SOME FACTORS AFFECTING THE DESIGN OF CROSSFLOW HEAT EXCHANGERS FOR GAS TURBINES

BY

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Heat exchangers, because of their very beneficial effect on the efficiency of gas turbines, have become an integral part of all gas turbine installations. It is well known, now, that the addition of an efficient heat exchanger will improve the overall efficiency of a gas turbine plant by about 25% or more.

Unfortunately, until now, the design of heat exchangers is not based on solid theoretical rules; trial and error and perhaps some empirical rules are the only guidance. As a result, the design of heat exchangers is generally a tedious job; this is mainly due to the multiplicity of factors involved. The main aim of this paper is to narrow down the scope of trial and error by introducing some rules and relations between the various items involved in heat exchanger design with the object of making design easier.

Heat exchanger design should be guided by three main factors:—

(*a*) The maximum heat gain from hot gases to cold gases.

(*b*) The minimum pressure drop in both hot and cold gas passages.

(*c*) The weight and volume of exchanger should be taken into consideration so as to attain optimum economical gain.

(*a*) and (*b*) affect the efficiency of the turbine, that is to say, will result in fuel saving, while (*c*) affects the initial cost of the heat exchanger. It should be borne in mind that the optimum economical design of an exchanger is necessarily a compromise between (*a*) and (*b*) on one side and (*c*) on the other side. This naturally depends upon locality, in other words, it depends upon the cost of fuel, cost of material, on labour, and the working conditions, that is to say, number of hours the plant is run per year. In short, the designer has to be careful that the gain due to fuel saving should not be more than offset by the depreciation of the heat exchanger itself.

Again, optimum gain in efficiency is a compromise between (*a*) and (*b*).

Looking at a heat exchanger as an energy saver, from the turbine point of view, general rules for their design can be obtained. The energy involved in this saving process appears in two distinct forms :—

1. Heat energy, which was a part of the total heat energy contained in the exhaust gases as they leave the turbine, transferred through the heat exchanger to the combustion chamber of the turbine. This energy is actually a gain for the turbine.

2. The total pressure drop in both the air charge and the exhaust gases in the exchanger passages. This represents an energy lost from the turbine, since if this pressure drop took place in the turbine itself, it would result in extra work performed by the turbine.

The net gain to the turbine is a function of the difference between (1) and (2). The energy gained, as in-put to the turbine in (1) is $\eta\ c_p\ \Delta\ T_1$, where $\Delta\ T_1$ is the rise in the temperature of the compressed air in the exchanger and η is the turbine efficiency. The energy lost, as output of the turbine in (2) is $c_p\ \Delta\ T_2$, where $\Delta\ T_2$ is the temperature drop due to a pressure drop $\Delta\ P$ which takes place in the exchanger instead of in the

turbine. The relation between Δi_2 and ΔP can be deduced as follows :—

Assuming $PV = RT$ . . . . . (1)

$$\therefore \quad PdV + VdP = RdT \quad . \quad . \quad . \quad . \quad (2)$$

Since the pressure drop in a well-designed exchanger is usually very small, the change in specific volume will be negligible and dV is taken as equal to zero. Therefore, equation (2) becomes :—

$$VdP = R\,dT$$

or

$$dT = \frac{dP}{R\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The net gain in an exchanger is thus equal to

$$G = c_p\,dT_1 \times \eta - c_p\,dT_2$$

$$G = c_p\left(dT_1 \times \eta - \frac{dP}{R\rho}\right) \quad . \quad . \quad . \quad . \quad (4)$$

Where dP is the total pressure drop in both cold air and hot exhaust gas passages. η is also a function of dT_1 and dP ; but if we try to put η in terms of dT_1 and dP, the equation (4) will become very complicated. It will be, therefore, assumed that η is constant. Such an approximation is, of course, basically in-correct but it is a simple mathematical study of the problem. It will be also assumed that both the hot and the cold fluids flowing in the exchanger are ideal air ; this will result in the following :—

1. Viscosity, specific heat and conductivity do not vary with the temperature.

2. The mass of the heated cold fluid is equal to the mass of the hot exhaust gases. In other words, the amount of fuel added per lb. of air is neglected ; this is justified by the fact that the fuel-air ratio in gas turbines is very small.

Equation (4) can be put thus:

$$\frac{G R\rho}{c_p} = R\rho\,dT_1 \times \eta - dP = G' \quad . \quad . \quad (5)$$

If the gain is to be calculated for one mean temperature of the exchanger, then ρ could be assumed constant at that mean temperature, and G′ will be maximum when $R\ \rho\ dT_1 \times \eta - dP$ is maximum.

It will be appreciated that it is very difficult to tackle all types and arrangements of heat exchangers in a single article, and this study will therefore be confined to the crossflow one-pass type of exchanger, in which the tubes are in an "in-line" layout with equal cross and longitudinal pitches of the value $S_q = S_l = 1{\cdot}5\ d_2$ where d_2 is the outer diameter of tubes (Fig. 1). Cold air is assumed to flow inside the tubes while the hot gases flow across them on the outside.

The heat transfer and other formulae used in this treatment as well as various coefficients given below are taken from Prof. C. H. Lander's Paper (1).

$$N_u = 0{\cdot}023\ R_e^{0{\cdot}8}\ P_r^{0{\cdot}4} \quad \text{For cold gases flowing inside tubes} \qquad (6)$$

$$N_u = 0{\cdot}29\ C_H R_e^{0{\cdot}61} \quad \text{For hot gases flowing across tubes} \qquad (7)$$

where N_u is the Nusselt number $\left(\frac{h\,t}{k}\right)$, R_e is the Reynolds number $\left(\frac{v\,d\,\rho}{\mu}\right)$, P_r is the Prandtl number $\left(\frac{c_p\,\mu}{k}\right)$ and C_H is a coefficient depending upon R_e as well as on the arrangement of the tubes. In the following, an average constant value of C_H was assumed to cover the whole range from laminar flow up to $R_e = 20000$ in the turbulent flow. C_H was assumed to be equal to 0·955. The factor h is the conductance due to forced convection, t the mean temperature of the fluid and k is the conductivity ; v is the velocity of the fluid, d the diameter, ρ is the density, μ is its viscosity and c_p is the average specific heat at constant pressure of the flowing fluid.

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(1) "A Review of Recent Progress in heat transfer", by C. H. Lander. I.M.E. Vol. 148. p. 81. 1942.

