NUMERICAL MODELING OF COMPOSITE BEAMS WITH WEB OPENINGS

A.F.M. HASSAN\textsuperscript{1} AND S.A. MOURAD\textsuperscript{2}

ABSTRACT

Although current design codes include detailed methodology for the design of beams, no provisions for the design of composite beams with web openings are incorporated. In this research, an elaborate finite element model for composite beams with web opening is developed. The model is verified by comparing its behavior with available experimental tests. The finite element model is used to conduct a parametric study to investigate the effect of opening location, depth, width, and vertical eccentricity on the behavior of the beam. The model is extended to take into consideration the material non-linearity, the shear connector non-linearity, and the geometric non-linearity. The extended model is used to determine the failure load for beams with different web opening geometry. Simplified approaches for modeling composite beams with web openings are presented and results are compared with available experimental tests and finite element model and limitations are investigated.

KEYWORDS: Web opening, composite beams, finite element modeling, non-linear analysis, stress distribution, shear connectors.

1. INTRODUCTION

The demand for more economical and efficient structures has led designers to reduce or eliminate the space below beams and girders provided for utility passage in buildings. The use of web opening (hole in the vertical portion of the steel section) will allow the passage of utilities through beams, thereby eliminating the need for space beneath them, maximizing the clear height and reducing the depth of the floor

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system. However, web openings reduce the stiffness of the beam at the opening and eliminate strain compatibility between top and bottom parts. In addition, openings reduce the cross sectional area available for resisting shear forces resulting in differential or vierendeel deflections across the opening. Several analytical and experimental investigations were conducted to study the actual behavior of composite beams with rectangular web openings. These investigations included experimental work [1, 2, 3], closed form analytical solutions [4], and finite element modeling [5]. This research investigates the various effects of web openings on the behavior of composite beams. Detailed finite element models as well as simplified models are used to conduct a parametric study. The nonlinear behavior of various components is considered to determine ultimate capacity of beam with opening.

2. FINITE ELEMENT MODEL

A finite element program [6] is used to prepare a three-dimensional model to represent the composite beam. Shell elements are used to model the concrete slab as well as the web and flanges of the steel beam. The connection between the concrete slab and the steel beam is modeled using a combination of three line elements; two beam elements, and one spring element. Figure 1 illustrates the main components used in the finite element model. Several trials were performed in order to determine the optimum mesh size for the model [7]. Figure 2 shows a typical mesh used for the analysis; where the mesh is chosen to be fine around the opening, and coarse away from the opening.

Fig. 1. Main components of the finite element model.
2.1 Shear Connector Stiffness

The effect of shear connector stiffness ($K_s$) on the behavior of the model is illustrated in Fig. 3 by plotting the deflected shape of a single beam modeled using different values of shear connector stiffness per unit length ($K_s/L_s$). It is clear that the deflection increases with the reduction in the shear connector stiffness. It is noted that the variation in the deflection when $K_s/L_s > 15$ t/cm$^2$ becomes negligible. This result is in agreement with other investigations [5].

2.2 Model Verification

Clawson and Darwin [2] conducted an experimental investigation of composite beams with web openings. Six beams were tested (B1 to B6). The results of these experiments were used to verify the finite element model. It is noted that the value $K_s/L_s$ for all the beams tested was larger than 15 t/cm$^2$, and thus the concrete slab was assumed fully bonded to the steel section and no slippage was considered. Values of deflection obtained from the model were within 5% of those obtained experimentally [2], whereas the differences in maximum stresses did not exceed 21%.
3. LINEAR BEHAVIOR OF COMPOSITE BEAMS WITH WEB OPENINGS

3.1 Configuration of Beams Used

The finite element model developed was used to study the effect of different parameters on the behavior of composite beams with web openings. Four different beams were used as shown in Table 1. The beams were chosen to provide a variety of loading conditions, and include beams that were critical in flexure (beam AA), shear (beam AD), and combined shear and moment (beam AB). Beam AC was taken identical to B6 of the parametric study by Bentiez et al. [1]. All beams were assumed to be fully restrained laterally, neglecting lateral buckling and local buckling.
Construction of the composite beams was assumed shored and no slippage between the concrete slab and the steel beam was considered.

Table 1. Configuration of the beams used in the parametric study.

