AXIAL AND ECCENTRIC BUCKLING WITH SPECIAL APPLICATION TO REINFORCED CONCRETE

BY

Prof. Dr. W. S. HANNA, Dr. MICHEL BAKHOUM
and LABIB RIAD, (*MSc.*)

INTRODUCTION

The main purpose of this investigation was to make an analysis and experimental study for the problem of reinforced concrete columns subject to eccentric compressive forces which cause buckling. Most specifications either give no information about it at all or give rough imperical rules which are not based on sound theoretical or experimental work. Research work on eccentric buckling is very scarce. Even the case of axial buckling has not been given as much attention as it deserves.

In the analysis of axial and eccentric buckling, the main difference between the case of a reinforced concrete column and that of a column of homogeneous elastic materials as steel lies in the term E I. For a reinforced concrete column both E and I vary continuously during loading. Consequently, it has been found essential to precede the study of the main problem of buckling in reinforced concrete by a study for the problem of buckling in homogeneous elastic materials.

The investigation consists of two parts :—

Part I.—*Axial and Eccentric Buckling in Homogeneous Elastic Materials :*

In this part the following points are dealt with :—

1. A brief review of the mathematical solutions for axial and eccentric buckling.

2. Systematised graphical procedures for the general solution of problems on axial and eccentric buckling for any variation in moments of inertia and end conditions.

3. Development of a simple apparatus adaptable for use in the design office which simplifies the solution considerably.

4. Two different procedures for solution by successive trials are given by which a complete picture of the variation of maximum deflections and stresses with the load up to failure can be obtained.

5. The use of an approximate simple equation for eccentric buckling which gives the maximum deflections and stresses to a good degree of accuracy.

Part II.—*Buckling in Eccentrically Loaded Reinforced Concrete Columns :*

A report is given, in this part, on an experimental investigation made on fourteen eccentrically loaded R.C. columns. The columns had the same concrete dimensions but were loaded at different eccentricities on the longer and shorter axes of symmetry of their rectangular cross-sections. The actual stresses and deflections obtained experimentally were compared with those computed by analytical procedures developed for the checking of stresses and deflections under eccentric buckling. It has been shown that the use of the ordinary assumptions of the standard theory for reinforced concrete design leads to wide discrepancies between the computed and actual values, while if more accurate assumptions are made a satisfactory agreement was obtained.

PART I

AXIAL AND ECCENTRIC BUCKLING IN HOMOGENEOUS ELASTIC MATERIALS

I.—Axial Buckling

1.— *Mathematical Methods for Determining the Buckling Load :*

a) *Exact Method :*

The exact mathematical solution for axial buckling is based on solving the differential equation of the slightly deflected column for the boundary conditions of the problem.

For example in (Fig. 1), the differential equation is $\frac{d^2y}{dx^2} = \frac{P(\delta - y)}{EI}$ giving a buckling load

$$\left(P_{cr} = \frac{\pi^2}{4} \frac{EI}{l^2} \right)$$

For complicated cases of loading or variations of the moment of intertia of the column, the solution of the differential equation becomes rather difficult.

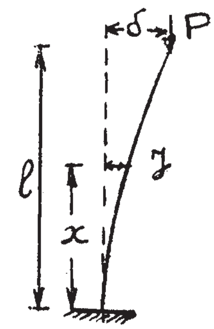


Fig. 1

b) *Approximate Method :*

The approximate mathematical solutions are based on assuming a simple equation for the deflected column and correcting it successively until we obtain the required degree of accuracy for the configuration of the column.

This may be effected either :

i) by considering the shape of the elastic line. For example in (Fig. 2), if we assume the elastic line a parabola $y_1 = \frac{4\delta_1}{l^2} x (l - x)$, then the deflection at the center by the

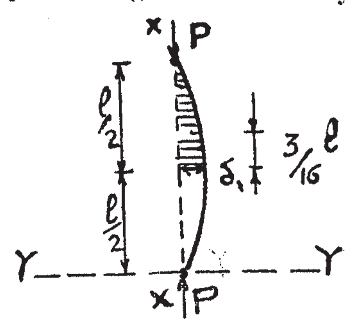


Fig. 2

moment area method is δ center $= \frac{5}{48} P \delta_1 l^2$. Equating this to the assumed deflection at center δ_1 we get the buckling load $P_{cr} = \frac{48}{5} \frac{EI}{l^2}$. A better approximation is obtained if the deflection curve obtained from the first assumption is assumed as an initial curve and so on or

ii) by considering the strain energy during the configuration of the column. The decrease in the potential energy due to the lowering of the top end is equal to the work done by the load during the lowering action.

In (Fig. 1) if we assume the initial curve as a parabola we obtain $P_{cr} = 2.5 \frac{EI}{l2}$ from the first assumption with an error of $1\frac{1}{3}\%$. Better results could be obtained successively.

2.—*Numerical Methods by Successive Trial:*

A practical solution could be made for any condition of axial loading and any variation of I avoiding differential or complicated equations using the following procedure:—

a) Assume a reasonable curve for the deflection curve.

b) Compute the deflections at different points of the column.

c) Equating either the maximum deflection or the average deflection in cases (*a*) and (*b*) we get an approximate value for the buckling load.

d) Taking the curve obtained in (*b*) as an initial curve and repeating the same as in (*c*) we get a better value for the critical buckling load.

In the case shown in (Fig. 3) which is solved before we will assume that the initial deflection curve is (i) Triangle (ii) Trapezium, (iii) Parabola, (iv) Sine Curve. Although the first assumption is a rude one, yet the result obtained was satisfactory. The table shown below gives the results obtained for the four assumptions.

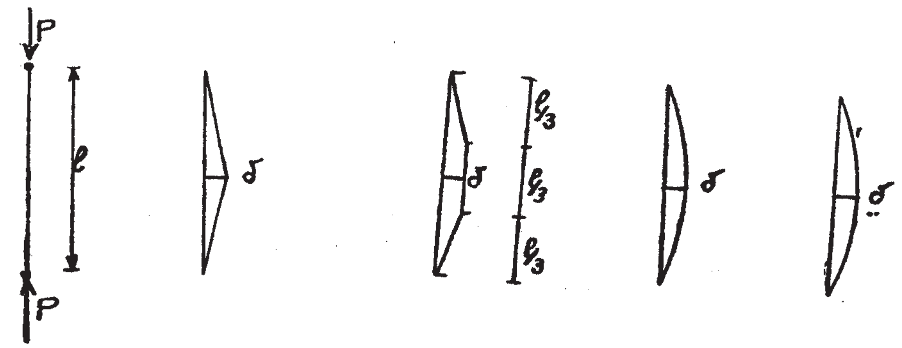


Fig 3 Triangle Trapezium Parabola Sine curve

Assumed Deflection Curve	Critical Buckling Load			Error %
	For Assumed Curve	After First Correction	Exact	
i. Triangle . .	$P_{cr} = \frac{12\ EI}{l^2}$	$P_{cr} = \frac{10{\cdot}13\ EI}{l^2}$	$P_{cr} = \frac{\pi^2\ EI}{l^2}$	+2·72
ii. Trapezium .	$P_{cr} = \frac{9{\cdot}4\ EI}{l^2}$	$P_{cr} = \frac{10{\cdot}05\ EI}{l^2}$	"	+1·92
iii. Parabola . .	$P_{cr} = \frac{9{\cdot}6\ EI}{l^2}$	$P_{cr} = \frac{9{\cdot}84\ EI}{l^2}$	"	—0·2
iv. Sine Curve .	$P_{cr} = \frac{\pi^2\ EI}{l^2}$	No correction	"	0

The accuracy of the results depends on the initial assumption of the deflection curve.

Prof. Newmark has developed an ingenious numerical procedure for determining deflections, based on parabolic distribution.

3.—*Graphical Methods by Successive Trial:*

Since in buckling problems the whole elastic line is needed, graphical methods are believed to be more adaptable, simpler and quicker. The graphical solution becomes much simpler if the initial assumption for the deflection curve is not very far from the actual configuration of the column and if a systematized method for the scales is used (see examples in the thesis).

An approximate method developed for the determination of the initial deflection curve:

This is a very rude experimental method in which a simple cardboard model represents the distribution of I of the column along its length. The model is held by both hands in a manner which represents the end conditions and pushed so as to give initial deflection curve (Fig. 4). The critical buckling load

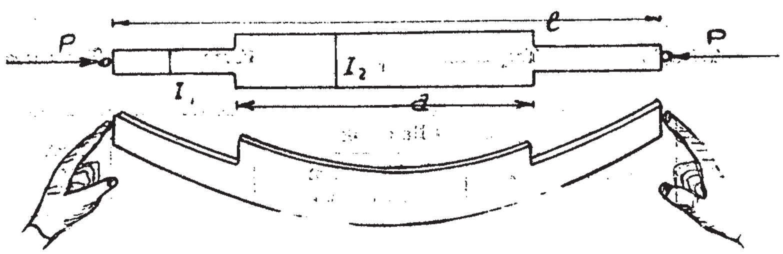


Fig. 4

[illegible] method was [illegible] accurate [illegible] design especially after one or two corrections. The error in many examples was less than 1 % after the first correction.

