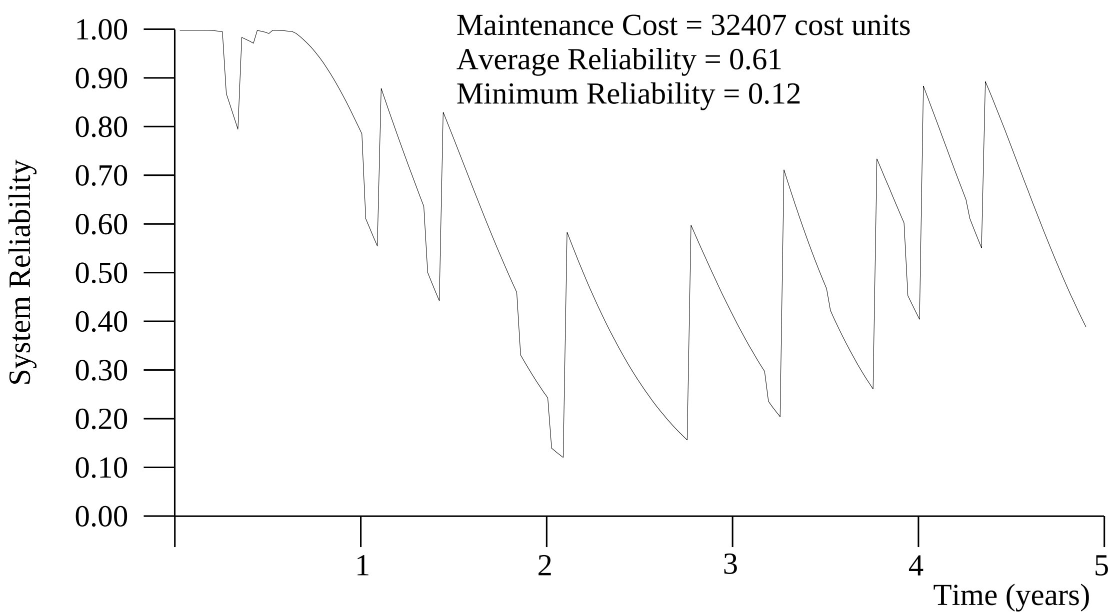

Fig. 5. System Reliability for the Existing Maintenance Schedule (1996-2000).

4.2 Proposed Maintenance Plans

The proposed plans are introduce to compare their performance with the existing one. As shown in Table 4, key reliability parameters for any maintenance plan are R_{S1} and R_{S2} where the first is to start the PM action and the other to not to let the system reliability decrease beyond. Thus each plan is characterized by these parameters. The GA's parameters shown in Table 5 are used.

Table 4. Reliability Constraints.

Proposed Plans		R_{S1}	R_{S2}
1st plan	Pc	0.5	0.1
2nd plan	AP1	0.6	0.2
3rd plan	AP2	0.7	0.3
4th plan	AP3	0.8	0.4
5th plan	AP4	0.9	0.5

Table 5. GA's Parameters.

Variable description	Abbreviation	Value
Population size	N_{Pop}	150
Number of generations	N_{Gen}	25
Number of variables	N_{Var}	20
Maximum variable's value	V_{max}	12
Uniform mutation	N_{UM}	4
Whole uniform mutation	N_{WNUM}	4
Non-uniform mutation	N_{WNUM}	4
Whole non-uniform mutation	N_{NUM}	4
Simple cross-over	N_{SC}	10
Uniform cross-over	N_{UC}	25
Renewal number	N_{new}	10
Stopping criterion value	N_{Stop}	7

A proposed maintenance plan "Po" is introduced to compare the reliability performance with the existing plan. In such industry, the system reliability is of great importance. Hence, to obtain better reliability performance, four alternatives plans (AP1 to AP4), less in cost than the existing maintenance plan, are introduced trying to satisfy the PM interval constraints. In the proposed maintenance plans, the PM reliability level R_{S1} at which the PM action should be applied and the minimal allowable reliability R_{S2}, were selected as shown in Table 4.