<table>
<thead>
<tr>
<th>Label</th>
<th>L (m)</th>
<th>W (t/m')</th>
<th>P (t)</th>
<th>section</th>
<th>B_conc (cm)</th>
<th>T_s (cm)</th>
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<td>2.1</td>
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<td>10</td>
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<tr>
<td>AB</td>
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<td>0</td>
<td>45</td>
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<td>16</td>
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3.2 Opening Geometry

Four parameters were chosen in order to investigate their effects on the behavior of the beam. The range of values for these parameters was chosen to be within the practical range. The four parameters were:

- The ratio between the opening depth and the section depth, n, which was varied from 0.3 to 0.7.
- The ratio between the opening width and the opening depth, m, which was changed from one to three.
- The ratio between the vertical distance between the opening centroid and the steel section centroid divided by the steel section depth, e, which was taken from −0.3 to +0.3 relative to the steel section depth.
- The ratio between the moment and shear force at the centerline of the opening divided by the steel section depth, M/VD.

While investigating each one of the parameters, the remaining three were kept constant.
3.3 Overall Behavior

Figure 4 shows the deflection curve of beam AA with a web opening at the mid span using different values for n from 0.3 to 0.7. The other parameters were; m = 1, e = 0.0, M/VD = ∞. It is noted that the deflection curves are smooth throughout the beam length except at the opening. The increase in deflection due to the opening reached about 15% for n = 0.7 which is generally acceptable except in beams where deflection governs the design. Figure 5 shows the deflection curve of beam AD having a web opening at the middle of the span using different values of n. The other parameters were; m = 1, e = 0.0, M/VD = 0.83. It is noted that the deflection curves have a very clear kink at the opening position making an S-shape curve, which extended along the whole beam. The increase in deflection due to the opening reached about 80% for n = 0.7 which is not accepted especially in beams where deflection governs the design. Figure 6 shows the in-plane rotation of the cross-section for both top and bottom tee of beam AA with and without opening, where n = 0.7, m =1, M/VD = ∞, e = 0. It can be seen from the figure that each of the top and bottom tee has different values of in-plane rotation, which is an indication that each tee behaves independently.

4. NON-LINEAR BEHAVIOR

Many current design codes are shifting to ultimate design philosophies [8]. In such concepts, the ultimate capacity of the composite section is determined based on post-elastic behavior while considering the appropriate load factor and strength reduction factors. The nonlinear behavior of composite beams with web openings is investigated by considering:
Fig. 4. Effect of opening depth on the deflection for beam AA.

Fig. 5. Effect of opening depth on the deflection for beam AB.

Fig. 6. Variation of angle of rotation for beam AA (n=0.7).
- The shear connectors non-linearity, using $p-\Delta$ curve for the spring element (Fig. 7a).
- The material non-linearity, using material curves for the steel and concrete as shown in Figs. 7b and 7c.
- The geometric non-linearity, by including the effect of large deformations by updating the stiffness matrices for all elements in the model after each solution step.

Ultimate capacity ($P_u$) is reached either when the beam fails or when the maximum deflection exceeds L/35. Both lateral and local buckling was not considered. Composite beams were assumed shored until full interaction between steel and concrete is achieved. Dead load was considered included within the distributed load, w.

4.1 Parametric Study

The four parameters chosen for the parametric study with the linear model were also considered to investigate their effects on the ultimate behavior of the beam. The beam AA was used to perform the parametric study. In order to assess the effect of opening size and location, the maximum deflection and stresses were computed for the beam both with and without opening. The values $\delta_1$ and $\delta_2$ were defined as the maximum deflection of the beam with and without opening, and $\sigma_1$ and $\sigma_2$ as the maximum stresses in the beam with and without opening, respectively. The ratio $\delta_1/\delta_2$ and $\sigma_1/\sigma_2$ were computed for the various parameters.

4.1.1 Effect of opening depth

The relation between the mid span deflection of beam AA and the corresponding values of load, w, on the same beam with different values of $n$ ($m = 2$) is shown in Fig. 8.
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a) Force/displacement relationship for shear stud.

b) Stress/strain relationship for steel element.

c) Stress/strain relationship for concrete element.

Fig. 7. Stress/strain relationships used in the finite element model.
It is clear from the figure that the ultimate deflection remained the same for $n = 0.3$ and $n = 0.5$, whereas for $n = 0.7$ the post-elastic stiffness was less for the same load, resulting in displacement about 40% larger.

4.1.2 Effect of opening width

The relation between the mid span deflection and the corresponding values of $w$ on the same beam with different values of $m$ ($n = 0.7$) is shown in Fig. 9. It is clear from the figure that the maximum deflection increases as the opening width increases.

