STUDY OF THE EFFECT OF STREAM LINE CURVATURE ON FLOW OVER WEIRS

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INTRODUCTION

The object of this work is to study the coefficient of discharge especially for broad crested weirs taking into consideration the effect of the stream line curvature.

Unfortunately, the effect of the curvature on the pressure and velocity distribution was always neglected and that led to the result that the theoretical value of the discharge $Q_{theor.}$ was not equal to the effective value $Q_{eff.}$ A correction factor (called the coeff. of discharge) had to be introduced in the theoretical equation for Q in order to get $Q_{eff.}$ (where $Q_{eff.} = \mu \; Q_{theor.}$). This coefficient μ includes the neglected effect of the curvature.

Since the curvature differs from case to case, according to the shape of the structure, the head, the discharge, the downstream conditions, etc., the value of the coeff. of discharge must therefore be variable.

A mathematical relation between the coeff. of discharge and these different variable factors is practically impossible. But since the shape of the water surface includes always the effect of the mentioned factors, it will be more logical to find out a direct relation between the coeff. of discharge and the curvature of the flow.

In this work we have succeeded in getting such a relation which has shown that the coeff. does not depend only on the curvature of water surface as some hydraulic authors believe, but also on the inclination of the tangent to water surface at the critical section.

It was also found that the results obtained from experiments carried on scale models of weirs cannot be applicable on the prototype. This led to the conclusion that: (it is not necessary, in case of geometrically similar models, that they will be kinematically similar).

The equations obtained from this work are, therefore, important because they are dimensionless and can be applied to any broad crested weir, whatever its size may be.

I.— *The relation between the curvature of the stream lines and the coefficient of discharge:*

Some authors gave theoretical relations between the curvature of the stream lines and the velocity distribution on vertical sections. Amongst those are Schorghuber, H. Rouse, and Ch. Jaeger. Using Euler equations, they came to the same following result:

$$v = v_o . e^{\int \frac{dn}{\rho}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where v = the velocity at any point on the section

v_o = „ „ the surface at the same section.

dn = height of an element.

ρ = radius of curvature of the stream line passing through the centre of the element.

This equation gives the distribution of the velocity over a vertical section. The distribution is not uniform so long as the stream lines are curved.

Substituting in this equation for $\rho = \infty$, then $v = v_o$, *i.e.* the velocity distribution will be uniform. This means that the coefficient of discharge equals 1.

This is not always the case, because the tests carried on all broad crested weirs, showed that in all cases where the water surface was straight (where $\rho = \infty$) and when it was inclined with an angle $= \phi$ to the horizontal sill, the coefficient of discharge was not equal to 1. Also the pressure distribution on vertical sections was not hydrostatic.

The inclination of the tangents to the stream lines plays, therefore, a role beside the curvature in affecting the coefficient of discharge.

Consider the general case of a curved flow over a curved bed as shown in Fig. 1. The water surface describes a curve with radius $= \rho_{\text{surface}}$. Consider an element with the dimensions dn and ds. The radius of the stream line passing through its centre $= \rho$. The z axis is normal to the bed and the n axis is normal to the stream line passing through the centre of the element (Fig. 1). From that figure it is clear that

$$\frac{dn}{dz} = \cos\phi \text{ and } \frac{ds}{dz} = \sin\phi$$

also:

$$dp = \frac{\partial p}{\partial n} dn + \frac{\partial p}{\partial s} . ds$$

and

$$\frac{\partial p}{\partial n} = -\gamma \cos(\phi + \Theta) + \frac{\gamma}{\rho} \frac{v^2}{g}$$

$$\frac{\partial p}{\partial s} = -\gamma \sin(\phi + \Theta) - \frac{\gamma\, dv}{gdt}$$

or

$$\frac{dp}{dz} = \cos\phi \left[-\gamma\cos(\phi + \Theta) + \frac{\gamma}{\rho} \cdot \frac{v^2}{g}\right] + \sin\phi \left[-\gamma\sin(\phi+\Theta) - \gamma \frac{d\, v_s}{g.dt}\right] \qquad (2)$$

In the special case of the broad crested weirs, the sill is horizontal and $\theta =$ zero.

Equation (1) will be :

$$\frac{dp}{dz} = \cos\phi\left(-\gamma\cos\phi + \frac{\gamma}{\rho}\cdot\frac{v^2}{g}\right)$$

$$+ \sin\phi\left(-\gamma\sin\phi - \frac{\gamma}{g}\cdot\frac{dv}{dt}\right)$$

or

$$\frac{dp}{dz}\quad\left(-\gamma\cos^2\phi + \frac{\gamma}{\rho\cdot g}\cdot\frac{v^2}{}\cos\phi\right)$$

$$+\left(-\gamma\sin^2\phi - \frac{\gamma}{g}\cdot\frac{dv_s}{dt}\sin\phi\right)$$

or

$$\frac{dp}{dz} = \left(\frac{\gamma}{\rho\cdot g}\frac{v^2}{}\cos\phi - \gamma\right) - \left(\frac{\gamma}{\rho}\frac{dv}{dt}\sin\phi\right) \quad . \qquad (3)$$

This equation gives four possible cases according to :

(1) ϕ = zero (Fig. 2a).

(2) $\rho = \infty$ (Fig. 2b).

(3) ϕ = zero and $\rho = \infty$ (Fig. 2c).

(4) $\phi \neq$ zero and $\rho \neq \infty$ (Fig. 2d).