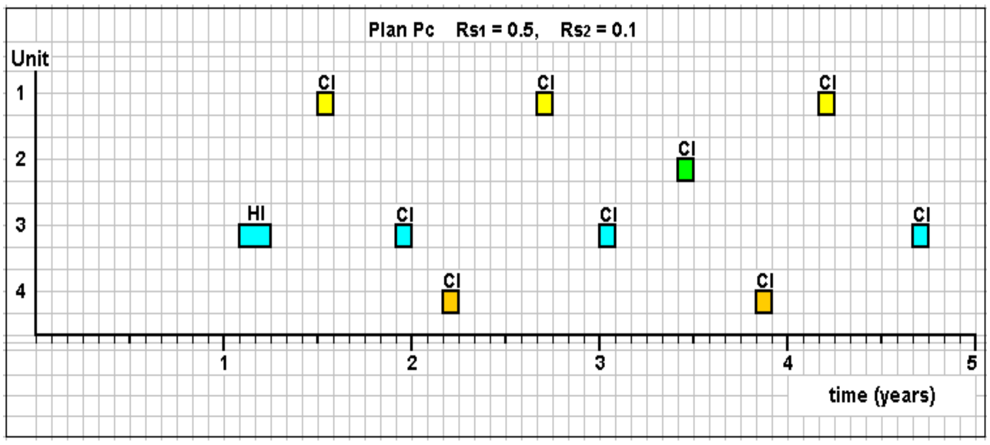

Fig. 6. Developed Maintenance Schedule for Plan Pc.

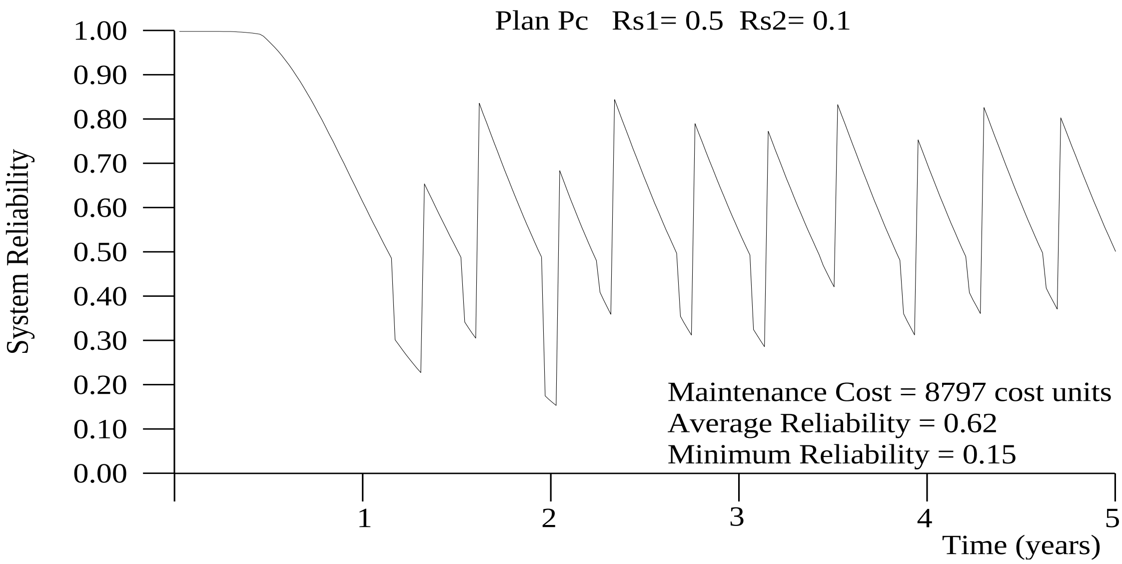

Fig. 7. System Reliability Curve for Plan Pc.