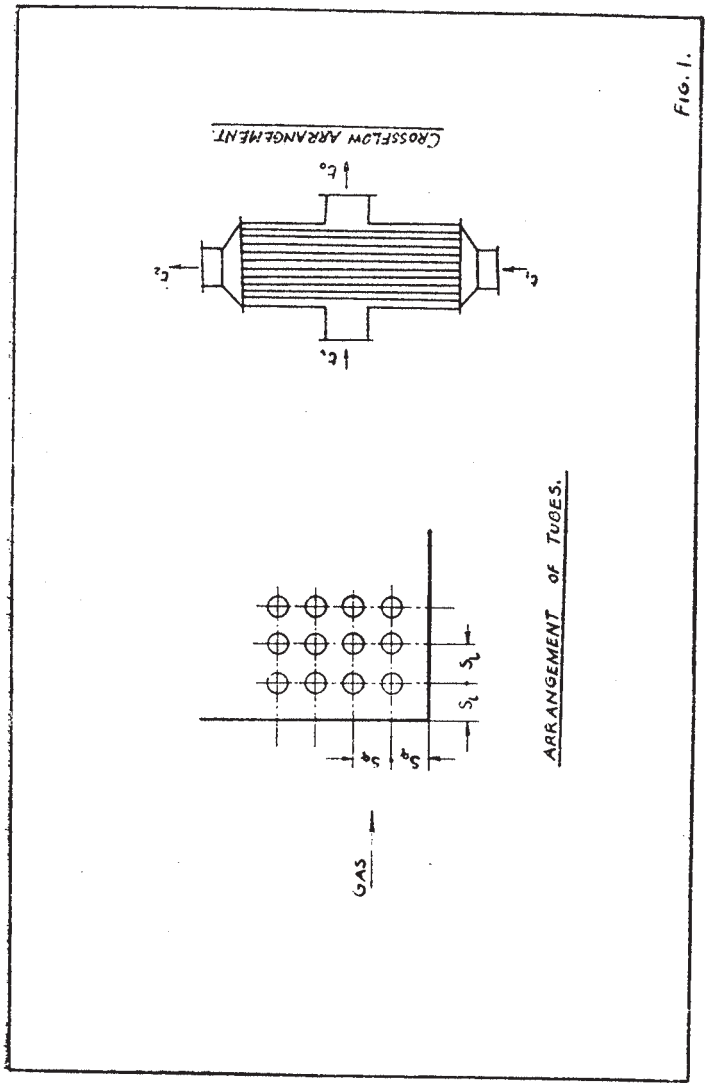
GAS
S_q
S_q
S_l
S_l
ARRANGEMENT OF TUBES.
t_1
t_2
t_0
CROSSFLOW ARRANGEMENT.
FIG. 1.

Fig. 1

$$\Delta \text{ p inside tubes} = \frac{2 \text{ f } \rho \text{ v}^2}{\text{g d}} \quad . \quad . \quad . \quad . \quad (8)$$

$$\Delta \text{ p across tubes} = 10^{-3} \text{ c}_\text{f} \rho \text{ v}^2 \text{ N}_\text{R} \quad . \quad . \quad (9)$$

where f is the known friction coefficient, N_R is the number of rows and C_f is a coefficient depending upon the tube arrangement and is taken as equal to 0·77.

The relation between the friction and heat transfer is taken as : $N_u = \frac{f}{2} (R_e P_r)$ . . . . . (10)

The transmittance or the overall heat transfer coefficient is given by the formulae :

$$U = \frac{\pi}{\frac{1}{h D_1} + \frac{D_2 - D_1}{D_2 + D_1} \times \frac{\phi}{K} + \frac{1}{h' D_2}} \quad . \quad . \quad (11)$$

where h is the conductance due to convection in the cold side.

h′ is the conductance due to convection in the hot side.

K is the conductivity of metal of tubes as is taken in this work as equal to 27 B.t.u/hr. °F. ft. (1).

$$\text{and } \phi = \frac{\left(\frac{D_2}{D_1} + 1\right)}{2\left(\frac{D_2}{D_1} - 1\right)} \times \log_e \frac{D_2}{D_1} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

D_2 and D_1 are the outer and inner diameters of the tubes in feet and an approximate value of the ratio $\frac{D_2}{D_1}$ was found, for the range of tubes used in heat exchanging apparatus, to be 1·159.

Then $\frac{D_2 - D_1}{D_2 + D_1} \times \frac{\phi}{K}$ in equation (11) will be equal to 0·00547

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(1) " Industrial heat transfer ", by Schack, 1934.

Therefore we have:

$$U = \frac{\pi}{\frac{1}{h\,D_1} + \frac{1}{h'\,D_2} + 0{\cdot}00547} \quad . \quad . \quad . \quad . \quad (13)$$

From equation (13); and using the above given formulae and factors, the following equations are deduced.

$$\frac{L\,N}{M_a\,C_{pa}}\left(\frac{\Delta t\,m}{t_2 - t_1}\right) - 0{\cdot}001742 = \frac{1{\cdot}742}{K_a}\left(\frac{N\mu_a d_1}{M_a}\right)^{0{\cdot}8}$$

$$+ \frac{0{\cdot}759}{K_g}\left(\frac{L\mu_g\,(N_B + 3)}{M_g}\right)^{0{\cdot}61} . \quad . \quad . \quad (14)$$

where L is the length of tubes in feet, N is the total number of tubes, M_a and M_g are the mass of air gas flowing per unit time, $\Delta\ t_m$ is the mean temperature difference (1), d_1 is the inner diameter of tube in inches and N_B is the number of banks.

$$\text{and } \Delta\ p_a = 10^{-7} \times 4{\cdot}37 \times \frac{(\mu_a)^{0{\cdot}2}}{\rho_a} \times \frac{M_a}{d_1^{4{\cdot}8}} \times \frac{L}{N}\ \text{lb/in}^2 \quad (15)$$

$$\Delta\ p_g = 10^{-7} \times 0{\cdot}00926 \times \frac{M^2{}_g}{L^2\,\rho_g}\left(\frac{1}{d_1\,(N_B + 3}\right)^2 N_R\ \text{lb/in}^2 . \quad (16)$$

where $\triangle\ p_a$ and $\triangle\ p_g$ are the pressure drop on the air side and that on the gas side successively.