4.—*A Simple Apparatus for the Determination of the Buckling Load:*

A simple apparatus was developed to improve the method given before by satisfying more accurately the end conditions. A cardboard model was used and fitted in elements A and B of the apparatus (Fig. 5), so as to satisfy the required boundary conditions. Six examples solved by the use of the apparatus, three of which were for columns of variable I with hinged ends and the other three were for the same columns with fixed ends. The error in the results was negligible.

The method is very simple, avoids elaborate mathematical solutions and saves much time.

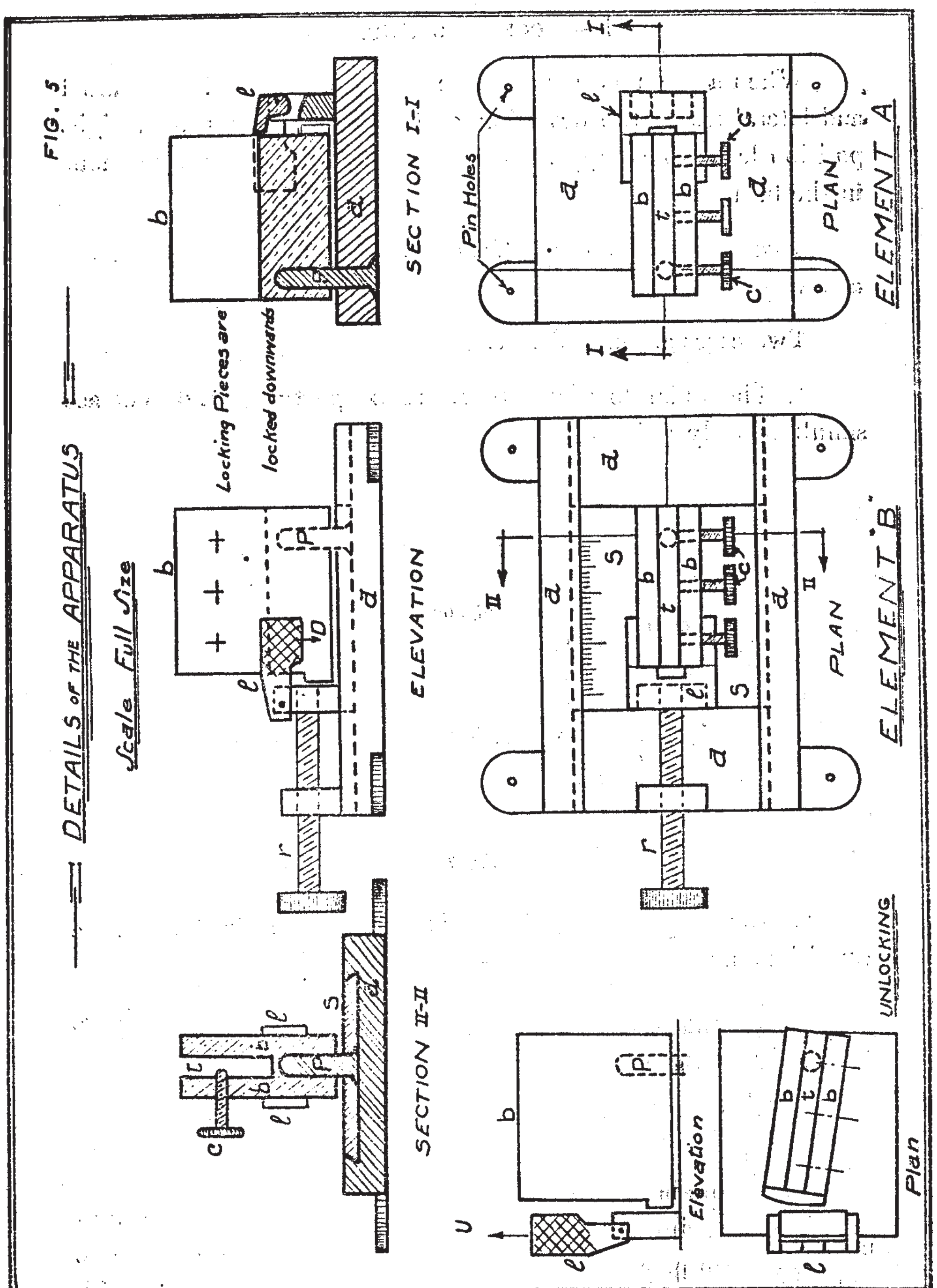


FIG. 5
DETAILS OF THE APPARATUS
Scale Full Size
Locking Pieces are locked downwards
SECTION I-I
ELEVATION
SECTION II-II
Pin Holes
PLAN
ELEMENT "A"
PLAN
ELEMENT "B"
UNLOCKING
Elevation
Plan

II.—Eccentric Buckling

When a beam is subjected to simultaneous action of axial and lateral loads, the deviation of the beam from the straight position leads to an appreciable increase in the straining actions in the beam.

Modes of change of Moments, Deflections and Stresses with the Load:

Two cases are considered :

1. The axial load and B.M. are proportional and increase simultaneously (Fig. 6).

Fig. 6

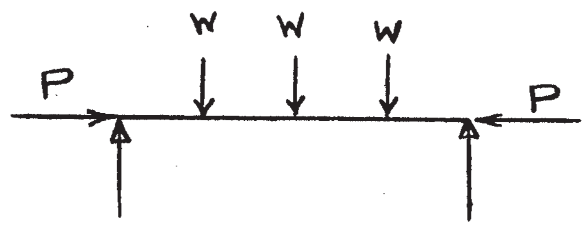


Fig. 7

2. The axial load and B.M. are independent (Fig. 7). Three conditions may be considered :—

a) Axial load P is constant and lateral loads W increase to failure.

b) W is constant while P increases up to failure.

c) Both P and W increase simultaneously.

In case *a*) the moments, deflections and stresses are approximately proportional to the lateral loads. In the other two conditions, they are increasing at a rapidly increasing rate with the increase of the axial load.

Mathematical solutions for eccentric buckling :

The mathematical solution is based on writing the differential equation for the deflected beam due to the combined action of bending and axial loading and solving for boundary conditions.

For example in Fig. 8 :

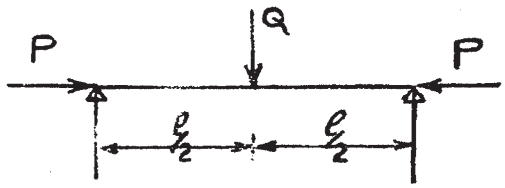


Fig 8.

$$\delta_{max.} = \frac{Q\, l^3}{48\,EI} \cdot \frac{3\,(\mathrm{Tan}\,\mu - \mu)}{\mu^3}$$

$$\text{and } M_{max} = \frac{Q\, l}{4} + P\,\delta_{max.}$$

In Fig. 9 :

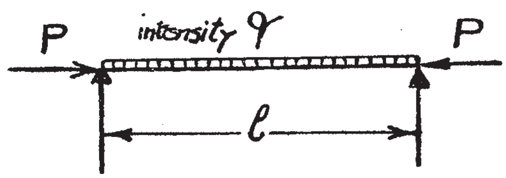


Fig 9.

$$\delta_{max.} = \frac{q\, l^4}{16\,EI} \cdot \frac{\left(\sec\mu - 1 - \frac{\mu^2}{2}\right)}{\mu^4}$$

$$M_{max.} = \frac{q\, l^2}{8} + P\,\delta_{max.}$$

In Fig. 10 :

$$\delta_{max.} = \frac{e\,(1 - \cos\mu)}{\cos\mu}$$

$$M_{max.} = Pe\,\sec\mu$$

In each case :

$$f_{max.} = \frac{P}{A} + \frac{M_{max}\, y}{I} \quad \text{and } \mu = \frac{l}{2}\sqrt{\frac{P}{EI}}$$

Fig 10

The load which causes a certain stress is obtained by trial. The manner of variation of the stress with the load is obtained by plotting the stress corresponding to the gradually increasing load.

5.—*General Solution for Eccentric Buckling by Successive Trial:*

For complicated cases the direct mathematical solution is too elaborate for practical design. A simple solution based on successive trial is given which applies for any condition of loading, any variation of I and any end condition.