4.3 Comparison of Maintenance Plans

Table 6 presents a comparison of key attributes for the proposed maintenance plans (Po, AP1, AP2, AP3, and AP4) and the existing one. The existing maintenance plan induced high cost and it doesn't satisfy the constraints. In Po, the model was built to obtain a lower cost schedule, such that it provides comparable performance to the developed maintenance plan while satisfying the constraints. It achieved cost savings about 72% of the existing one with higher reliability. Then, four alternative schedules (AP1 to AP4) are developed to give much better reliability performance measures, and yet less in cost than the existing plan.

Table 6. Maintenance Plans Comparison.

Plans		Minimal allowable reliability	Average reliability	Minimum reliability	Maintenance cost (cost unit)	Cost saving %
Existing plan		(0.1)*	0.61	0.1	32407	-
Proposed plan	Po	0.1	0.62	0.15	8797	72%
Alternative plans	AP1	0.2	0.67	0.26	11520	64%
	AP2	0.3	0.72	0.32	14280	56%
	AP3	0.4	0.77	0.41	20884	35%
	AP4	0.5	0.84	0.58	32093	1%

* Obtained from data analysis of the existing plan.

It is evident that as the reliability level increases the maintenance cost increases. And for almost the same cost, a better reliability plan could be obtained “AP4”. Moreover, the developed plans give the maintenance management department the opportunity to choose alternative plans with higher system reliability depending on the affordable finance for maintenance actions.

5. CONCLUSIONS

The objective of this study was to develop a general optimization search procedure to find a good, and hopefully approximately optimal maintenance schedules, for a multi-unit, multi-state system with multi-levels of preventive maintenance actions. The developed model contained realistic constraints such as system reliability, PM intervals, and maintenance crew’s availability. Optimization of the model is achieved by using constrained genetic algorithm as a global optimization technique to give significant information on the objective function. Two limits of system reliability were predefined, the first necessitated to start a PM action, and the other did not allow letting the system reliability decrease beyond a predefined value. Simulation was used to simulate a given plan of maintenance actions for the system during a period of study. The genetic optimization algorithm is tested through a case study of an existing power station. Algorithm results exhibited an evident expected improvement for both total maintenance cost and system reliability performance measure. The proposed modeled algorithm has the potential to solve maintenance scheduling problems with a reducible system structure.

REFERENCES

1. Huang, S.J., "Generator maintenance scheduling: a fuzzy system approach with genetic enhancement", Electric Power Systems Research, Vol. 41, pp. 233-239, 1997.
2. Kralj, B., and Petrovic, R., "A multi-objective optimization approach to thermal generating units maintenance scheduling", European Journal of Operational Research, Vol. 84, pp. 481-493, 1995.
3. Levitin, G., and Lisnianski, A., "Optimization of imperfect preventive maintenance for multi-state systems", Reliability Engineering and System Safety, Vol. 67, pp. 193-203, 2000.
4. Yamashina, H., and Otani, S., "Optimal preventive maintenance planning for multiple elevators", Journal of Quality in Maintenance Engineering, Vol. 7, No. 2, pp. 128-150, 2001.
5. Gomaa, A., H., "Maintenance scheduling for a multi-group system", Journal of Engineering and Applied Science, Vol. 46, No. 3, pp. 515-528, 1999.
6. Lai, K.K., Leung, F.K.N., Tao, B., Wang, S.Y., "Practices of preventive maintenance and replacement for engines: A case study", European Journal of Operational Research, Vol. 124, pp. 294-306, 2000.
7. Dekker, R., and Roelvink, I.F.K., "Marginal cost criteria for preventive replacement of a group of components", European Journal of Operational Research, Vol. 84, No. 2, pp. 467-480, 1995.
8. Sandve, K., and Aven, T., "Cost optimal replacement of monotone repairable systems", European Journal of Operational Research, Vol. 116, pp. 235-248, 1999.
9. Sergaki, A., and Kalaitzakis, K., "A fuzzy knowledge based method for maintenance planning in a power system", Reliability Engineering and System Safety, Vol. 77, pp. 19-30, 2002.
10. Haleem, B.A., and Yacout, S., "Simulation of maintenance policies for a fleet of forklift trucks", 22nd International Conference on Computers and Industrial Engineering, Dec. 20-22, Cairo, Egypt, pp. 76-79, 1997.