Assuming ideal air to be the working medium and that $M_a = M_g = M$ (as mentioned before), equation (14) can be reduced to:—

$$\frac{\Delta\ t_m}{t_2 - t_1} = \frac{1{\cdot}742}{K} \times \frac{M\,C_p}{LN}\left(\frac{\mu\ d_1\ N}{M}\right)^{0{\cdot}8} + \frac{0{\cdot}759}{K} \times$$

$$\frac{M\,C_p}{LN}\left(\frac{L\ \mu\ (N_B + 3)}{M}\right)^{0{\cdot}61} + 0{\cdot}001742\ \frac{M\,C_p}{LN}$$

For simplicity, assuming:

$N_B + 3 = N_B$ and substituting $N_B\ N_R = N$ and putting

(1) Schack proposed $\triangle\ t_m = \frac{(t_i - t_1) + (t_o - t_2)}{2}$

$$K_3 = \frac{1{\cdot}742}{K} C_p \mu^{0{\cdot}8}, \quad K_4 = \frac{0{\cdot}759}{K} C_p \mu^{0{\cdot}61} \text{ and}$$

$$K_5 = 0{\cdot}001742\, C_p$$

where K_3, K_4 and K_5 are all constants for the assumed conditions, we get :—

$$\frac{R\rho_a(t_2-t_1)}{144} = \frac{R\,\rho_a\,\Delta\,t_m}{144\left(K_3\frac{M^{0{\cdot}2}d_1^{\,0{\cdot}8}}{L\,N_B^{0{\cdot}2}N_R^{0{\cdot}2}}+K_4\frac{M^{0{\cdot}39}}{L^{0{\cdot}39}N_B^{0{\cdot}39}N_R}+K_5\frac{M}{L\,N_BN_R}\right)} \quad (17)$$

In the same way equations (15) and (16) can be expressed as :

$$\Delta\, p_a = K_1 \frac{M^{1{\cdot}8}L}{N_B^{1{\cdot}8}N_R^{1{\cdot}8}d_1^{4{\cdot}8}} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

$$\text{where } K_1 = 10^{-7} \times 4{\cdot}37 \times \frac{\mu^{0{\cdot}2}}{\rho_a}$$

$$\text{and } \Delta\, p_g = K_2 \frac{M^2\,N_R}{L^2\,d_1^{\,2}\,N_B^{\,2}} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$\text{where } K_2 = \frac{10^{-7} \times 0{\cdot}00926}{\rho_g}$$

Putting the denominator in equation (17) equal to "C" and $K_6 = \eta \frac{R\,\rho_a\,\Delta\,t_m}{144}$, we get—

$$C' = \frac{K_6}{C} - K_1 \frac{M^{1{\cdot}8}L}{N_B^{1{\cdot}8}N_R^{1{\cdot}8}d_1^{4{\cdot}8}} - K_2 \frac{M^2\,N_R}{L^2\,d_1^2\,N_B^2} \quad . \quad . \quad (20)$$

In equation (20) K_1, K_2 and K_6 can be assumed constant for one given air and gas temperature.

For a certain constant mass of fluid passing the exchanger to gain a certain temperature rise, M will be constant as well as $\Delta\, t_m$.

$$\therefore \quad G' = f\,(F_R\,,\,N_B\,,\,L\,,\,d_1)$$

G' is a function of four totally independant variables and for maximum values of G', the following relation should be fulfilled.

$$\frac{\partial G'}{\partial N_R} = \frac{\partial G'}{\partial N_B} = \frac{\partial G'}{\partial L} = \frac{\partial G'}{\partial d_1} = 0$$

Differentiating G' partially with respect to each independant variable and equating to zero as well as putting :

$$x = \frac{M}{\mu\, L\, N_B\, N_R} \text{ a dimensionless number.}$$

$$y = \frac{L}{d_1} \qquad \text{a dimensionless number.}$$

and $\quad K_7 = K_3\,\mu^{0\cdot 2}\,, \quad K_8 = K_4\,\mu^{0\cdot 39}\,, \quad K_9 = K_5\,\mu$

$$K_{10} = K_1\,\mu^{1\cdot 8}\,, \quad K_{11} = K_2\,\mu^2.$$

we get the four following equations :—

$$0\cdot 2\, K_6\, K_7 \frac{x^{0\cdot 2}\, d_1^2}{y^{0\cdot 8}} + K_6\, K_8 \frac{x^{0\cdot 39}\, d_1^2}{N_R^{0\cdot 61}} + K_6\, K_9\, d_1^2$$

$$+ 1\cdot 8\, K_{10} C^2\, x^{1\cdot 8}\, y^{2\cdot 8} - K_{11}\, C^2\, x^2\, N_R^3 = 0 \qquad (21)$$

$$0\cdot 2\, K_6\, K_7 \frac{x^{6\cdot 2}\, d_1^2}{y^{0\cdot 8}} + 0\cdot 39\, K_6 K_8 \frac{x^{0\cdot 39}\, d_1^2}{N_R^{0\cdot 61}} + K_6\, K_9\, x\, d_1^2$$

$$+ 1\cdot 8\, K_{10} C^2\, x^{1\cdot 8}\, y^{2\cdot 8} + 2\, K_{11}\, C^2\, x^2\, N_R^3 = 0 \qquad (22)$$

$$K_6\, K_7 \frac{x^{0\cdot 2}\, d_1^2}{y^{0\cdot 8}} + 0\cdot 39\, K_6 K_8 \frac{x^{0\cdot 39}\, d_1^2}{N_R^{0\cdot 61}} + K_6\, K_9\, x\, d_1^2$$

$$- K_{10} C^2\, x^{1\cdot 8}\, y^{2\cdot 8} + 2\, K_{11}\, C^2\, x^2\, N_R^3 = 0 \qquad (23)$$

and $\quad 0\cdot 8\, K_6\, K_7 \dfrac{x^{0\cdot 2}\, d_1^2}{y^{0\cdot 8}} - 4\cdot 8\, K_{10}\, C^2\, x^{1\cdot 8}\, y^{2\cdot 8}$

$$-2\,K_{11}\,C^2\,x^2\,N_R^3 = 0 \qquad . \quad . \quad . \qquad (24)$$

From those four equations and assuming that

$$x^{0\cdot19} \simeq x^{0\cdot2} \quad \text{and} \quad N_R^{3\cdot61} \simeq N_R^{3\cdot6} \quad \text{we get:—}$$

$$y = \psi\,N_R \quad \text{or} \quad \frac{L}{d_1} = \psi\,N_R \qquad . \quad . \qquad (25)$$

where $$\psi = \left(\frac{1\cdot405\,K_{11}\,K_7}{K_{10}\,K_8}\right)^{\frac{1}{3\cdot6}}$$

Substituting for the values of constants K_{11}, K_7, K_8 and K_{10} as well as substituting for $P_r = \frac{C_p\,\mu}{K} = 0\cdot74$ (1)

$$\therefore \qquad \psi = 0\cdot2529\left(\frac{\rho_a}{\rho_g}\right)^{0\cdot278} \qquad . \quad . \quad . \qquad (26)$$

Another relation is also obtained other than the equation (25)

$$\frac{K_7}{0\cdot61\;K_8} + \frac{0\cdot39}{0\cdot61}\,\psi^{0\cdot8}\left(\frac{d_1}{L}\right)^{0\cdot2} Z_{N_B}^{\;0\cdot2}$$
$$+ \frac{K_9}{0\cdot61\;K_8}\,\psi^{0\cdot8}\left(\frac{d_1}{L}\right)^{0\cdot8} Z_{N_B}^{\;0\cdot8} = \frac{K_{10}}{K_{11}}\,\psi^{3\cdot6} \qquad (27)$$

where $Z_{N_B} = \frac{M}{\mu\,d_1\,N_B}$ . . a dimensionless number.