Case 1: ϕ = zero, thus equation (3) will be :

$$\frac{dp}{dz} = \left(\frac{\gamma}{\rho}\cdot\frac{v^2}{g} - \gamma\right) \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

Since $H = z + \frac{\rho}{\gamma} + \frac{v^2}{2g}$ (Bernolli's equation)

after differentiation.

$$0 = 1 + \frac{1}{\gamma}\cdot\frac{dp}{dz} + \frac{v\,dv}{g\,dz} \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

substitute for $\frac{dp}{dz}$ its valve from equation (4) thus

$$0 = 1 + \frac{1}{\gamma}\cdot\frac{\gamma}{\rho}\cdot\frac{v^2}{g} - 1 + \frac{v\,dv}{g\,dz}$$

or

$$-\frac{v}{\rho} \quad \frac{dv}{dz}$$

$$\frac{dv}{v} \quad \frac{-dz}{\rho}$$

or

$$v = v_o \cdot e^{\int\frac{dz}{\rho}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

which is identical with equation (1).

Therefore equation (1) is applicable for this special case. It is not a general equation.

Case 2: $\rho = \infty$ equation (3) will be:

$$\frac{dp}{dz} = -\gamma - \left(\frac{\gamma}{g} \cdot \frac{d\,v}{dt} \sin \phi\right) \quad . \quad . \quad . \quad (7)$$

substituting this value in equation (5) we get

$$0 = 1 + \frac{1}{\gamma}\left[-\gamma - \left(\frac{\gamma}{g}\frac{d\,v_s}{dt} \sin \phi\right)\right] + \frac{v\,dv}{g\,dz}$$

$$0 = -\left(\frac{\gamma}{g} \cdot \frac{d\,v_s}{dt} \sin \phi\right) + \frac{v\,dv}{g\,dz}$$

$$\frac{v\,dv}{g\,dz} = \frac{\sin \phi}{g} \cdot \frac{d\,v_s}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The solution of this equation is difficult. Considering equation (7), the effect of the curvature on the pressure is eliminated by using $\rho = \infty$, while the effect due to the inclination of the surface to the horizontal is represented by the term

$$\left(\gamma + \frac{\gamma}{g} \cdot \frac{d\,v_s}{dt} \sin \phi\right)$$

Case 3: $\phi =$ zero and $\rho = \infty$ equation (3) will be

$$\frac{dp}{dz} = -\gamma.$$

It is the case of parallel stream lines, uniform velocity distribution and hydrostatic pressure.

Case 4: $\phi \neq 0$ and $\rho \neq \infty$

This is the general case in which the velocity as well as the pressure at any point in any vertical section are effected by both the curvature and the inclination of the stream lines to the horizontal.

Since the solution of the equation (3) is very difficult, it is intended in this work to separate both effects and study each of them separately.

II.—*The effect of the inclination of the tangents of the stream lines on the coefficient of discharge:*

The special case of horizontal bed will be studied in this paper. It will be represented by the broad crested weirs with $\rho_{surface} = \infty$. In order to solve this case two assumptions were made:

Assumption (1):

The curved planes of equal velocities and equal $\left(\frac{p}{\gamma} + z\right)$ are always perpendicular to the boundaries of the flow (Fig. 3).

Assumption (2):

ϕ is proportional to z

i.e. $\frac{z}{h} = \frac{\phi_z}{\phi_{surface}}$ or $\phi_z = \phi_{surface} \cdot \frac{z}{h}$ (Fig. 3)

where ϕ_z is the inclination of any stream line at any point in a vertical section, and $\phi_{surface}$ is the water surface inclination at the same section.

These two assumptions were justified by accurate measurements using a movable pitot-tube which measures the velocities and pressures in the direction of the stream lines (A Khafagi, Venturi-Kanal, Theorie und Anwendung, Zurich 1942).

The results showed that the assumption $\phi_z = \phi_{sur} \cdot \frac{z}{h}$ is true for vertical sections, as well as for the curved plane L.

In the equation $Q = b.h.v_0 = \quad . \sqrt{2g(H-h)}$; b = the length of the weir. The rest of notations are shown in Fig. 3.

This equation is correct by using the coefficient of discharge k_0 in order that the calculated Q will be equal to the actual discharge. The equation will be:

$$Q = k_0 \,.\, b.\, h.\, v_0 = k_0 .\, b.\, h \sqrt{2g\,(H-h)} \quad . \qquad (9)$$

The actual discharge $Q = b.\, h.\, v_m$

Equating this equation with equation (9) we get

$$k_o = \frac{v_m}{v_o} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Also $\quad Q = b.\ L.\ v_o = b.\ h.\ v_m$

Therefore $\frac{v_m}{v_o} = \frac{L}{h}$ but $\frac{v_m}{v_o} = k_o$

Therefore $\frac{L}{h} = k_o \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$

The coefficient of discharge was calculated first according to equation $Q = k_o.\ b.\ h.\ \sqrt{2g\ (H\text{-}h)}$ and second according to equation (11) by measuring L and h. The results tabulated in Table I showed that both values of k_o are equal:

TABLE I

Weir (*)	Q	hcr	ϕsur.	R	L	$\frac{L}{hcr}$	ko
	Lt/sec.	cm.	o	cm.	cm.		
6	10·00	4·84	13	∞	4·93	1·018	1·019
7	20·00	7·68	11	∞	7·79	1·014	1·015
7	30·00	10·07	14	∞	10·28	1·020	1·021

ϕsur. = Inclination of surface at critical section.
R = Radius of curvature of water surface at critical section.