Two different procedures are proposed. They are illustrated here for the case shown in (Fig. 10) :—

A.—*First Procedure:*

a) Assume the deflection curve a parabola with maximum ordinate δ.

b) Using the moment-area method, find P corresponding to assumed δ.

c) Correct the deflection curve due to P at other points of the column.

d) Find the corrected value of P due to final deflection curve.

e) Find $f_{max.} = \frac{P}{A} + \frac{P\ (e + \delta)\ y}{I}$

f) Assume a higher value of δ middle and repeat steps (a) to (e).

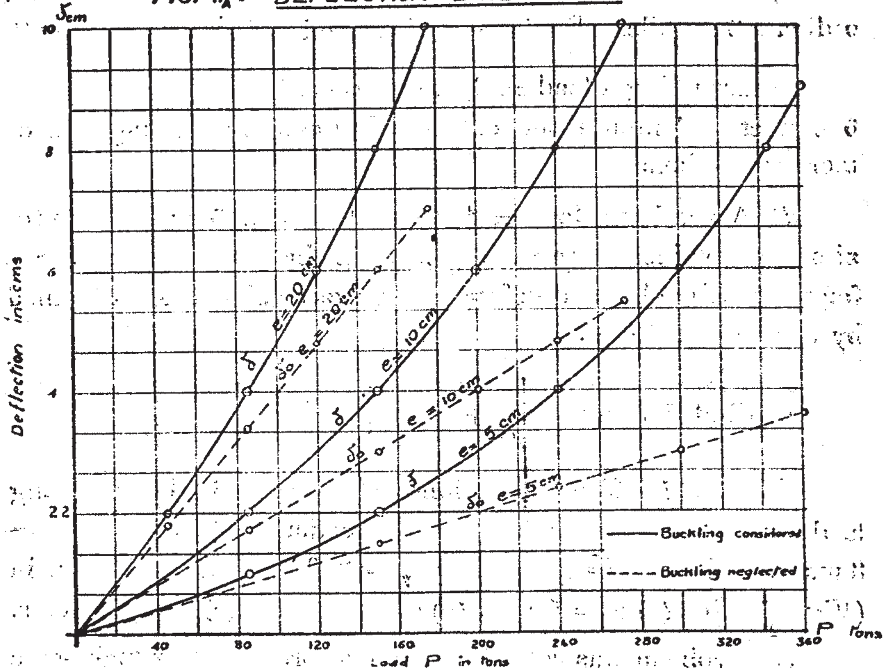
g) Plot P against $f_{max.}$ and P against δ.

The curves give a complete picture for the variation of stresses, deflections with the load, from which the safe and failure loads can be easily obtained.

B.—*Second Procedure:*

a) Assume a small value of P and find δ middle neglecting the effect of buckling.

FIG. II_A - DEFLECTION - LOAD CURVE



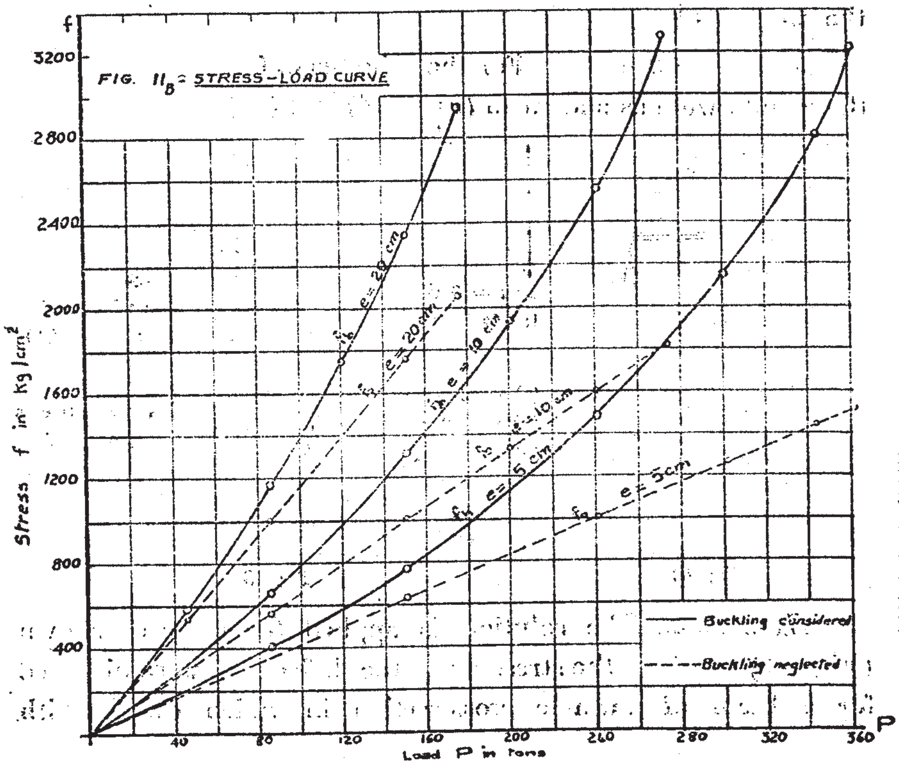


FIG. II_B - STRESS - LOAD CURVE

b) Assume the deflection curve a parabola with initial max. ordinate δ middle. Find the exact deflection δ correct due to P.

c) Assuming a load = 2p and corresponding deflection = 2 δ correct. Compute the exact deflections by making one or more corrections.

d) Assuming a load = 4 P the deflection δ can be approximately obtained from the extension of the load deflection curve for the results obtained before. The exact value of δ is obtained by one or more corrections and so on.

e) Find f max. $= \frac{P}{A} + \frac{P\,(e + \delta)y}{I}$

Example 1: The deflections, stresses, and the buckling load for a column of constant cross-section 20 × 30 cms. l = 8 ms. for which E = 2000 t/cm² f_y = 3000 kg/cm² are shown in (Fig. 11), for (a) e = 5 cms. (b) e = 10 cms. (c) e = 20 cms.

The problem was solved by the two procedures which gave the same result.

Examples 2 and 3: For the columns of (Figs. 12) and (13), the results were as shown in (Figs. 14) and (15).

Ex. 2

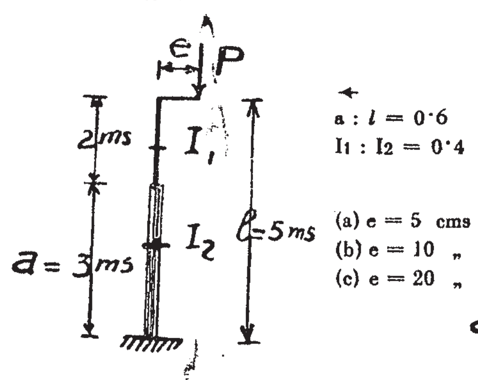


Fig. 12

Ex. 3

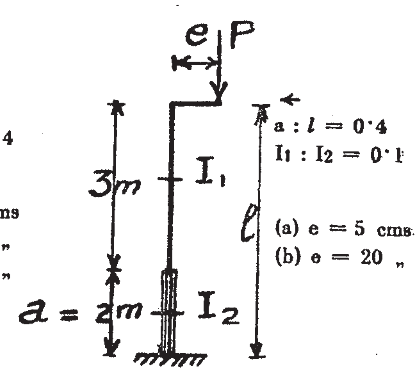


Fig. 13

Example 4: The solution is applied for the case shown (Fig. 16) in which the stresses and the deflections were obtained for a beam of variable cross-section in which the breadth