4.1.3 Effect of opening eccentricity

The relation between the mid span deflection of beam AA and the corresponding values of $w$ on the same beam for different values of $e$ while $n = 0.3$ is shown in Fig. 10. It is clear from the figure that the deflection decreases as the value of $e$ increases towards the concrete slab and vise versa. However, the model is less sensitive to the variation of eccentricity.

4.1.4 Effect of opening location

The relation between the mid span deflection of beam AA and the corresponding values of $w$ on beam AA with different values of $M/VD$ while $n = 0.5$ is shown in Fig. 11. It is clear from the figure that the ultimate deflection decreases with the increase in $M/VD$ ratio.

4.2 Failure Load

In order to investigate the effect of opening on the ratio between the ultimate load capacity, $P_u$, and the elastic load capacity, $P_e$. Models with different parameters were analyzed. The results obtained from the models are tabulated in Table 2. From this table, it is clear that the ultimate capacity of the composite beams with web openings could be reduced by about 25% as compared to the capacity of the beam
Fig. 8. Effect of opening depth on the deflection of beam AA (m=2).

Fig. 9. Effect of opening width on the deflection of beam AA (n=0.7).
Fig. 10. Effect of opening eccentricity on the deflection of beam AA (n=0.3).

Fig. 11. Effect of opening location on the deflection of beam AA (n=0.5).
without opening. In general, openings with large dimensions resulted in a larger reduction in the ultimate capacity. Within the parameters investigated, the eccentricity was the least effective.

Table 2. Ratio between ultimate load and elastic load.

<table>
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<tr>
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<th>m</th>
<th>n</th>
<th>e</th>
<th>x</th>
<th>M/V</th>
<th>P&lt;sub&gt;u&lt;/sub&gt;</th>
<th>P&lt;sub&gt;r&lt;/sub&gt;</th>
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<td>4.28</td>
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5. SIMPLIFIED MODELS

Several simplified models are often used to model beams with web openings. Two of these models are illustrated in this section and results are evaluated.

5.1 First Simplified Model

Bentiez et al. [1] proposed a simplified model (Sm0dell) to predict the deflection of the beam using the stiffness method. The proposed model was used to predict the axial stress. Results of this model were compared with that of the finite element model. The simplified model for the web-opening element was constructed using four rigid links and two beam elements as shown in Fig 12. The nodes of the web-opening elements were connected to the ends of the top and bottom tee elements by rigid links of length l<sub>t</sub> and l<sub>b</sub>, respectively. The web adjacent to the opening was considered infinitely rigid. If the portions of the beam above and below the opening
(the top and bottom tees) were modeled as uniform beam elements, a web-opening element consisting of two beam elements connected by rigid links could be developed.

Fig. 12. Web opening element geometry and axes orientation.

Fig. 13. Comparison between Smode1 and experimental deflection for beam B6.
5.1.1 Comparison between the first simplified model and experimental results

Beam AC (identified as B6 in the earlier experimental tests) was chosen as an example for the comparison between the experimental results and the first simplified model. The variation of deflection along the beam length is shown in Fig. 13. From the figure, it is clear that there is good agreement between experimental and simplified model results. The values of axial stresses of the beams tested by Clawson and Darwin [5] is plotted versus the distance from the opening edge. The values are plotted for the beam at high and low moment ends in Figs. 14 and 15, respectively. It is clear from the figures that there is good agreement between experimental tests results and Smodel1 results in the top tee at high and low moment end and the bottom tee at high moment end. However, there is a significant difference in the direction of the stresses and distribution at the bottom tee in the low moment, which is the main deficiency in Smodel1.

5.1.2 Comparison between finite element model and first simplified model

The deflection and the stresses from the parametric study done by using the finite element model, and the deflection and the stresses from Smodel1 for beam AD are tabulated in Table 3. The ratio between the values of finite element model and the first simplified model was calculated. The table also shows the average and standard deviation for the deflection ratio and stresses ratio. Form the tables it is clear that Smodel1 provides a good estimation for the deflection. The average of the ratio between the deflection of the finite element model and the deflection of the first simplified model was 0.99 for the four beams considered while the standard deviation was 0.05. Smodel1 gives a good estimation for the maximum stresses; the average of the ratio between the maximum stresses of the finite element model and the maximum stresses of Smodel1 was 1.02 while the standard deviation was 0.104. For the case of a concentrated load located above the opening and the parameter (M/VD) smaller than one, Smodel1 underestimates the deflection and the stresses - for example beam AD9 in Table 3.
Fig. 14. Comparison between finite element model, Smodel1 and experimental stresses for beam B6 at high moment end.