Equation (27) shows that for optimum conditions, there is a relation between the dimensionless numbers $\frac{L}{d_1}$ and Z_{N_B}.

From equation (26), it is clear that ψ is a function of pressure and temperature of both fluids flowing through the exchanger.

Within the range met with ordinarily in combustion gas turbines, that is to say, assuming that the pressure ratio used lies between 3 and 4 and that the maximum temperature is 1600°F abs., the variation in ψ with the mean temperature difference is given in (Fig. 2) and average value can be assumed to be 0·355.

(1) " Heat Transmission ", Mac. Adams, 1942.

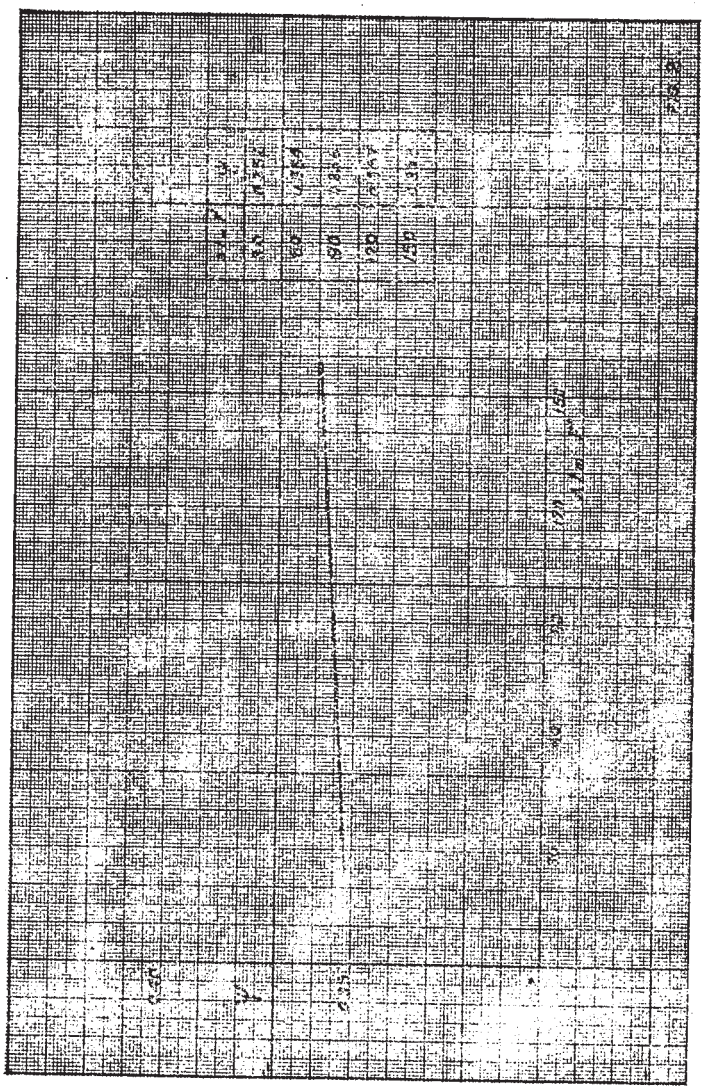

Fig. 2

This factor can be considered as an important guide in the design of exchangers, if optimum conditions are aimed at. However, this guide is not sufficient since we have another condition, represented by the equation (27), which has to be fulfilled. Equation (27), as it stands, does not give any practical guide or solution, but although of no direct application in its present form, is valuable since it directs the attention to the fact that there must be a relation between $\frac{L}{d_1}$ and Z_{N_B}.

In support of these results and in order to verify the two general rules given, a complete solution was worked out, by the trial and error method in the case of a heat exchanger for the following gas turbine unit of an average output in relation to units so far constructed.

Rated Power = 4000 K.W. Pressure ratio = 3

Suction Temperature = 55°F. and Max. abs. temp. — 1600°F.

Calculations were carried out with the number of tubes ranging from 4000 to 24000, lengths ranging from 8′ to 28′, and diameters ranging from $\frac{1}{8}''$ to 3″. Alternatives of designs were also carried out for 4 values of Δt_m, namely, 60°F. 90°F. 120°F. and 150°F. with corresponding values of $\frac{\Delta t_m}{t_2 - t_1}$ equal to 1·152, 0·2384, 0·337 rnd 0·434 respectively.

Fig. 3 shows the relation between ψ and the gain G', giving an idea about the value of ψ at which the maximum gain is attained for different lengths, at different values of Δt_m.

It is clear that ψ ranges obout a constant average value, but yet by itself it does not give a single established design for optimum conditions. Figure (4) shows the plotting of d_1 against ψ so as to get the value of d_1 at $\psi = 0{\cdot}355$, fulfilling the condition of optimum gain, in order to calculate the corresponding values of Z_{N_B}. Fig. 5 shows the curves connecting $\frac{L}{d_1}$ against