FIG. 11a - DEFLECTION - LOAD CURVE

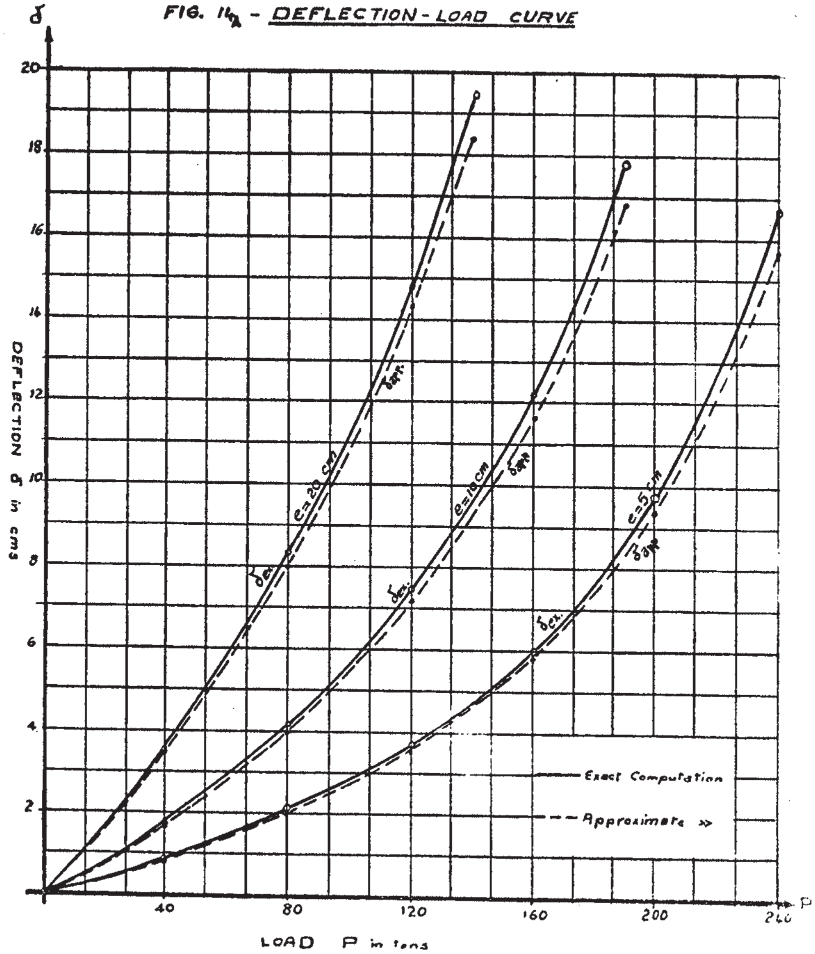
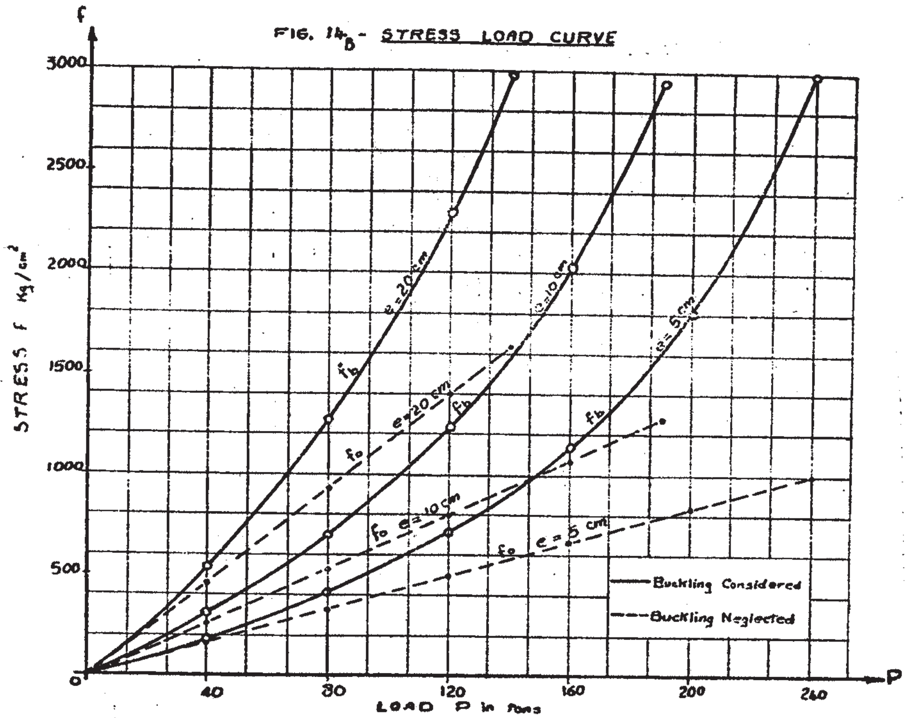


FIG. 14B - STRESS LOAD CURVE



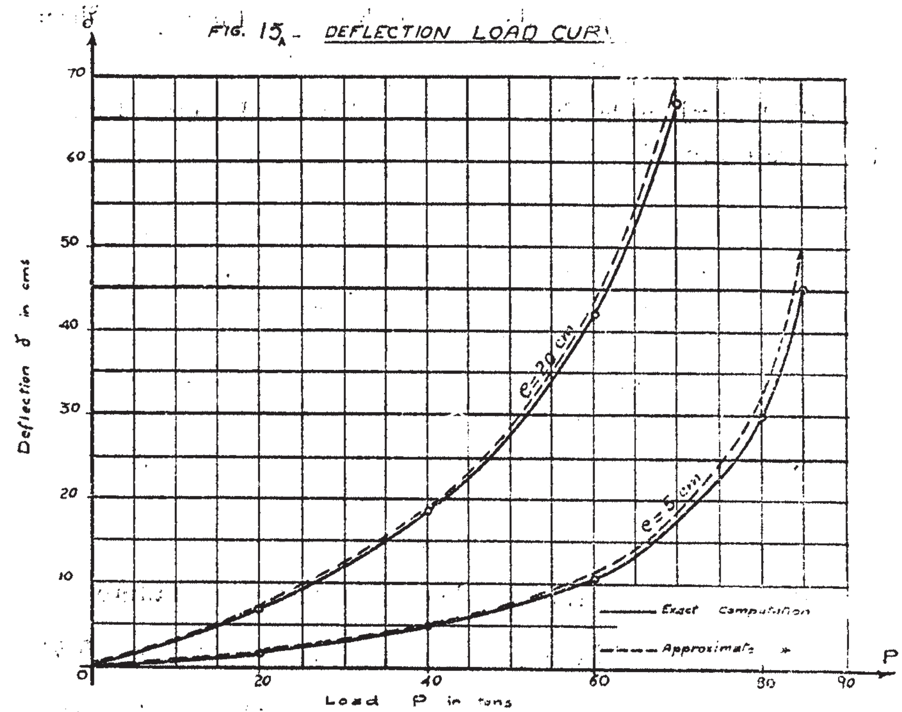


FIG. 15A - DEFLECTION LOAD CURVE
Deflection δ in cms
Load P in tons
e=20 cm
e=5 cm
Exact Computation
Approximate »

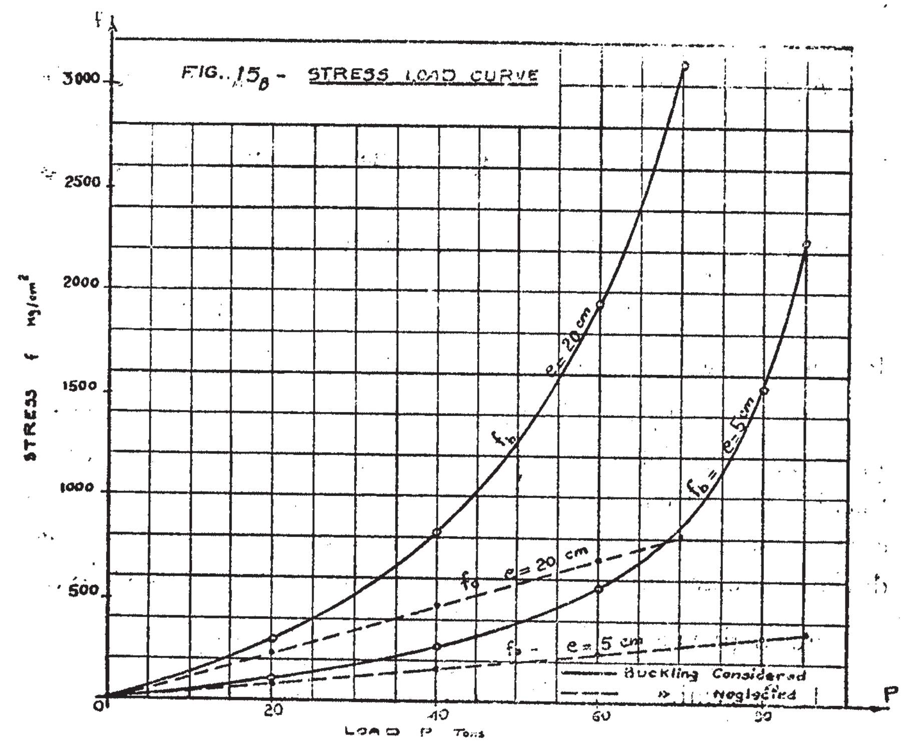


FIG. 15B - STRESS LOAD CURVE
STRESS f Kg/cm²
LOAD P Tons
f_b e=20 cm
f_b = e=5 cm
f_o e=20 cm
f_o - e=5 cm
Buckling Considered
» Neglected

is constant = 40 cms, while the depth varies linearly. Three cases were considered.

a) Lateral loads W = const = 6 t while axial load P increases up to failure.