By integrating the length of an element along any curve, it is easy to show that the length of this curve (L)

$$L = \int \sqrt{1 + tg^2\ \phi}\ .\ dz \quad . \quad . \quad . \quad . \quad . \quad (12)$$

substituting this value in equation (11) thus:

$$k_\phi = \frac{L}{h} = \frac{\int_o^h \sqrt{1 + tg^2\ \phi}\ .\ dz}{h} \quad . \quad . \quad . \quad (13)$$

Since $\quad \phi_z = \phi_{sur.} \frac{z}{h}$ assumption (2)

$$\therefore \quad \frac{dz}{d\phi} \left(\frac{h}{\phi_{sur.}}\right) \text{ or } dz = \frac{h\ d\ \phi}{\phi_{sur.}}$$

(*) See Figs. 6, 7.

$$\therefore \qquad k_{\phi} = \frac{\int_{0}^{\phi_{sur.}} \sqrt{1 + tg^2 \phi} \, . \, d\phi}{\phi_{sur.}}$$

or
$$k_{\phi} = \frac{1}{\phi_{sur.}} \left[\text{Ln} \;\; tg \left(\frac{\phi_{sur.}}{2} + 45^{\circ} \right) \right] \qquad \qquad (14)$$

The values of k_{ϕ} can be calculated from equation (14). They are given in Table II :

TABLE II

$\phi_{sur.}$	k_{ϕ}	$\phi_{sur.}$	k_{ϕ}	$\phi_{sur.}$	k_{ϕ}
10	1·014	40	1·09	70	1·42
20	1·030	50	1·16	80	1·74
30	1·05.	60	1·26	90	∞

This gives the coefficient of discharge in the case of $R = \infty$. If $R \neq \infty$, the coefficient of discharge will be consisting of two parts. The first one is that due to the inclination of the tangents of the stream lines to the horizontal and can be calculated from equation (14), and the second part is that due to the curvature only.

It should be noted that the effect of the inclination is not k_{ϕ} itself, but it is the difference between k_{ϕ} and unity ($\Delta \; k_{\phi} = k_{\phi} - 1$). Unity is the coefficient of discharge in the case of parallel-horizontal stream lines.

III.—*The effect of the curvature of the stream lines on the coefficient of discharge:*

In this paragraph, the inclination of the stream lines will be excluded. Only the cases where ϕ = zero will be considered (Fig. 4).

The velocity distribution corresponding to these cases follows the equation (6).

$$v = v_o . e^{\int \frac{dz}{\rho}} \qquad \qquad (6)$$

In order to solve the **integral** $\int \frac{dz}{\rho}$, **a relation between** ρ and z **must be found.**

Let the relation be $\rho = R\left(\frac{h}{z}\right)^n$ (assumption)

R is the radius of water surface.

For $z = h, \ \rho = R$

and for $z = 0, \ \rho = \infty$

$$\frac{1}{\rho} = \frac{1}{R}\left(\frac{z}{h}\right)^n \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

differentiating equation (6) $\therefore \frac{dv}{v} = \frac{-dv}{\rho}$

$$\text{Therefore } \frac{dv}{v} = -\frac{z^n \,.\, d_z}{R \,.\, h^n} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$\therefore \log v = -\frac{z^{(n+1)}}{(n+1).\ R.\ h^n} + C \quad . \quad . \quad . \quad . \quad (17)$$

$$\text{For} \quad z = 0 \quad \therefore \quad \log v_u = C \quad . \quad . \quad . \quad . \quad (18)$$

(v_u is the velocity at bed level).

$$\text{and for} \quad z = h \quad \therefore \quad \log v_o = -\frac{h}{(n+1).\ R} + C \qquad (19)$$

(v_o is the velocity at the surface).

From equations (18 and 19):

$$\log \left(\frac{v_o}{v_u}\right) = -\frac{h}{(n+1)\ R} = -K = \text{constant} \qquad (20)$$

K is a constant value for a certain profile.

$K = f\ (R, n)$

From equation (20)

$$v_o = v_u \,.\, e^{-K} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

From equations (19 and 20)

$$\log \left(\frac{v}{v_n}\right) = -\frac{z^{(n+1)}}{(n+1)\ R.\ h^h} = -B \quad . \quad . \quad (22)$$

or $\quad v = v_u \, . \, e^{-B}$ . . . . (23)

where $\quad B = f(z) = -K \left(\frac{z}{h}\right)^{n+1}$

Finally the velocity at any point in a certain profile follows the equation:

$$v = v_u \, . \, e^{-B} = v_o \, . \, e^{K-B} \quad . \quad . \quad . \quad . \quad . \quad (24)$$

K and B contain the value "n" which is still unknown and must at first be determined in order to let equation (24) be practically applicable.

Determination of "n":

"n" depends on the shape of the velocity distribution.

The measurements of the velocity at different points for different discharges and different shapes of the water surface have shown that in the case of the convex water surface, v_u is always greater than v_o; where in the case of concave water surface v_u was always less than v_o (Figs. 5 to 8).

The mathematical explanation of this phenomena is given in the following paragraph:

From equation (24), it is clear that the velocity distribution diagram describes almost a parabola whose axis is at the bed in the cases where the water surface is convex, *i.e.* R is positive (Fig. 9). From this figure:

$$X = v_u - v_o, \text{ but } v_u = v_o \, . \, e^K$$

$$\therefore \; X = v_o \, . \, e^K - v_o = v_o \, (e^K - 1).$$

The mean value of $X = \frac{2}{3} X = \frac{2}{3} v_o \, (e^K - 1)$

therefore $v_m = v_o + \frac{2}{3} v_o \, (e^K - 1) = v_o \left(\frac{1 + 2 \, e^K}{3}\right)$

$$\therefore \; \frac{v_m}{v_o} = \frac{1 + 2 \, e^K}{3} = k_R \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Thus the effect due to the curvature only

$$\Delta \, k_R = k_R - 1 = \frac{1 + 2 \, e^K}{3} - 1 = \frac{2}{3} \, (e^K - 1) \quad (26)$$

k_R is the coefficient of discharge due to curvature

1 is the coefficient of dischorge for parallel horizontal stream lines.