Fig. 15. Comparison between finite element model, Smodel1 and experimental stresses for beam B6 at low moment end.
5.1.3 Effect of the shear deformation

The effect of shear deformations on the deflection of composite beams with web openings is studied. The deflection for beam AD1 in Table 3 decreased from 0.27 to 0.213, when shear deformations were neglected.

5.2 Second Simplified Model

The second simplified model (Smodel2) is the most commonly used model among designers in which the top and bottom tee are assume to be working together with net properties of the cross section. In this model neglecting the effect of shear deformations on the model results in reduction about 10% in the deflection, so during the analysis shear deformations must be considered.

5.2.1 Comparison between the second simplified model and finite element model

This model is used to predict the maximum deflection along the beam span and the maximum stresses along the section depth of the composite beams with web openings. The results were compared to the finite element model, as shown in Table 3. From the table it is clear that the results of Smodel2 have the same accuracy of the results of First simplified model, as the average mean ratio for stresses prediction is 1.02 while the corresponding standard deviation equal 0.105. However, the second simplified model results are not affected by the opening width, which for example can be seen from Table 3. The beams AD1, AD4, and AD7 have the same values of the maximum deflection and the maximum stresses.

5.2.2 Comparison between the first simplified model and second simplified model

It is clear from Table 3 that these two simplified models give the same results for the maximum deflection and maximum stresses especially in the case of opening subjected to bending only. However, Smodel2 is not affected by the width of the opening. Due to the fact that Smodel2 is simpler than Smodel1, it is recommended for design usage.
6. CONCLUSIONS

Within the range of models and parameters studied, the following conclusions were reached:

- The top tee and the bottom tee have different rotation values, which indicates that each tee behaves independently.

- Material nonlinearity has the dominant effect on the post-elastic behavior as compared to the shear stud nonlinearity.

- The ultimate deflection is almost the same for the models having ratio between opening depth and section depth equals 0.3 to 0.5, but for models with ratio equals 0.7 the post elastic stiffness was considerably less, resulting in displacement about 40% larger.

- The ultimate deflection increases as the opening width increases, whereas the ultimate deflection decreases as the ratio between the vertical eccentricity and section depth increases, and also with the increase in the moment to shear ratio divided by the section depth.

- The ultimate capacity of the composite beams with web openings could be reduced by about 25% as compared to the capacity of the beam without opening. In general, openings with large dimensions resulted in a larger reduction in the ultimate capacity. Within the parameters investigated, the eccentricity was the least effective.

- There was good agreement between experimental testing and simplified model results in the top tee at high and low moment, and also in the bottom tee at high moment end. However, there was a difference in the direction of the stresses for the bottom tee at the low moment end.

- For the case of a concentrated load located above the opening and the parameter M/Vd (moment to shear ratio divided by the section depth) smaller than one, the simplified models underestimated the deflection and the stresses.
REFERENCES


النمذجة الرقمية للكميات المركبة ذات الفتحات في الأغصان

على الرغم من أن كودات التصميم الحالية تضمن تفاصل لتصميم الكميات، إلا أنها لا تتضمن اختيارات لتصميم الكميات المركبة ذات الفتحات، تم تقديم نموذج رياضي باستخدام برنامج النماذج المحددة لتمثيل الكميات المركبة ذات الفتحات، وتحقيق نتائج التمثيل مع نتائج أبحاث عملية مسبقة لتأكيد من نفته وتم استخدام النمذج في دراسة متغيرات تتعلق بالعوامل المختلفة لدراسة تأثير هذه المتغيرات على التشكلات والإجهاديات بالكميات المركبة، وتم تطوير النموذج الرقمي للكميات بحيث يأخذ في الاعتبار التصرف الłatwكي للكميات عند تخفيض الصلب أحد المرونة، وتخطيط الحمل على وصلات الفتحات حدوت المرونة، واستخدام البرنامج في تحديد الحمل الأقصي للكميات، وقد تم استخدام نموذج في مميزات لتمثيل الكميات المركبة ومقارنة هذه النتائج بالنتائج العملية ونتائج نماذج النماذج المحددة بهدف تقييم هذه النماذج المسبقة ومعرفة حدود استخدامها في تحليل وتصميم الكميات المركبة ذات الفتحات.