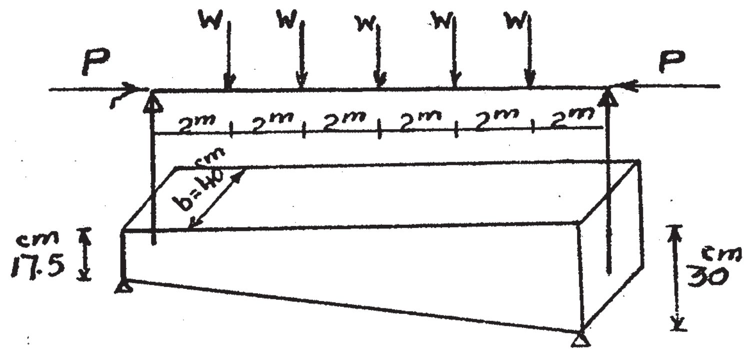


Fig. 16

b) P = const = 100 t while W increases up to failure.

c) Both P and W increase simultaneously so that P = 25 W.

6.—*An Approximate Simple Solution for Problems of Eccentric Buckling :*

A general simple solution can be made for problems of eccentric buckling by the use of the approximate formula $\delta = \dfrac{\delta_o}{1 - \alpha}$

where $\alpha = \dfrac{P}{P_{cr}}$ δ = deflection buckling considered

δ_o = " " neglected

P = axial load on member P_{cr} = critical buckling load.

This formula is given in Timoshenko for some special cases. It has been proved for some other cases. It has also been shown that it applies to a sufficient degree of accuracy, for the general case, if the shape of the deflection curve due to bending does not deviate considerably from the configuration due to axial loading.

For a given condition of loading, the procedure is as follows :

1. Determine the deflection δ_o due to lateral loads

2. Determine P_{cr} for axial loading.

3. The deflection due to combined action of axial and lateral loads is given by $\delta = \delta_0 \frac{1}{1-\alpha}$

Application of the approximate formula on the examples solved before:

Applying this approximate formula on the examples solved before, the error in the stresses and deflections was very small. The results obtained by this approximate formula were very satisfactory. The use of the apparatus for the determination of P_{cr} gives a further appreciable simplification for the problem.

PART II

EXPERIMENTAL INVESTIGATION ON REINFORCED CONCRETE COLUMNS SUBJECT TO ECCENTRIC BUCKLING

I. Introduction

Purpose and Scope of Tests:

In this investigation a study was made on the stresses and deflections in fourteen R.C. columns of constant cross-section 20 × 25·8 cms. and constant height 6 ms., subject to eccentric buckling when the load increased up to failure.

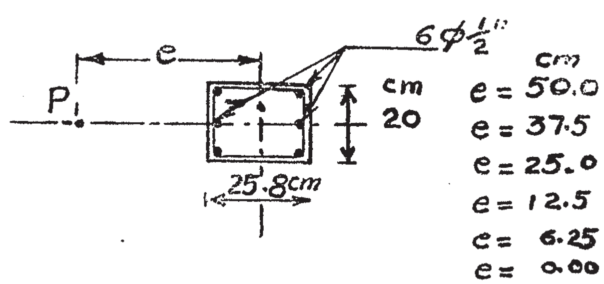


Fig. (17a)

Figs. (17a) and (17b) show the reinforcing of the columns and the manner in which they were loaded.

Strain measurements and deflections were taken at the middle and quarter points of the height. Actual stresses and deflections obtained from tests were compared with those computed according to the ordinary assumptions of the standard theory, and according to some amended assumptions which gave much better results.

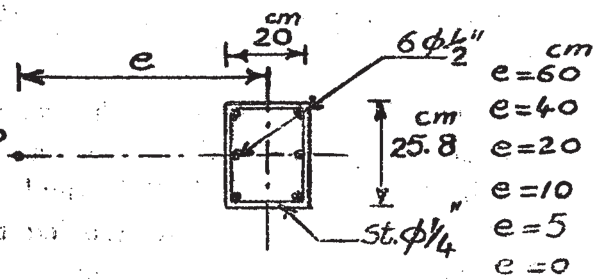


Fig. (17b)

II.—Scheme of Investigation

The materials used were, Tourah Portlant Cement, Pyramids sand and gravel thoroughly washed, and ordinary reinforcing bars of mild steel. Proportions of mix used were 1 : 2 : 4 by weight and water cement ratio 0·5.

Mixing was carried out in a mechanical mixer. Steel moulds were used for formwork. A vibrator was used in placing concrete.

The specimens were kept in the ordinary atmosphere of the laboratory. They were sprinkled with water for the first week only. All columns were tested at the age of 28 days in a 500 t Amsler Machine.

A Huggenberger deformeter (gage 25 cm and reads to $\frac{1}{400}$ mm) was used for obtaining strains, and a deflectometer ($\frac{1}{400}$ mm) was used for measuring deflections. For the stress—strain curves of the concrete prisms used as control specimens a Huggenberger extensometer measuring to $\frac{1}{1000}$ mm was used.

III.—Test Results

The control specimens gave the following resnlts :

Concrete :

Cube strength ($15{\cdot}8^3$ cm.) 334 kg./cm²

Standard prisms ($15{\cdot}8 \times 15{\cdot}8 \times 47{\cdot}4$ cm.) were tested at four different rates of loading.

For the standard rate of loading	$f_p = 250$ kg./cm²
For a rate of 4 kg/cm²/min.	$f_p = 245$ „
For a rate of 2 kg/cm²/min.	$f_p = 233$ „
For a very slow rate	$f_p = 240$ „

Steel :

A ½″ diam. bar is tested in tension and the results of testing were $E_s = 2000$ t/cm²

yield stress $f_y = 2800$ kg/cm

tensile stress $f_{max.} = 4250$ kg/cm²

For the fourteen main columns, the thesis gives tables and curves for the Load—ε c, Load—ε s and Load—δ as obtained from tests, (ε c = max. strain on compression side, ε s = max. strain in tension steel and δ = deflection).

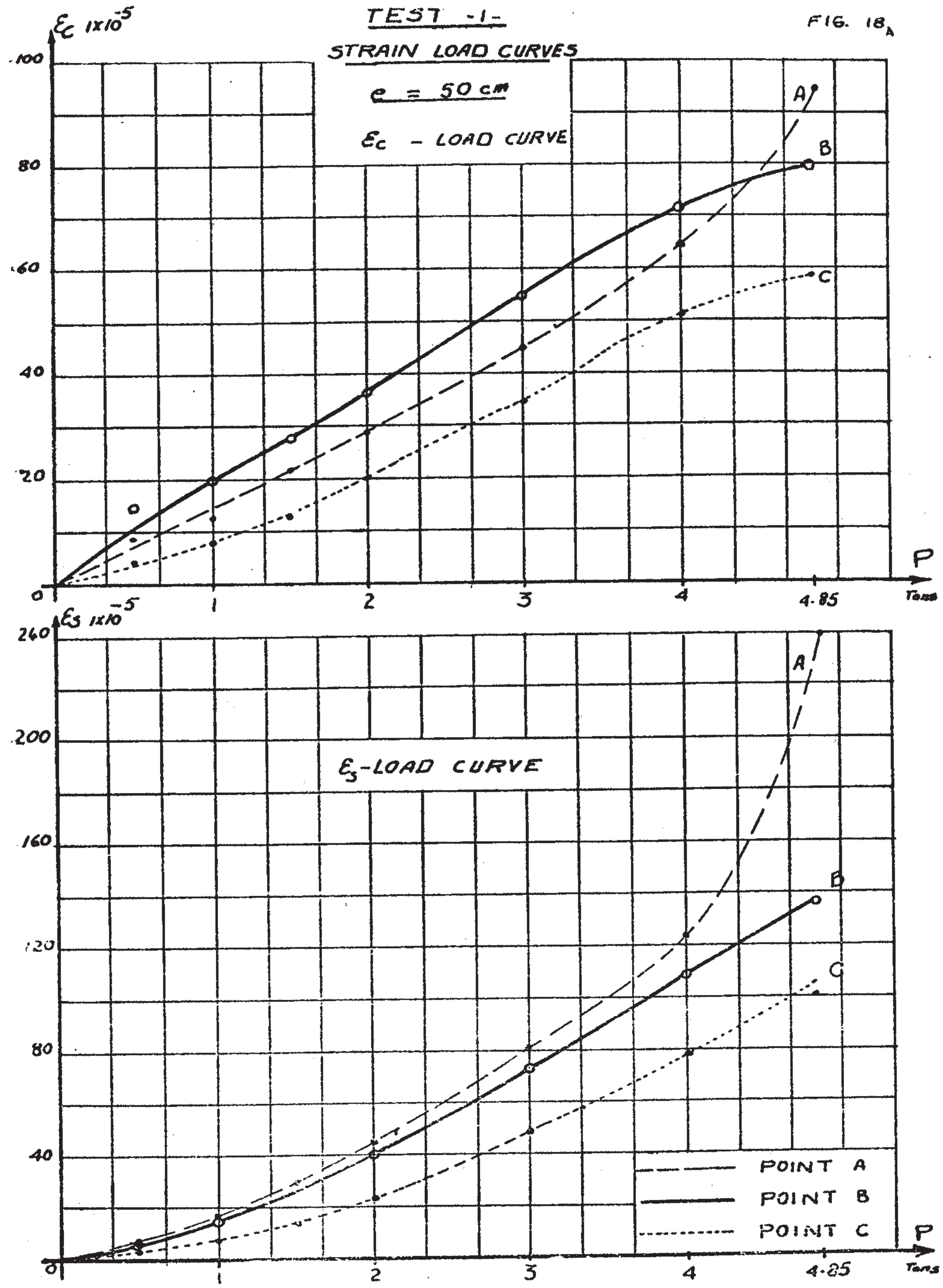
Then from the stress-strain curves of the control specimens the load—f_c and the load—f_s curves were obtained.