On the other hand, if the water surface describes a concave surface, *i.e.* R is negative, the axis of tha parabola is at the surface (Fig. 10).

$$X = v_o - v_s = v_o - v_o e^K = v_o (1 - e^K)$$

The mean value of $X = \frac{2}{3} X = \frac{2}{3} v \ (1 - e^K)$

$$\therefore \ v_m = v_s + \frac{2}{3} X = v_o e^K + \frac{2}{3} v_o (1 - e^K) = v_o \frac{(e^K + 2)}{3}$$

$$\therefore \ \frac{v_m}{v_o} = \frac{e^K + 2}{3} = k_R \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Thus the effect due to curvature only :

$$\Delta \ k_R = k_R - 1 = \frac{e^K + 2}{3} - 1 = \frac{1}{3} (e^K - 1) \quad (28)$$

The coefficient of discharge k_o was calculated from the equation : $Q = k_o \left[\left(\frac{2}{3}\right)^{3/2} b \sqrt{g} \ H^{3/2} \right]$ for general cases where water surface was curved and inclined to the horizontal bed.

The coefficient due to the inclination only was calculated from equation (14), then subtracted from unity to get the effect due to inclination only ($\Delta k_\phi = k_\phi - 1$). This effect Δk_ϕ was then subtracted from the total coefficient k_o to get the coefficient due to curvature only ($k_R = k_o - k_\phi$).

The values of k_R were substituted either in equation (25) or (26) as the case may be, in order to get K from which "n" was determined. It was noticed that the value of "n" was nearly constant and its mean value in the case of the convex water surface was double its value for the case of the concave one. In the first case, the mean value of "n" is 1·12 and in the second case the mean value of "n" is — 0·56.

The final equation for the coefficient of discharge can, therefore, be written as follows:

$$k'_o = k_\varnothing + \Delta k_R$$

For cases of convex water surface, *i.e.* where R is positive then:

$$\boxed{k'_o = \frac{1}{\varnothing_{sur.}} \left[Ln\ tg \left(\frac{\varnothing_{sur.}}{2} + 45^\circ \right) \right] + \frac{2}{3} \left(e^K - 1 \right)} \qquad (29)$$

where $$K = \frac{h_{cr}}{(n+1) R} = \frac{h_{cr}}{2{\cdot}12\ R} \quad . \quad . \quad . \qquad (30)$$

Values of k'_o for different $\frac{h_{cr}}{R}$ and different $\varnothing$ according to equation (29) were drawn in Fig. 12.

For cases of concave water surface, *i.e.* where R is negative then:

$$\boxed{k'_o = \frac{1}{\varnothing_{sur.}} \left[Ln\ tg \left(\frac{\varnothing_{sur.}}{2} + 45^\circ \right) \right] + \frac{1}{3} \left(e^{K'} - 1 \right)} \qquad (31)$$

where $$K' = \frac{h_{cr}}{(n+1) R} = \frac{h_{cr}}{(-0{\cdot}56 + 1) R} = \frac{h_{cr}}{0{\cdot}44\ R} \qquad (32)$$

$$h_{cr} = \sqrt[3]{\frac{Q^2}{gb^2}}$$

$\varnothing_{sur.}$ = the inclination of the tangent to the water surface at the position of the critical section ($h_{cr.}$).

R = the radius of the water surface at the critical section (positive sign for convex surface, and negative sign for concave surface).

Also k'_o for different $\frac{h_{cr}}{R}$ and different $\varnothing$ according to equation (31) is drawn in Fig. 13.

IV.—*Discussion of the General Equations 29 and 31:*

From Fig. 14, it is noticed that for very small values of h, the coefficient of discharge is big, decreases rapidly till it reaches a minimum at values of h between 0·10 and 0·20 m., and then begins to rise again, but slowly.

This irregular variation of "μ" with respect to "h" leads to the result that the values of μ obtained from experiments on models should never be applied on Prototypes. To make this clear, we suppose that we have a model for a sharp-edged weir scale 1 : 10, and that the max. head on the prototype is 80 cms. The corresponding head in the model should be 8 cms. From Fig. 14, for h = 8 cms. μ is 0·637, where for h = 80 cms. (Prototype) μ = 0·788.

Practical examples which support this explanation are the cases of regulators: Kembs, Klingnau and Rupperswil built across the Rhein and Aare rivers in Switzerland.

Table III after W. Eggenberger gives the dimensions of the openings and gates as well as the maximum discharges and heads for the mentioned works.

TABLE III

Regulator	River	Gates				Max. drop of upper gate	Max. Q over upper gate (total)	Max. Q through regul. (total)
		Height	Breadth	No. of openings	Total width			
		ms.	ms.		ms.	ms.	m^3/sec.	m^3/sec.
Kembs . .	Rhein	11·50	30	5	150	3·00	1455	4200
Klingnau .	Aare	7·00	30	4	120	2·25	736	2110
Reckingen .	Rhein	12·00	20	3	60	3·75	810	2250
Rupperswil.	Aare	8 00	22	3	66	2·50	467	1230

According to the statements published in S.B.Z. Bd. 95 April 1930, the maximum possible capacity of the water-structure-Research lab. in Zurich (Wasserbau Laboratorium der E.T.H., Zurich) is 750 lit./sec. where 250 lit./sec. for the high pressure head works and 500 lit./sec. for moderate heads.

Supposing that we use the whole value of 500 lit./sec. for one model only, the scale λ, should not exceed the value given in Table IV.

TABLE IV

Regulator	Max. Q nature	Max. Q model	$\lambda^{5/2}$	λ	$1:\lambda$
	m^3/sec.	m^3/sec			
Kembs	4200	0·500	8400	37	1 : 40
Klingnau	2110	0·500	4220	28	1 : 30
Reckingen	2250	0·500	4500	29	1 : 30
Rupperswil	1230	0·500	2460	23	1 : 25

Using these values in calculating the max. heads in the models, we get the values given in Table V.