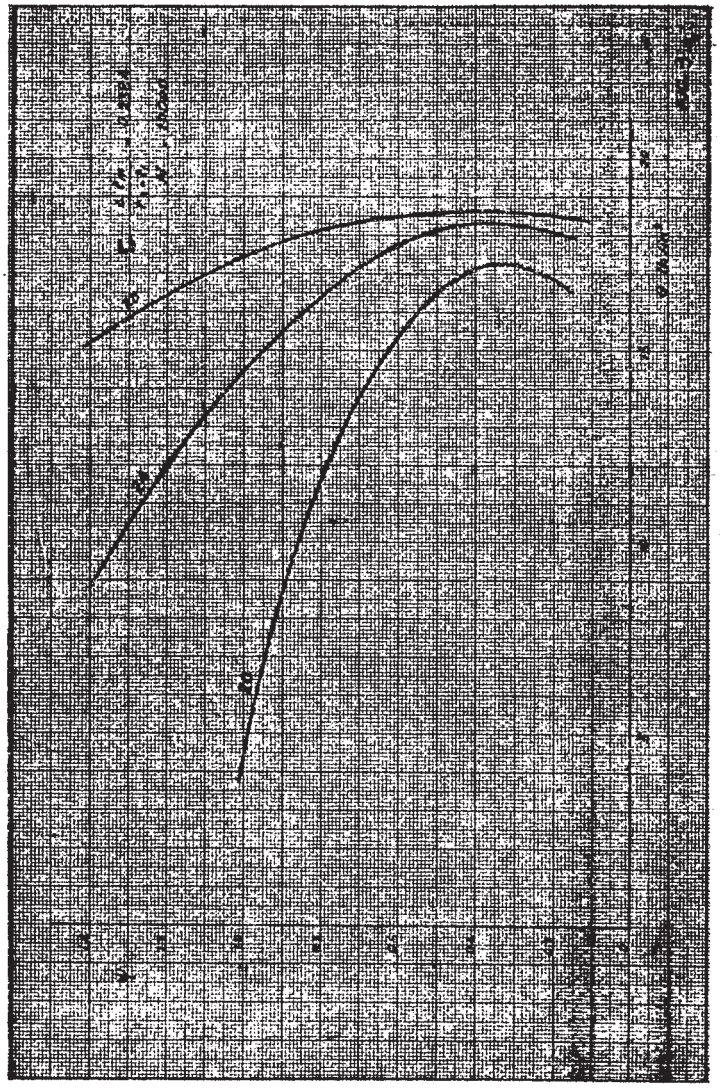

Fig. 3

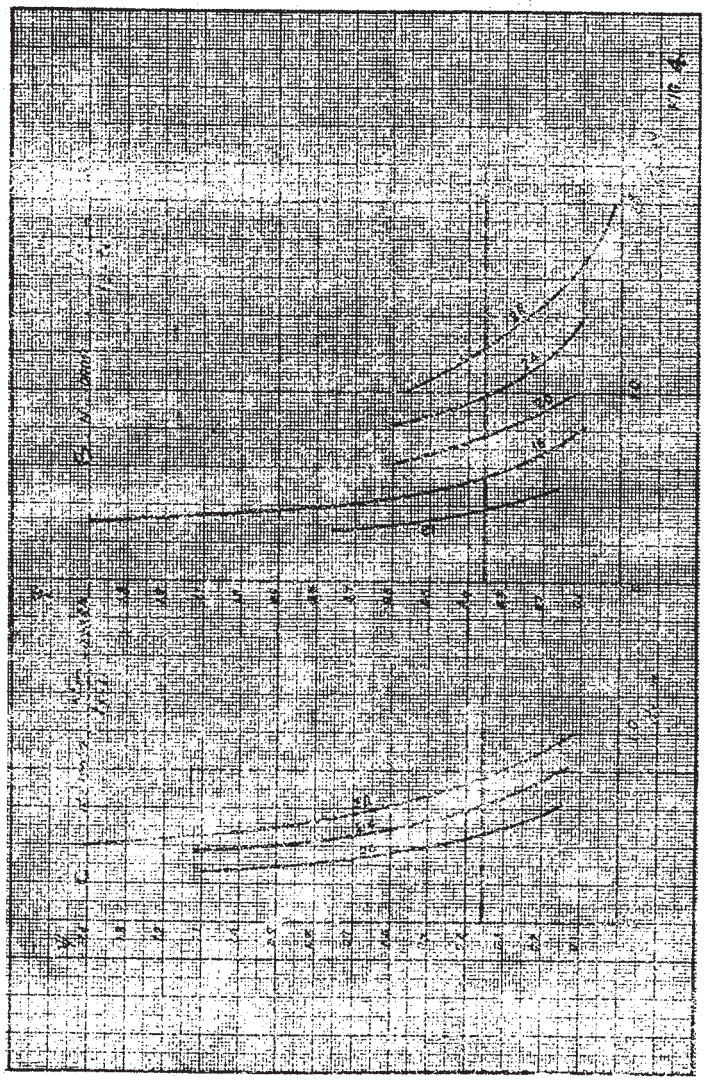

Fig. 4

Z_{N_B}, where $\psi = 0{\cdot}355$. The four curves refer to the pre-mentioned four values of Δt_m.

Again for each value of Δt_m, a group of intersecting curves, representing the relation between the heating surface H and the net gain G′, is obtained using varying lengths ; the inspection of these curves shows that the optimum length for each case lies very nearly on the envelope of all those groups. This envelope represented by the curve "E" in Fig. 6 is extended to the origin by a line of dashes, since no gain will be obtained with zero heating surface. For practical purposes, the region RR′ is recommended, since the point R′ shows the limit of turbulent flow, beyond which the flow will be viscous and a very large heating surface would therefore be essential ; however, the gain obtained beyond this point will be nearly negligible and does not justify the relative increase in heating surface.

On the same Fig. 6, the relation between $\frac{L}{d_1}$ and the heating surface H is represented in four groups of curves; each of them refers to one value of $\frac{\triangle t_m}{t_2 - t_1}$. The four points of tangency of the envelope "E" were marked on the lower groups, each on its corresponding value of $\frac{\triangle t_m}{t_2 - t_1}$ and length, giving the curve "F" which in turn represents the relation between $\frac{L}{d_1}$ and the heating surface H for optimum conditions as given by the envelope.

Fig. 7 shows the curve connecting the value of $\frac{\triangle t_m}{t_2 - t_1}$ with the heating surface H. Again the values of $\frac{L}{d_1}$ marked, for optimum conditions, are transferred to Fig. 5 so as to define the curve "F_1", which connects the values of Z_{N_B} to $\frac{L}{d_1}$ for optimum conditions as given by the envelope "E".