Fig. (18) shows curves for column (Fig 17a) e = 50 cms.

IV.—Theoretical Computations

In computing deflections and stresses, two different approaches were tried :—

(*A*) Computations applying simplified assumptions which follow the "Standard Theory of R.C.".



TEST -1-
STRAIN LOAD CURVES
e = 50 cm
ε_c - LOAD CURVE
FIG. 18A
ε_c 1x10^-5
100
80
60
40
20
0
1
2
3
4
4.85
P
Tons
A
B
C
ε_s 1x10^-5
240
200
160
120
80
40
ε_s-LOAD CURVE
POINT A
POINT B
POINT C

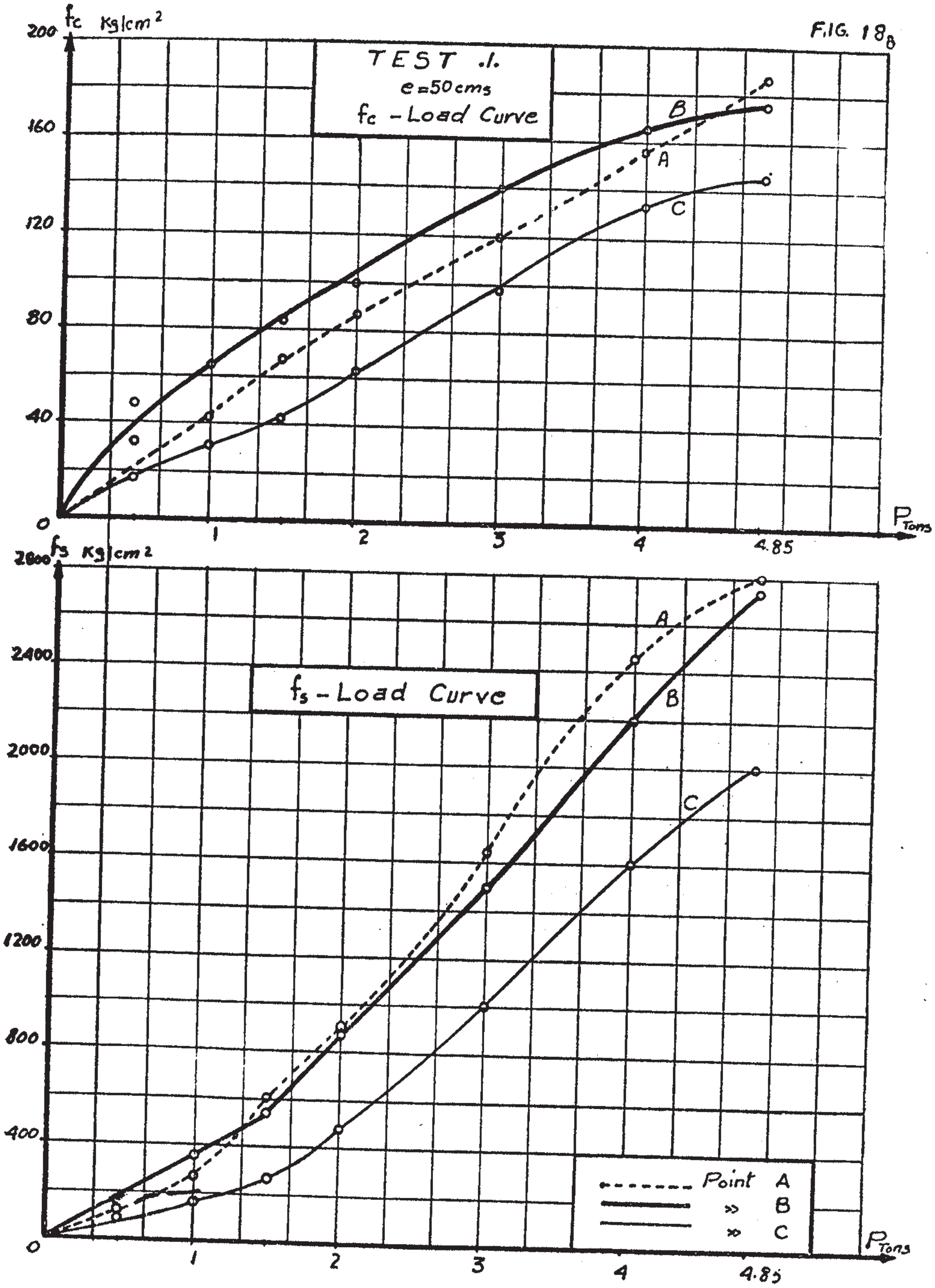
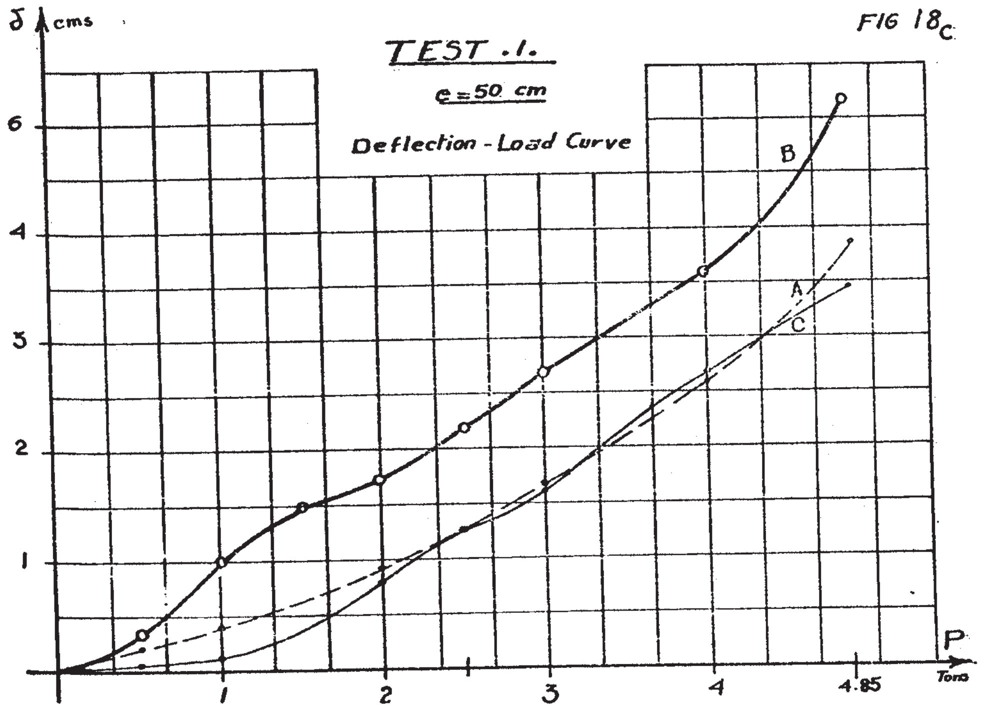

TEST .I.
e = 50 cms
f_c - Load Curve
f_c Kg/cm²
200
160
120
80
40
0
1
2
3
4
4.85
P Tons
A
B
C
f_s - Load Curve
f_s Kg/cm²
2800
2400
2000
1600
1200
800
400
Point A
» B
» C

FIG. 18B

FIG 18c


δ cms
TEST .1.
e = 50 cm
Deflection - Load Curve
B
A
C
P
Tons
1
2
3
4
4.85
6
4
3
2
1