TABLE V

Regulator	Max. head over the upper gste in nature (from Table III)	1 : λ (from Table IV)	Max. head over the upper gate in model
	ms.		cms.
Kembs	3·00	1 : 40	7·5
Klingnau	2·25	1 : 30	7·5
Reckingen . . .	3·75	1 : 40	9·4
Rupperswil . . .	2·50	1 : 25	10·0

From this table it is obvious that the max. head in all models of the mentioned works does not exceed 10·0 cms.

Taking the case of "Reckingen" as an example and considering an intermediate value of h (Prototype) equals say 50 cms. The corresponding h (model) = 1·25 cms. According to the μ curve (Fig. 14) we find that μ (Prototype) = 0·665 and μ (model) = 0·84. The difference is now clear and shows that the μ curves known up till now and the equations which are obtained from experiments carried out on models can never be applied on Prototypes. It is of course clear that the difference between μ (model) and μ (Prototype) decreases with the increase of the scale of the model. But since the scale of the model is dependent on the maximum water capacity of the research lab. as explained before (Table V), it will not be always possible to choose big sizes of models.

If exact values of $Q_{(Prototype)}$ are required, our equations 29 and 31 can, therefores be used. Since these equations contain the curvature and the inclination of the water surface, and since these factors can be easily measured in any weir whatever its size may be, our equations are, therefore, general equations for broad crested weirs.

V.—*Application:*

The discharges of a weir 15 ms. long were accurately measured by means of the current meter for h = 0·70 ms. and h = 0·90 ms.

The curved water surfaces were measured by means of an apparatus especially designed and constructed for this work. The coefficients of discharges for these two heads were calculated, once from the equation:

$$Q_{eff.} = k_o \left(\frac{2}{3}\right)^{3/2} b \sqrt{g}\, H^{3/2} \text{ and once from equation 29}$$

$$k_o' = \frac{1}{\varphi_{sur}}\left[\text{Ln tg}\left(\frac{\varphi_{sur}}{2} + 45^\circ\right)\right] + \frac{2}{3}\left(e^{K} - 1\right)$$

where $K = \dfrac{h_{cr}}{2{\cdot}12\ R}$

The results are tabulated in Table VI.

From these results it is noticed that k_o and $k^{o\prime}$ are very nearly equal.

The same measurement and calculations were carried on a model 1 :25 using model heads corresponding to those of the prototype given in Table VI. The results are tabulated in table VII.

TABLE VI

h	Q	$\frac{v_o^2}{2g}$	H	hcr	ϕsur	R	$\frac{hcr}{R}$	ko	k_o^1 eqn. (29)
ms	$m^3/m^1/$ sec.	ms,	ms,	ms,	o	ms			
0·70	1·095	0·005	0·705	0·497	21	2·60	0·191	1·086	1·091
0·90	1·647	0·0125	0·9125	0·652	$23\frac{1}{2}$	2·70	0·241	1·109	1·114

TABLE VII

h	Q	$\frac{v_o^2}{2g}$	H	hcr	ϕsur	R	$\frac{hcr}{R}$	ko	k_o^1 eqn. (29)
cms	$lt/m^1/$ sec.	cm.	cm.	cm.	o	cm.			
2·80	0·091	0·02	2·82	1·955	22	7	0·28	1·128	1·125
3·60	0·135	0·05	3·65	2·650	$23\frac{1}{2}$	8·5	0·31	1·136	1·138

Again here the values of k_o and k_o' are equal. Comparing the k_o values in both tables, we find that those for the prototype differ very much from those of the model. This again justifies our mentioned discussion which comes to the conclusion that it is not necessary, in case of geometrically similar models, that they will be kinematically similar.

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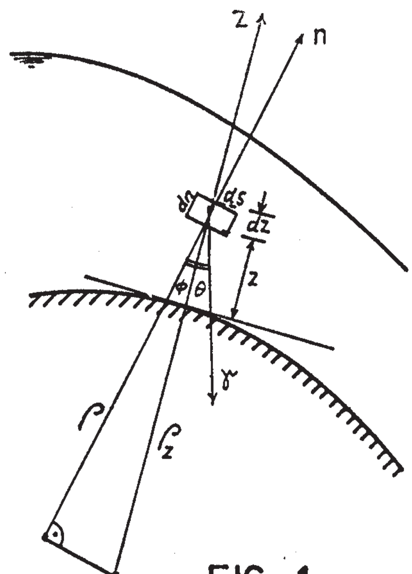