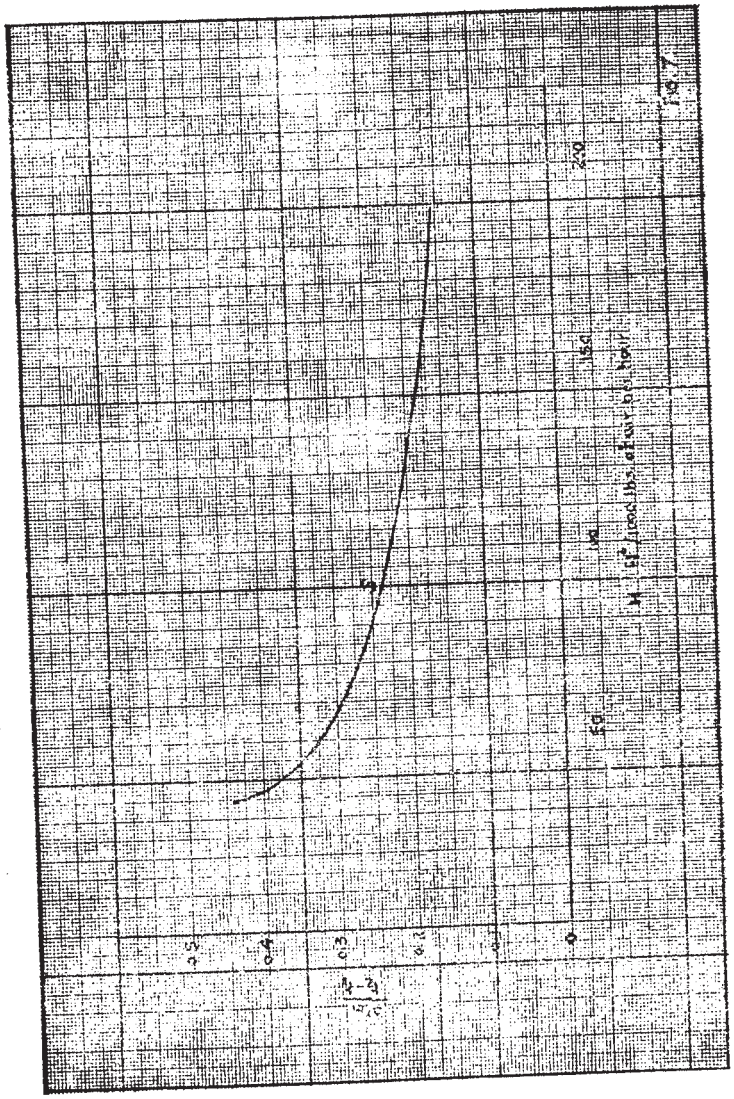

Fig 7

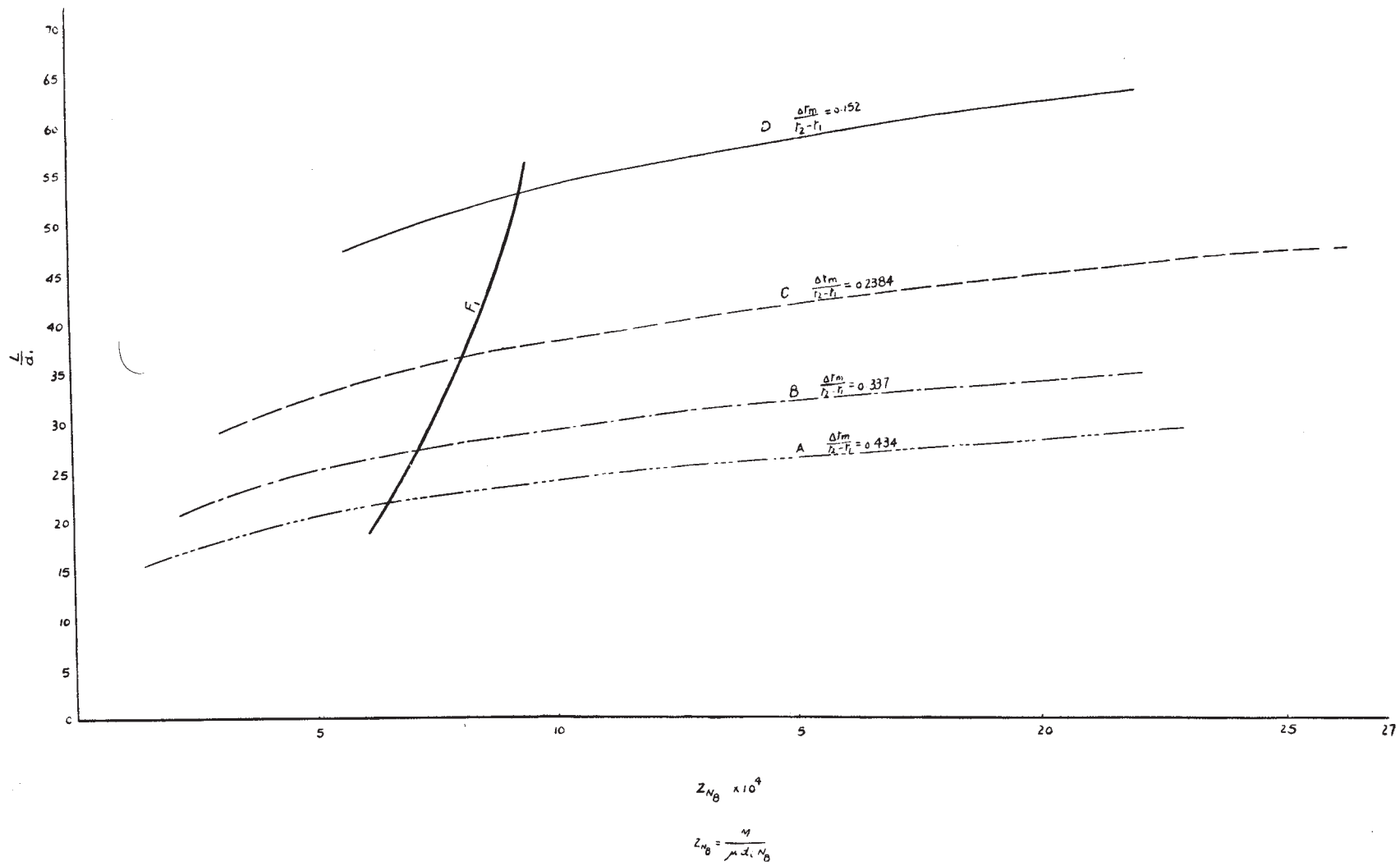
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0
L/d
D
$\frac{\Delta r_m}{r_2 - r_1} = 0.152$
C
$\frac{\Delta r_m}{r_2 - r_1} = 0.2384$
B
$\frac{\Delta r_m}{r_2 - r_1} = 0.337$
A
$\frac{\Delta r_m}{r_2 - r_1} = 0.434$
F_1
5
10
5
20
25
27
$Z_{N_B} \times 10^4$
$Z_{N_B} = \frac{M}{\mu d_i N_B}$

FIG. 5.