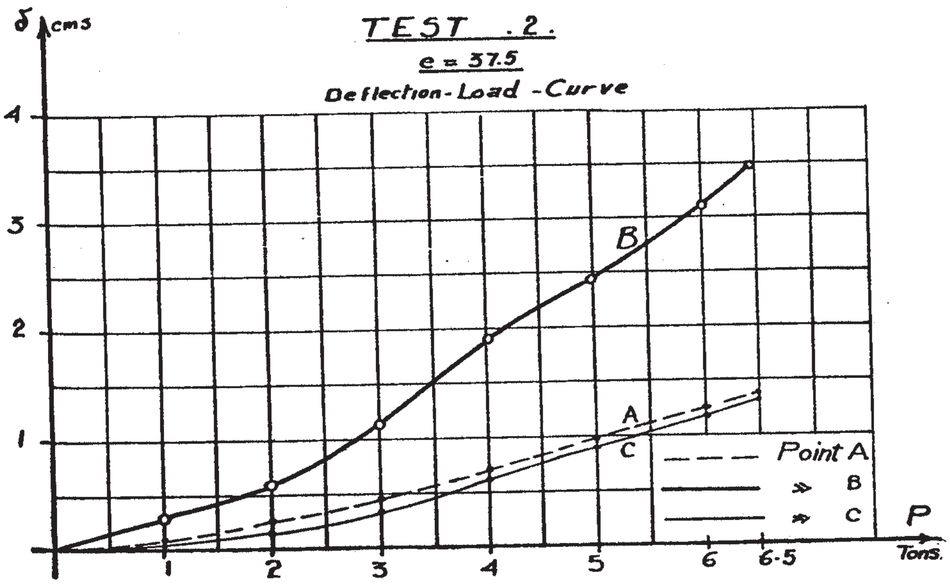
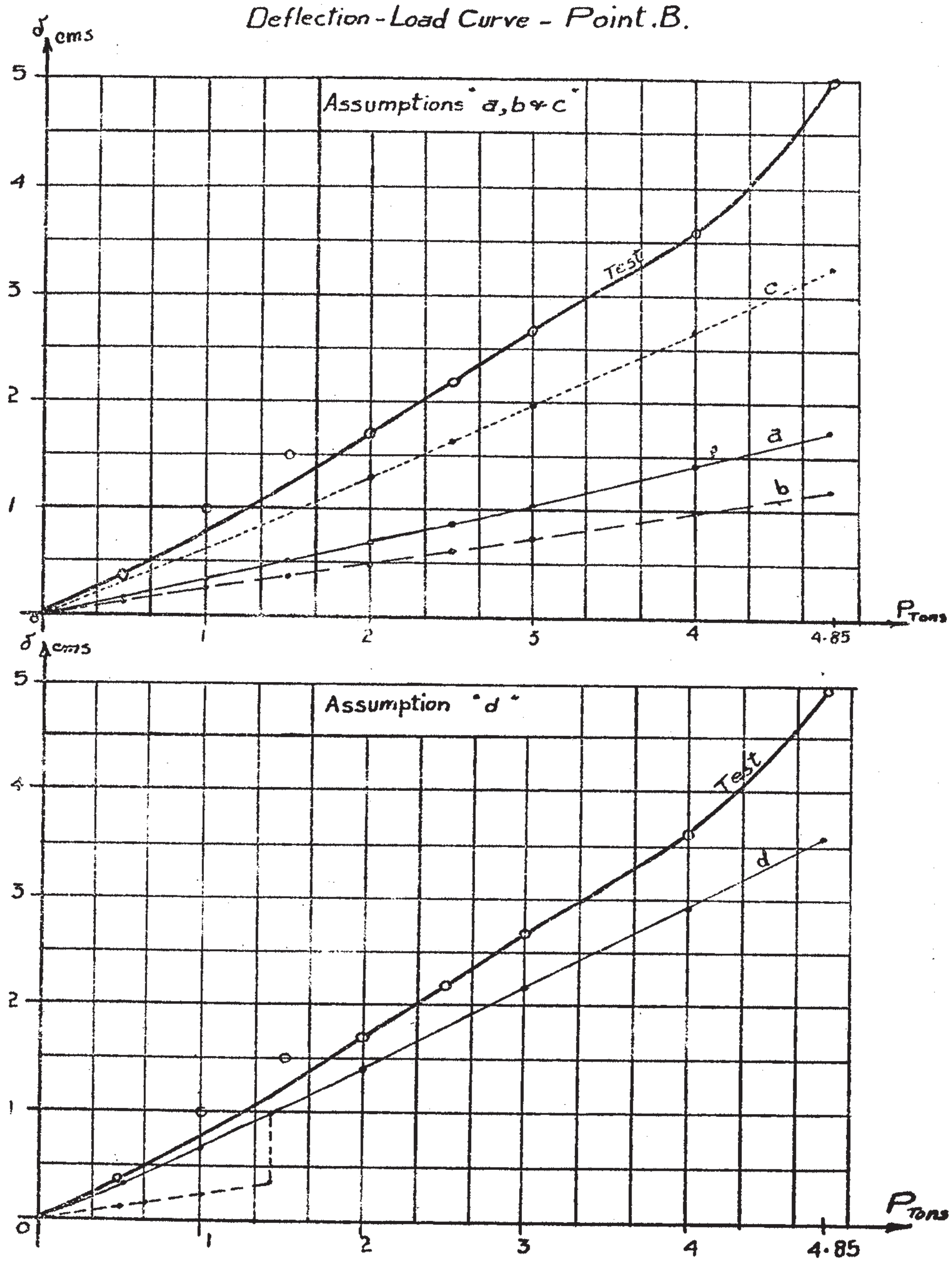

δ cms
TEST .2.
e = 37.5
Deflection - Load - Curve
B
A
C
Point A
» B
» C
P
Tons.
1
2
3
4
5
6
6.5

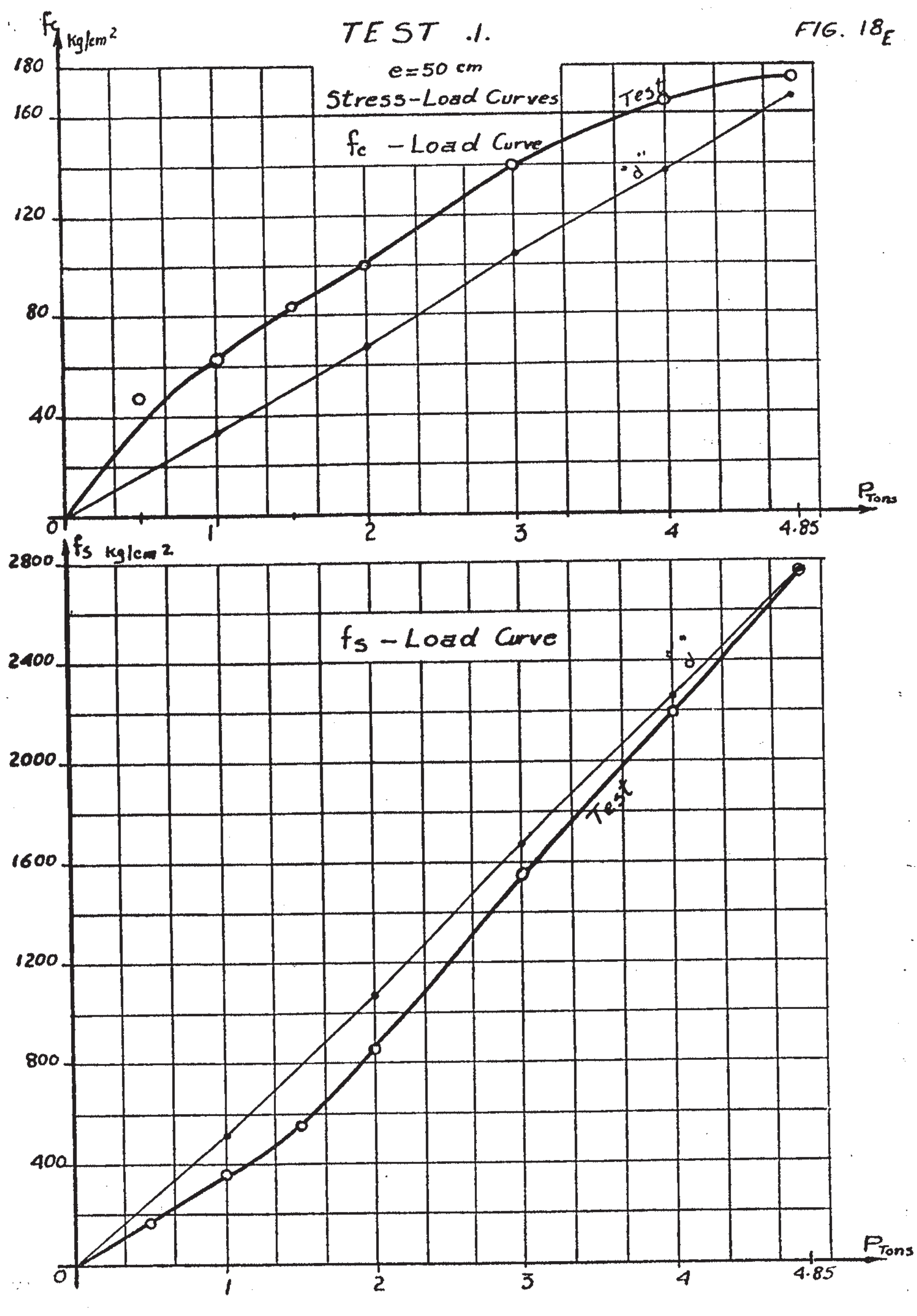
FIG. 18D

TEST .1.

e = 50 cm

Deflection - Load Curve - Point .B.