FIG.-1-

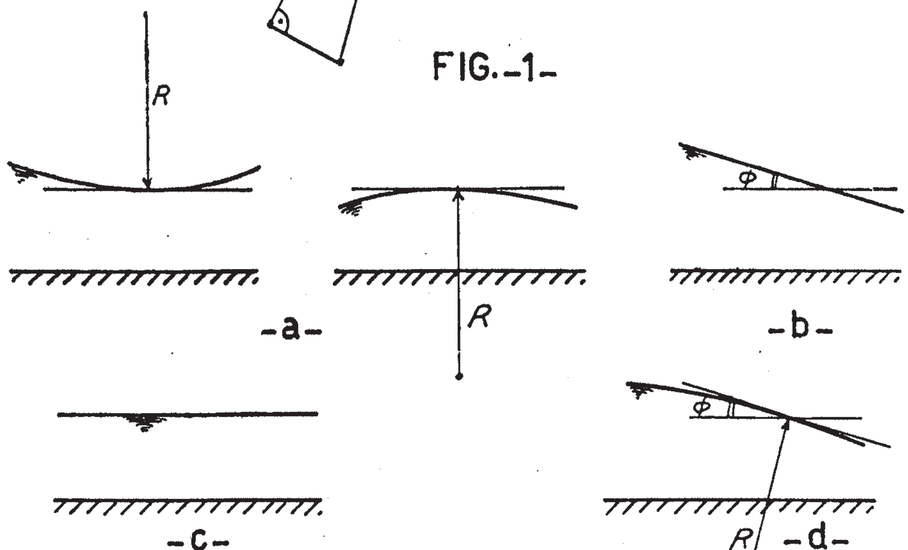


FIG.-2-

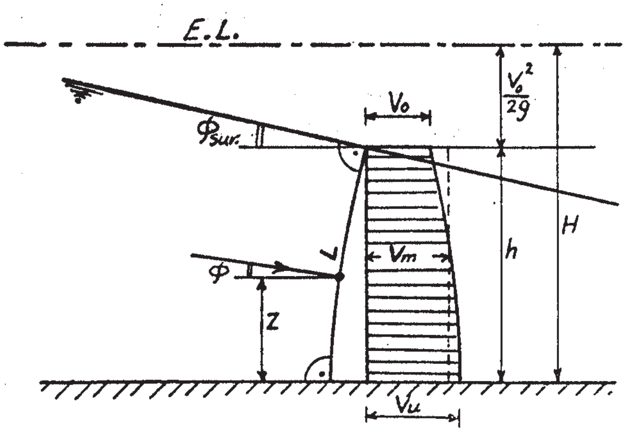


FIG-3-

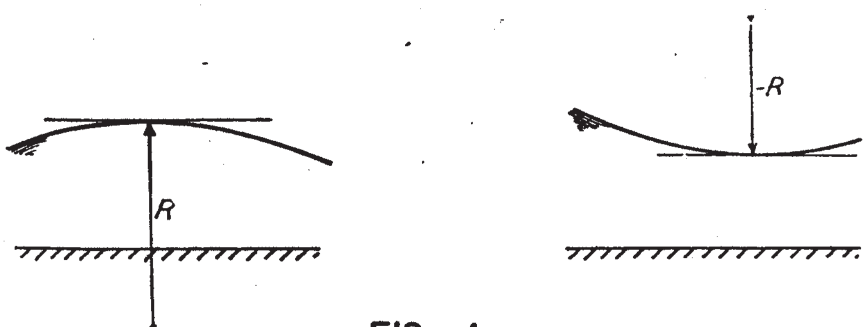
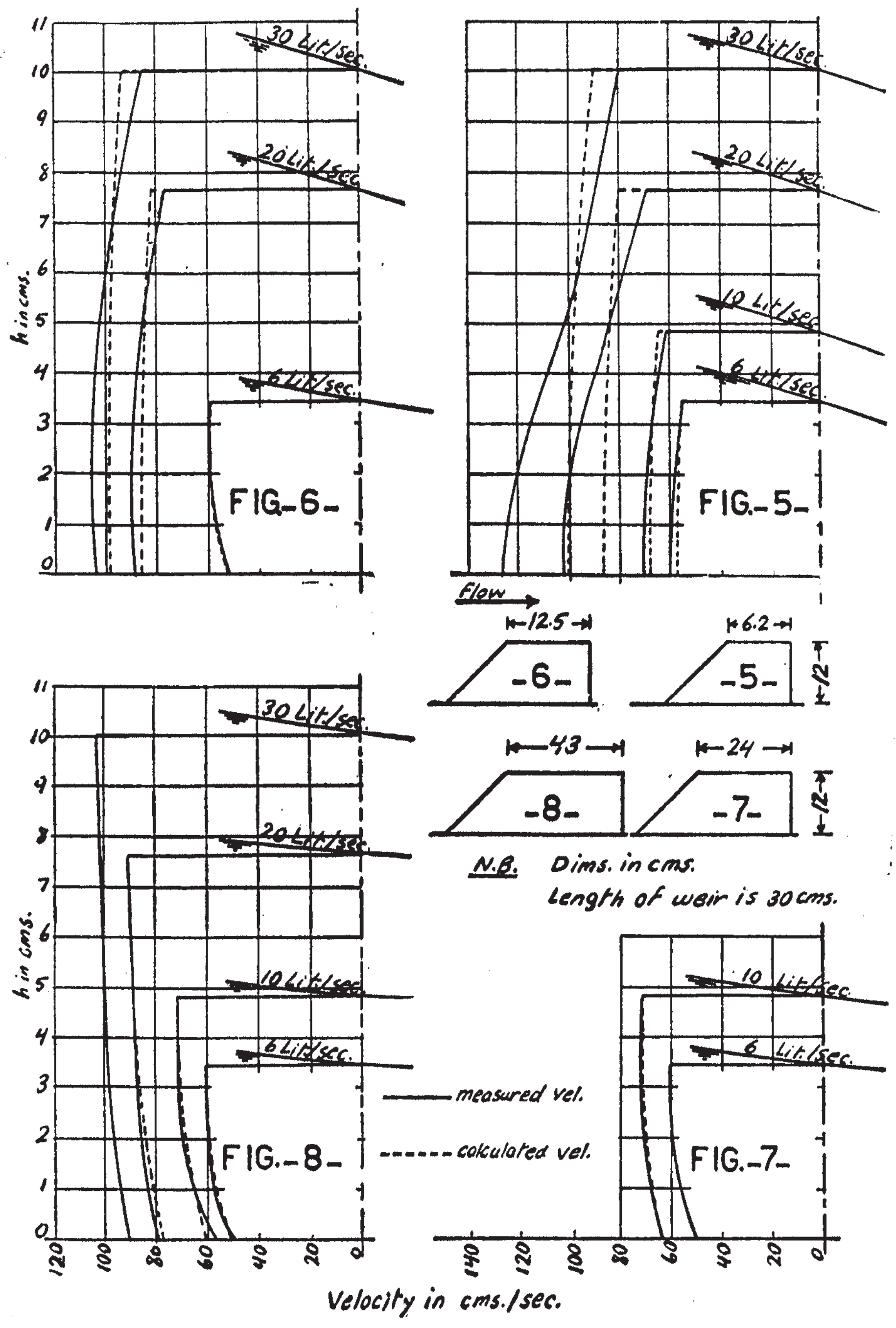


FIG. -4-


30 Lit/sec.
20 Lit/sec.
10 Lit/sec.
6 Lit/sec.
h in cms.
FIG.-6-
FIG.-5-
FIG.-8-
FIG.-7-
Flow
12.5
6.2
43
24
12
-6-
-5-
-8-
-7-
N.B. Dims. in cms.
Length of weir is 30 cms.
measured vel.
calculated vel.
Velocity in cms./sec.