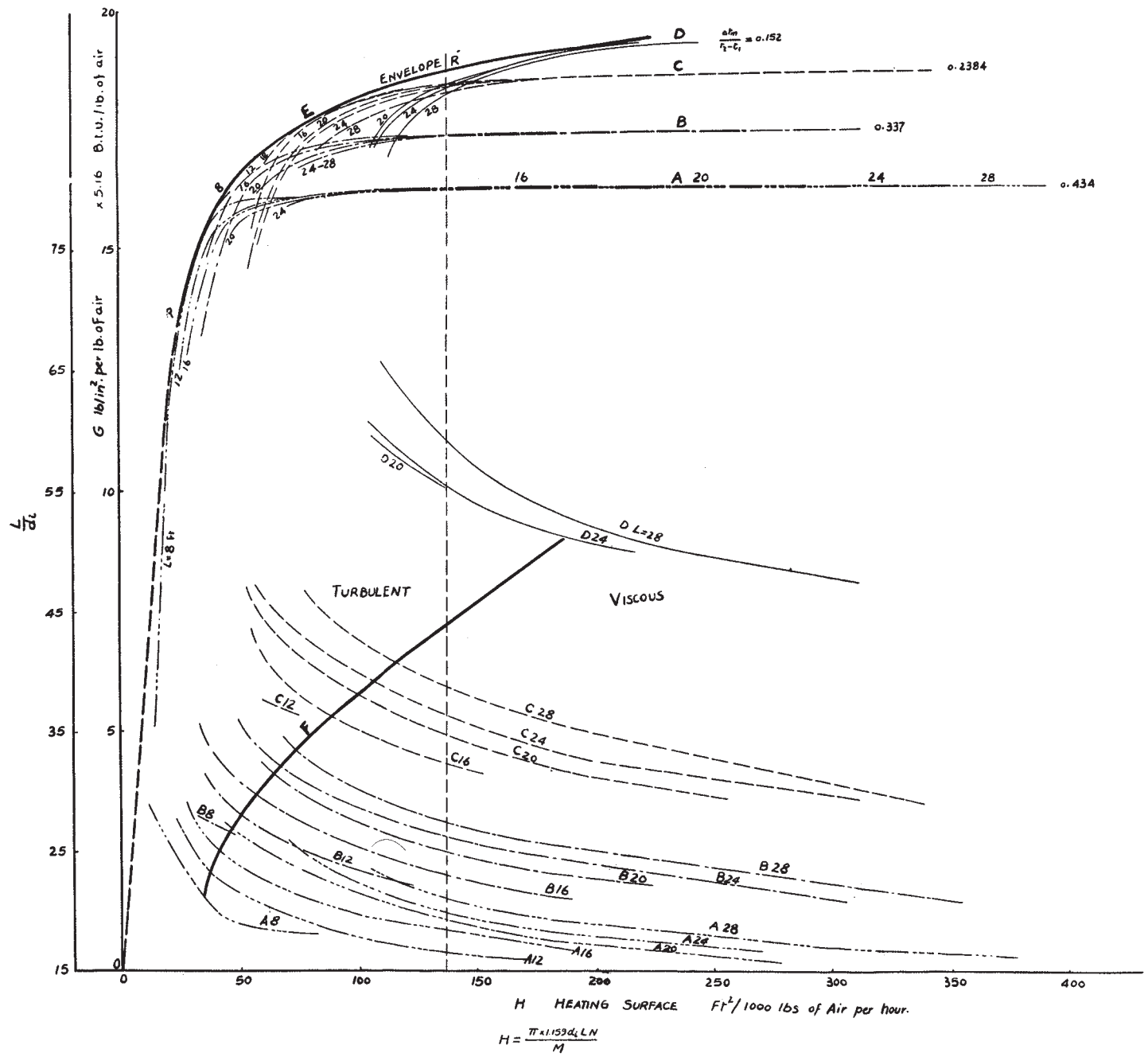
ENVELOPE R
TURBULENT
VISCOUS
G lb/in² per lb. of air
x 5.16 B.t.u./lb. of air
H HEATING SURFACE Ft²/1000 lbs of Air per hour.
H = π x 1.159 d_i L N / M
0.2384
0.337
0.434
D L=28
D24
D20
C28
C24
C20
C16
C12
B28
B24
B20
B16
B12
B8
A28
A24
A20
A16
A12
A8

FIG. 6.

Fig. 6

From the above, it is obvious that the result of this work is three curves F, F_1 and S as well as a relation $\frac{L}{d_1} = \psi N_R$. Those guides are very useful, and the procedure of design can be summarized and demonstrated as follows :—

1. The working point on the curve " E " (Fig. 6) should be determined. The choice of this point depends mainly upon the cost of fuel, running hours and cost of material and labour. In other words, the designer has to compromise between the saving in fuel as given in the gain curve " E ", and the initial cost and the depreciation of the plant as given by the heating surface H.

This choice differs according to locality.

2. Having decided upon the working point, the value $H = \frac{\pi d_2 LN}{M}$ is known or $L\, d_1\, N = C_1$ . . . (28)

3. With the value of H decided upon, $\frac{L}{d_1}$ is obtained directly from curve F (Fig. 6)

$$\frac{L}{d_1} = C_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

4. Since $\frac{L}{d_1} = \psi N_R$

$$\therefore \quad N_R = \frac{C_2}{\psi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

5. Multiplying (28) by (29), we get :—

$$L^2 N = C_1 C_2$$

But $\quad N = N_B\, N_R$ and $N_R = \frac{C_2}{\psi}$

$$L^2 N_B = \frac{C_1}{\psi} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

6. From Fig. 7, knowing H, $\frac{\Delta t_m}{t_2 - t_1}$ can be easily determined. Knowing t_1, the temperature of hot gases entering the exchanger

and $\Delta t_m = t_1 - t_2$

$$\therefore \quad \frac{\Delta t_m}{t_2 - t_1} = \frac{t_1 - t_2}{t_2 - t_1} = C_3 \quad . \quad . \quad . \quad . \quad (32)$$

From equation (32), the value of t_2 can be found as t_1 is already known and thus the average value of μ can be calculated.

7. Figure 5, will give the value of Z_{N_B} for the value of $\frac{L}{d_1}$ predetermined.

$$\therefore \quad Z_{N_B} = \frac{M}{\mu\, d_1\, N_B} = C_4 \quad . \quad . \quad . \quad . \quad (33)$$

$$\text{or} \quad d_1\, N_B = C_5 \quad . \quad . \quad . \quad . \quad . \quad (34)$$

8. Dividing (31) by (34) we get:—

$$\frac{L^2\, N_B}{d_1\, N_B} = \frac{C_1}{\psi\, C_5}$$

$$\text{or} \quad \frac{L}{d_1} \times L = \frac{C_1}{\psi\, C_5} \qquad \therefore \; L = \frac{C_1}{\psi\, C_2\, C_5}$$

9. With L known, d_1 can be fixed from equation (29) and consequently N_B can be determined from equation (34).

Thus L, d_1, N_B and N_R are fixed for the heating surface chosen to give optimum conditions.

To show the effect of locality as well as the validity of the curves and relations obtained in this paper, it may be of some value to mention a recommendation by Tom Sawyer (1) to use a value of heating surface of 2–3 ft² per horse power. He stated that this value of heating surface will be within the scope of a fairly simple heat exchanger equipment in order to make it possible to obtain 75% regeneration efficiency. Applying these figures to the unit under consideration a value of 28–42 ft² per 1000 lb. of air per hour is needed.

From Fig. 7, for H = 42 ft², $\frac{\Delta t_m}{t_2 - t_1} = 0{\cdot}36$

(1) "The modern gas turbine", by Tom Sawyer 1945.

Accordingly it follows that $t_2 - t_1 = 364°F$.