11. Lee, C.Y., and Chen, Z.L., "Scheduling jobs and maintenance activities on parallel machines", Naval Research Logistics, Vol. 47, pp. 145-165, 2000.
12. Deris, S., Omatu, S., Ohta, H., Kutar, L.C.S., and Samat P. A., "Ship maintenance scheduling by genetic algorithm and constraint based reasoning", European Journal of Operational Research, Vol. 112, pp. 489-502, 1999.
13. Mohib, A.M.N., "Preventive maintenance scheduling algorithm for multi-unit multi-state system with application to a power station", Master Thesis, Faculty of Engineering, Cairo University, 2003.
14. Ushakov, I.A., and Harrison, R.A., "Handbook of reliability engineering", John Wiley and Sons, New York, 1993.

LIST OF SYMBOLS

n	number of units j
j	unit indices
L_j	number of PM levels for unit j
l	PM level l for unit j
$\delta_{j,l}$	maximum allowable time interval of PM level l for unit j
Q	number of Maintenance crews in the system
V	sequence of PM actions v_i
T_m	sequence of times $t_{m\,i}$ at which the corresponding PM action v_i is performed
v_i	PM action number i
$t_{m\,i}$	time at which the corresponding PM action v_i is performed
$K_{j,l}$	number of repetition of PM action of level l to unit j
$D_{PM\ j,l}$	duration required to perform PM action of level l
$C_{PM\ j,l}$	PM cost generated from applying the PM action l
$C_{PM\ j}$	PM cost generated from applying PM actions on unit j
C_{PM}	PM cost generated from applying PM actions on the system
$C_{Dtr\ j}$	downtime cost rate generated from putting unit j out of work
$C_{Dt\ j}$	downtime cost generated from putting unit j out of work
C_{Dt}	downtime cost generated over planning period T
$C_{mrr\ j}$	minimal repair cost rate for unit j
$C_{mr\ j}$	minimal repair cost for unit j
$\varepsilon_{j,l}$	age reduction coefficient corresponding to PM level l
$\tau_j^{+}(t_{m\ j,l,i})$	age of unit j after the PM action of level l number i
$\tau_j(t)$	age of unit j at time t
$r_j(t)$	reliability of unit j at time t

b

$h_j(.)$	hazard rate of unit *j*
$H_j(.)$	cumulative hazard rate of unit *j*
G_j	nominal performance rate of unit *j*
$U_j(t,Z)$	U-function of unit *j* at time *t*
E	number of levels of output performance
W^*	required system output performance demand
R_{S2}	minimal allowable reliability
T	planning period or period of study
K	number of scheduled PM actions during the period of study T
$R_S(t,W^*)$	system reliability at time interval *t* for the required W^*.

خوارزم لجدولة الصيانة الوقائية لمنظومة متعددة الوحدات ومتعددة الحالات

فى هذا البحث تم إقتراح نموذج رياضى لجدولة عمليات الصيانة الوقائية لمنظومة متعددة الوحدات، ومتعددة الحالات (مستويات الإنتاج)، بالإضافة الى تعدد مستويات الصيانة يهدف الى تدنية تكاليف الصيانة التى تشمل تكلفة الصيانة الوقائية، وتكلفة اعمال الإصلاحات البسيطة والتكلفة الناتجة عن توقف المعدة عن الإنتاج، مع مراعاة درجة موثوقية النظام, وفترات تكرار مستويات الصيانة ومدى إتاحية مجموعات عمل الصيانة، ولحل النموذج، تم استخدام خوارزم جينى واسلوب محاكاة, وتمت البرمجة بواسطة برامج (MATLAB), ويمكن تطبيق النظام المقترح على المنظومات الصناعية المختلفة.