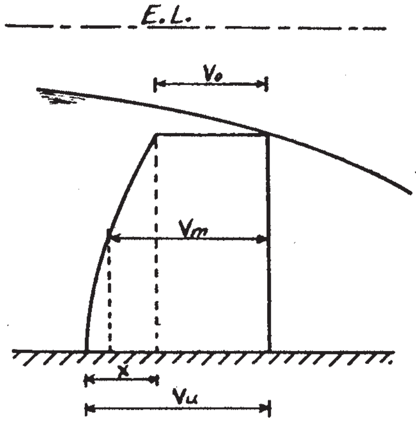


FIG.-9-

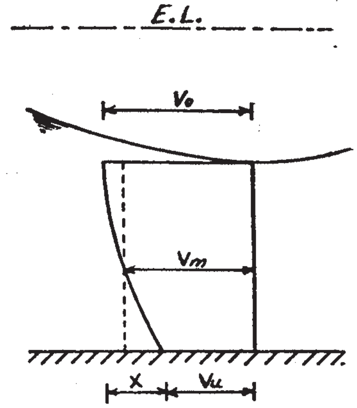


FIG.-10-

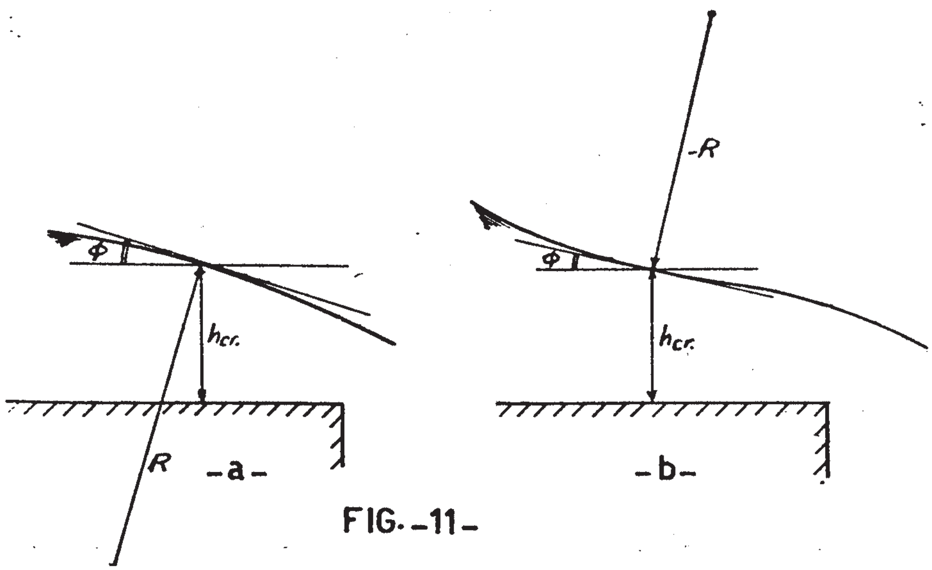
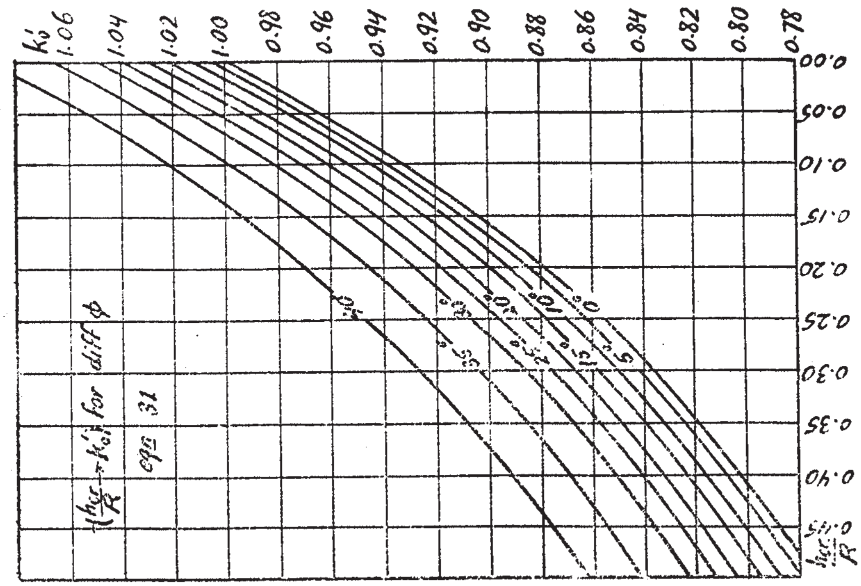


FIG.-11-


$(\frac{h_{cr}}{R} \rightarrow K_o')$ for diff ϕ
eqn 31
K_o'
1.06
1.04
1.02
1.00
0.98
0.96
0.94
0.92
0.90
0.88
0.86
0.84
0.82
0.80
0.78
$\frac{h_{cr}}{R}$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45