TEST .I.
FIG. 18E
e=50 cm
Stress-Load Curves
fc - Load Curve
fc kg/cm²
180
160
120
80
40
0
Test
"d"
P Tons
1
2
3
4
4.85
fs kg/cm²
2800
2400
2000
1600
1200
800
400
fs - Load Curve
Test
"d"
P Tons
0
1
2
3
4
4.85

(*b*) Computations according to amended more accurate assumptions.

A.—*Computation according to Ordinary Simplified Assumptions.*

1.—*Deflections at central sections:—*

In this assumption E_c and I are considered constant. Three different trials were applied to find out which will give better result.

(*a*) $E_c = 210$ t/cm²
I = moment of inertia of full concrete area neglecting steel.

(*b*) $E_c = 210$ t/cm² and $n = 15$
I = for full concrete area + n times the steel area.

(*c*) $E_c = 380$ t/cm² (from the initial slope of stress strain curve "d") and $n = \frac{Es}{Ec} = 5{\cdot}25$ and I is taken for the concrete area in compression + n times the steel area about the N.A. and is considered constant at each stage of loading.

The formula used was that derived for homogeneous materials.

$$\delta_{\text{max.}} = \frac{M\, l^2}{8\, EI} \; \frac{2\,(1-\cos\mu)}{\mu^2 \cos\mu} = \frac{e\,(1-\cos\mu)}{\cos\mu}$$

where $\mu = \frac{l}{2}\sqrt{\frac{P}{EI}}$ l = effective length = 5·75 ms.

e in assumptions (a and b) is taken to the c.g.

e in assumption (c) is taken to the N.A.

2.—*Stresses f_c and f_s:*

Rough assumptions (a and b) are not valid for computing stresses. Assumption (c) will give the stress load curve a straight line one as $f = \frac{P}{A_v} + \frac{My}{I_v} = P\left(\frac{I}{A_v} + \frac{ey}{I_v}\right)$

the second term is always constant.

B.—*Computation according to Amended Assumptions:*

E_c as obtained from secant line of the stress-strain curve of concrete $= 270$ t/cm² and $n = \frac{2000}{270} = 7{\cdot}42$ (constant).

E_c is considered constant for all stages of loading.

Two different stages of loading were considered :—

(i) *First stage before the development of cracks.*

Stresses and deflections were calculated in this stage by applying the formulae of homogeneous elastic bodies.

Assuming the deflection curve to be a parabola.

$$\therefore \delta_{max.} = \frac{P}{E_c\ I_v}\left(\frac{e\ l^2}{8} + \frac{5\ \delta\ l^2}{48}\right)$$

$$f_{max.} = \frac{P}{A_v} \pm \frac{My}{I_v}$$

where A_v = total virtual area = bt + n A_s

I_v = moment of inertia of full area of concrete + n times the steel area about the c.g.

M = P (e + δ)

e = eccentricity of the load to the c.g. of the section.

(ii) *Second stage after the development of cracks:*

After f_t has reached a value of 17 kg/cm², the effect of concrete in tension is neglected. The change from stage (i to ii) is considered abrupt, although the actual change is gradual. Deflections were computed by successive trial as a function of the stresses rather than a function of the moments and moments of inertia *i.e.* by the use of the equation $\frac{E}{\rho} = \frac{f}{y}$ instead of $\frac{E}{\rho} = \frac{M}{I}$

The procedure is: Two sections were considered, section I—I just below the leg, section II—II at the middle. For a given load P the stresses f_{cI} and f_{sI} at section I—I were obtained for an eccentricity e from the c.g. of section. Then a reasonable value of δ at center was assumed and the stresses f_{cII} and f_{sII} at section II—II were obtained due to P acting at an eccentricity $(e + \delta)$ from the c.g. of section. The value was checked as follows:

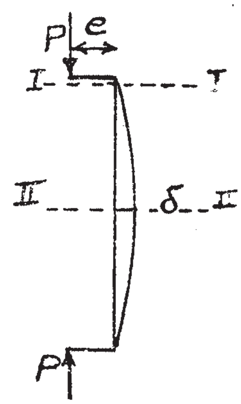


Fig. 19

$$\delta_{max.} = \frac{1}{E_c I}\left(\frac{Pe\, l^2}{8} + \frac{5}{48} P \delta l^2\right)$$

which could be written in the form

$$\delta_{max.} = \frac{l^2}{8 E_c}\left(\frac{f_{cI}}{z_I} + \frac{5}{6}\left(\frac{f_{cII}}{z_{II}} - \frac{f_{cI}}{z_I}\right)\right)$$

The work was repeated when necessary until the computed δ was equal to the assumed value of δ. The values of f_c and f_s for section I and II were obtained from the formulae:

$$f_c = \frac{M_n\, z}{I_n} \text{ and } f_s = n\, f_c \frac{d - z}{z}$$

where I_n is computed for the compressed area of concrete + n times the steel area about the N.A. (variable).

z = distance between the N.A. and extreme compressed fibres.

M_n = moment of P about the N.A. = $P \times H_n$

H_n = eccentricity of load from the N.A. and is calculated at each stage of loading at sections I and II.

The procedure was repeated for higher values of P until the failure load was reached, which load corresponds to a max. stress $f_s = f_y = 2800$ kg/cm² or $f_c = f_p = 240$ kg/cm².

V.—Conclusions

Part I:

1. Direct mathematical solutions of problems on axial buckling require elaborate differential equations particularly for complicated cases and variation of the moment of inertia.

2. Mathematical solutions by successive trial for the configuration of the deflected column give simple algebraic or trigonometric equations from which P_{cr} can be obtained.

3. Direct numerical solutions by successive trial, avoid the use of elaborate mathematical work and are much simpler.

4. Graphical solutions when systematized as proposed, are simpler ones.

5. The use of cardboard models representing the variation of I gives a very good initial assumption for the deflection curve.

6. The use of the proposed simple apparatus together with the cardboard model gives very satisfactory results.

7. In eccentric buckling direct mathematical solutions are very complicated.

8. Procedures for systematic solutions by successive trial for the determination of stresses and deflections were proposed for different modes of loading.

9. The use of the approximate formula $\delta = \frac{\delta_o}{1 - \alpha}$ gave satisfactory results and saves much time especially if the apparatus is used for the determination of P_{cr}.

Part II:

For the fourteen columns tested the following conclusions could be made.

Deflections :

1. It has been found that the use of assumptions (a) and (b) gave generally results much smaller than those obtained experimentally particularly for small eccentricities.

2. Assumption (c) led generally to more satisfactory results except for small eccentricities. Still, however, the discrepancy between the computed and experimental results was relatively big.

3. The amended assumptions proposed in (d) gave generally a more satisfactory result than assumptions (a, b, and c). In tests of small eccentricities that lie within the section, however, the results were not satisfactory. The discrepancy is believed to be due to the sensitivity to any smaller error in the point of application of the load or any small curvature in the center line which will be of pronounced effect in case of small eccentricity tests. An initial curvature of $\frac{l}{400}$ was allowed to cover the imperfections in small eccentricity tests and better results were obtained.

Stresses :

Stresses were computed according to assumption "d" only and the results obtained were in general satisfactory. The values of f_c were in general somewhat lower than test results particularly in small eccentricity tests, where the assumption of an initial curvature $\frac{l}{400}$ gave better agreement. The values of f_s were satisfactory.

Reference :

"Axial and Eccentric Buckling with Special Application to Reinforced concrete" M.Sc. Thesis by Labib Riad submitted at the Faculty of Engineering, Cairo University, 1953.