The regeneration η_r can be assumed approximately

$$\eta_r = \frac{t_2 - t_1}{t_i - t_1}$$

$$\therefore \eta_r = \frac{364 \times 100}{494} = 73.6 \%$$

which agrees nearly with that proposed by Tom Sawyer. From Fig. 6, the net gain $G' = 16 \times 5.16 = 82.5$ B.t.U./lb. of air.

The heat gained by air $= 0.24 \times 364 = 87.5$ B.t.U./lb. of air.

The loss of energy due to pressure drop in heat exchanger $= 87.5 - 82.5 = 5.$ B.t.U./lb. of air

The new η of the turbine $= \frac{35.15 - 5}{212.5 - 87.5} = 0.241$

Fitting the plant with a heat exchanger, as suggested by Tom Sawyer, raised the efficiency of the plant from 16.5 to 24.1%.

This estimation, of course, is based on local considerations of cost.

Again, referring to a paper written by H. Pfenninger (1), the minimum charges are registered with a thermal efficiency of 26-27 %, which again necessitates a specific heating surface of about 1 0 sq. metre per K.W.

Applying this to our work, a value of nearly 110.5 ft^2/1000 lb. of air per hour giving a net gain of $G' = 18.35 \times 5.16 = 94.6$ B.t.U./lb. Accordingly $\frac{\Delta tm}{t_2 - t_1} = 0.2125$

and $t_2 - t_1 = 406$ °F.

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(1) " Gas Turbine with waste heat utilisation by air preheating ". The Engineer's Digest, 1943 (From Die Warme Vol. 66, No. 17, August 1943).

Heat energy gained by air $= 0{\cdot}24 \times 406 = 97{\cdot}6$ B.t.U./lb.
Loss in energy due to pressure drop $= 97.6 - 94.6 = 3{\cdot}0$ B.t.U./lb.

The new efficiency of turbine plant $= \frac{35{\cdot}15 - 3}{212{\cdot}5 - 97{\cdot}6} = \frac{32{\cdot}15}{114{\cdot}9} = 0{\cdot}28.$

The new efficiency obtained, 28 % also agrees nearly with the value proposed by Pfenninger. This, of course, refers to a German practice.

The American practice, since cost of fuel is low, tends towards a smaller heating surface, while in Germany where saving of fuel is very important larger heating surfaces are recommended.

Traupel (1) in his paper entitled "Similarity in heat exchangers", drew the attention to the importance of the dimensionless ratio F which is the ratio of the area of the cooler fluid to that of the hotter fluid. Further he mentions that F would vary with the other factors involved in the design of heat exchangers as follows :—

$$F = \frac{N_R S^2}{4 n \beta \frac{L}{D_2} \left(\frac{s_q - D_2}{D_2} \right)} \quad . \quad . \quad . \quad (35)$$

where n is the number of passes, S is the ratio of internal diameter to outer diameter, when the hotter fluid flows outside the pipes, and S_q is the value of the pitching of tubes across the flow with respect to the outer diameter of the tube. β is a factor depending upon the arrangement. $\beta = 1$ for in line arrangements.

Equation (35) can be transferred to the following :

$$\frac{L}{D_2} \quad \frac{D_2 S^2}{4 F n \beta (s_q - D_i)} N_R \quad . \quad . \quad . \quad (36)$$

This equation can be assumed as analogous to that obtained before in equation (25) with the difference that D_2, in the term $\frac{L}{D_2}$, is the internal diameter and expressed in a more practical

(1) "Similarity in heat exchangers", by W. Traupel, Schweizerarchiv, 1943.

form from the point of view of units, namely, $\frac{L}{d_1}$ where L is in feet, d_1 in inches. This will result in a change in equation (36) which will become:

$$\frac{L}{d_1} = \frac{D_2\ S}{48\ F\ n\ \beta\ (s_q - D_2)}\ N_R \quad . \quad . \quad . \qquad (37)$$

For optimum conditions, and for the same assumptions preassumed, it follows that:

$$\psi = \frac{D_2\ S}{48\ F\ n\ \beta\ (s_q - D_2)} \quad . \quad . \quad . \quad . \qquad (38)$$

For the problem considered before, $n = 1$, $\beta = 1$, $sq = 1{\cdot}5\ D_2$ and S is taken approximately as $1/1{\cdot}159$.

Substituting these values in equation (38), we get:—

$$\psi = \frac{0{\cdot}1129}{F}$$

But from equation (26) we have:—

$$\psi = 0{\cdot}2529 \left(\frac{\rho_a}{\rho_g}\right)^{0{\cdot}278}$$

$$\therefore\ F = 0{\cdot}446 \left(\frac{\rho_g}{\rho_a}\right)^{0{\cdot}278} \quad . \quad . \quad . \qquad (39)$$

This means that, for optimum conditions, the ratio of flow of the cooler fluid to that of the hotter one will vary inversely as the densities to the power 0·278.

In the previous work, the average value of ψ was assumed as 0·355 for the conditions concerned, giving an average value of $F = 0{\cdot}318$.

As a confirmation of this value, some alternatives of the designs, previously calculated (1) to give optimum conditions, are

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(1) "Theoretical investigation of the design of heat exchangers for combustion gas turbines", by Ahmed Ezzat. Thesis for M.Sc. 1947.

tabulated, in the following table, as well as the relative areas of flow and the values of F.

N	L ft.	d_1 ins.	a' ft^2	N_B	a ft^2	F
4000	28	0·85	15·8	42·5	51·8	0·306
6000	24	0·82	22·0	71·6	71·6	0·308
8000	20	0·71	22·0	99·2	70·5	0·311
10000	16	0·57	17·8	124·2	56·1	0·316
2000	12	0·42	11·59	146·6	36·3	0·318
14000	8	0·355	9·63	217	30·1	0·319
16000	12	0·47	19·25	219	60·7	0·318
20000	12	0·52	29·5	304	92·7	0·318
24000	8	0·3	11·8	315·5	36·8	0·320

From this table it is clear that an average value of 0·315, for F, can be assumed for optimum condition for the problem concerned.

It is important to understand the physical meaning of the factor ψ, deduced before, as representing the ratio of the area of flow of the hotter fluid to that of the cooler one, that is to say the reciprocal of the factor F.

Again, it is clear that ψ is put in a more practical form for design, as it is expressed in the terms L, d_1 and N_R. However, the value of F may be of use as a guide for those who would prefer to carry out the calculations from the beginning. Furthermore, Traupel mentioned in his paper that Sulzer has widely developed this theory, giving the best value of "F" as well as taking into consideration the variable conditions along various types of exchangers. The author believes that for the type of exchanger in question, the value of "F" is probably consistent with the value pre-suggested.