FI . 1 -

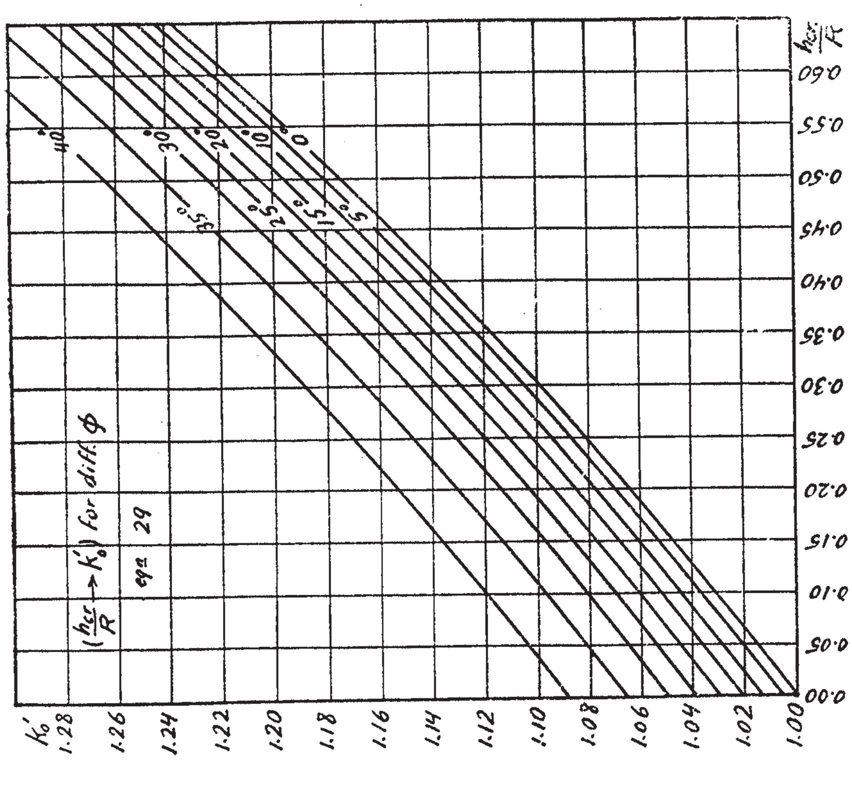

$(\frac{h_{cr}}{R} \rightarrow K_o')$ for diff. ϕ
eqn 29
K_o'
1.28
1.26
1.24
1.22
1.20
1.18
1.16
1.14
1.12
1.10
1.08
1.06
1.04
1.02
1.00
$\frac{h_{cr}}{R}$
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.50
0.55
0.60

FIG. -12-

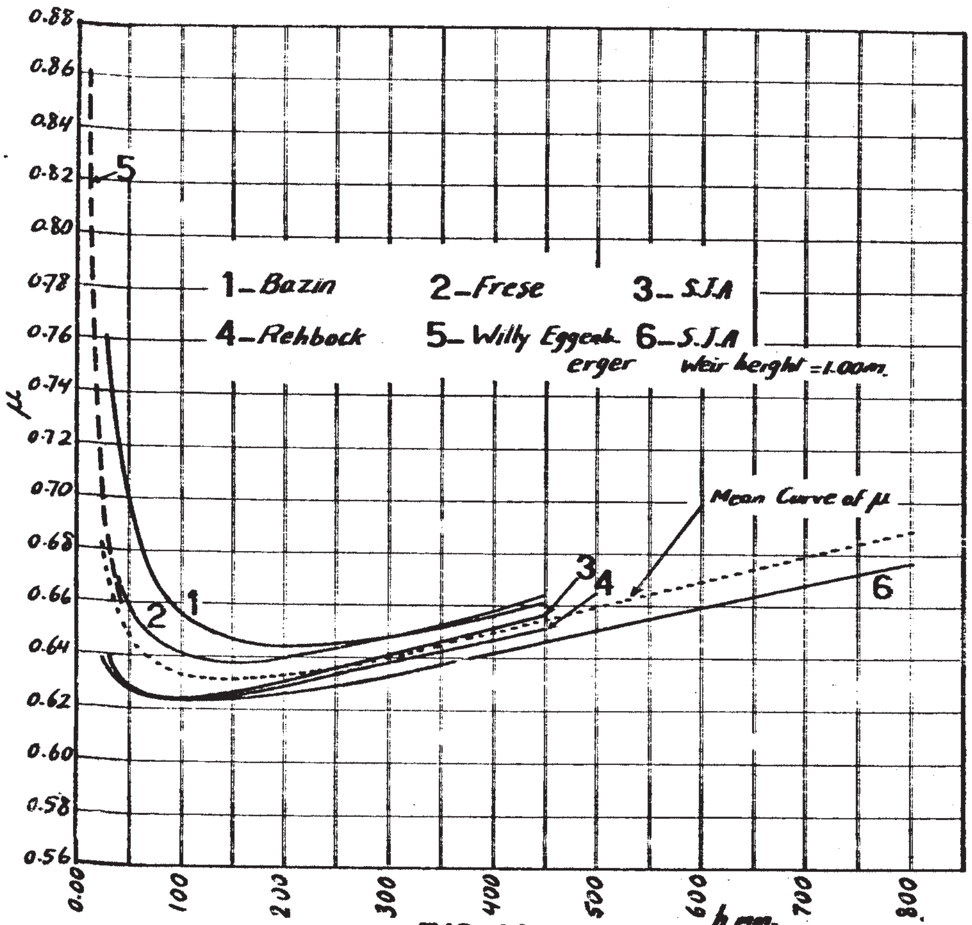

1- Bazin
2- Frese
3- S.I.A
4- Rehbock
5- Willy Eggeberger
6- S.I.A weir height = 1.00 m.
Mean Curve of μ
μ
h mm.
0.56
0.58
0.60
0.62
0.64
0.66
0.68
0.70
0.72
0.74
0.76
0.78
0.80
0.82
0.84
0.86
0.88
0.00
100
200
300
400
500
600
700
800

FIG-14-